

Financial Markets Microstructure

Lecture 1

Introduction and institutions
Chapters 0 and 1 of FPR

Egor Starkov

Københavns Universitet
Spring 2025

This slide deck:

- 1 Logistics
- 2 Why do we need financial markets?
- 3 Types of financial markets
- 4 Types of agents in financial markets
- 5 Next on...

Contact Information

Lecturer: Egor Starkov

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- Office hours: by appointment (or after lectures)

2

Schedule

2 hours of lectures once or twice a week

- **Wednesdays 13.00-15.00** (every week 6–20, except week 7 and week 16);
- **Fridays 10.00-12.00** (odd weeks 9–19).
- Same room (CSS 4.1.36) unless told otherwise
- Physical classes, no streams/recordings

3

Resources

- Get **the book** by Foucault, Pagano, and Röell.
 - We'll be using it a lot (incl. exercises).
 - I will be following edition 1 from 2013, but you can get ed.2 from 2024
 - Will switch to research articles towards the end of the course.
- Everything else (lectures, problem sets for exercise classes, etc.) is on **Absalon** – check regularly.
- I will upload the **slides** before the lectures for you to print out.
 - Those may contain omissions!
- **2020 lectures** are on youtube (not recommended).
- **Study groups** may be helpful. Email me before 17:00 Thu, Feb 05 if you are looking for a group.

4

Problem sets and Exam

- **readings and exercises** after lectures (esp. in earlier part of the course)
 - voluntary; will go through some in class
 - readings not always immediately related to theory, but always related to the course
- 2 problem sets (larger, cumulative, more open-ended?)
 - Voluntary too. Take them as an opportunity to check your progress
- **final exam**: 12-hr take-home exam.

5

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6

Why do we need markets?

- 1 **What** is a market?
- 2 **Why** do we need markets?

7

Why do we need Financial Markets?

- **Financial markets:** markets for financial assets
- **Financial assets:** move wealth across time and contingencies:
 - Financial assets provide 'contingent cash flows'
 - Stocks, bonds, derivatives...
- **Purposes** of financial markets:

- **Specifics** of financial markets:
 - Common-value setting with dispersed/private information
 - Because of info asymmetries, usually organized in a specific way and heavily regulated.

8

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- 5 Next on...

9

Types of Financial Markets

- **Primary markets:** “Allocate savings to investment”
 - Issues of new assets (typically by investment bank)
- **Secondary markets:** “Reallocate investments across savers”
 - Trade the existing assets on exchange
- This course: focus on secondary markets, such as
 - Stock and bond markets
 - Derivative markets
 - Currency (FX) markets
 - Most insights will apply to commodity markets that deal in futures (oil/gas, metals, food...)
 - (Corporate finance looks at primary markets)

10

Market Setups

Markets can be set up in different ways

- 1 Decentralized “markets”
- 2 Dealer markets
- 3 Order-driven markets
 - Continuous
 - Call
- 4 Hybrid (dealer+order) markets

11

Direct trading

- Buyers and sellers could interact **directly**.
- A useful benchmark, but not a very interesting case.

12

Dealer Markets

- A **dealer** quotes a bid and an ask price (valid up to certain number of shares)
- Bid-ask spread:
 - Narrow to fend off competitors
 - Wide to generate trading profits
- Dealer exchanges: Nasdaq

13

Order-driven markets

- Timing
 - 1 Orders are submitted
 - 2 Trades are arranged
- The markets can vary in different ways
 - Frequency of trading: how often?
 - Order precedence: price/time/public
 - Pricing rule: uniform/discriminatory price, determined from orders/taken from other exchange
 - Opening/closing: are there any special trading rules?

14

Order types

- There's a tremendous amount of different order types. We focus on two main ones:
- **Market order** specifies quantity: "buy 500 shares at best available price".
- **Limit order** specifies a quantity and a price: "will buy 500 shares at below price 40"
 - Limit orders sit in **the Limit Order Book** until they can be executed
- Trades occur either through
 - 1 a limit order 'match' (somebody posts a bid price above the lowest ask price), or
 - 2 through market orders against the LOB

15

Order-driven markets: Continuous auctions

- LOs together compose a **Limit Order Book**.
 - Any given LO can be **marketable** or not.
 - A marketable LO is expected to execute immediately given other orders in the LOB.
- Traders can choose orders best for them:
 - Patient traders are more passive: use limit orders (fill LOB)
 - Impatient traders are more aggressive: use market orders (deplete LOB)
- **Discriminatory price**: depends on how your order is matched
- Continuous LOB exchanges: NYSE (both electronic and 'outcry'), LSE, BATS, Euronext

16

Order-driven markets: Call (batch) auctions

- Orders are collected (in a 'batch') and cleared at a certain frequency
- The price is chosen so as to maximize the number of executed orders
- **Uniform price**: all orders in a batch trades at same price
- In theory, very good efficiency properties: all profitable trades are carried out
- But... slower than the continuous market
- Call auction exchanges: most of the major exchanges (e.g. Nasdaq, LSE, Euronext) operate call auctions together with other trading methods. Danish electricity markets work via call auctions afaik.

17

Exchange versus over-the-counter

Exchange trading

- Organized exchange venues (NYSE, NASDAQ, LSE, ...)
- Generally offer a lot of services
 - Liquidity and stability through *specialists/market makers*
 - Clearing and settlement
 - Transparency

Over-the-counter (OTC) trading

- Off-exchange venues
 - Use quotation systems to match dealers and traders
 - However, no guarantee of liquidity (no specialists)...
 - ...nor transparency: may not publish trade information

Dark liquidity: Liquidity in 'private exchanges' (not public)

18

Other dimensions

- Many other aspects in which markets can differ:
 - Transparency about current orders
 - Transparency about past trades
 - Fragmentation
 - Listing requirements
 - ...
- Who benefits from markets being organized in a particular way? Who loses?

19

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20

Players

- **Traders/investors:** the end- “consumers” or suppliers of the asset
- **Brokers:** the intermediaries who relay investors’ orders to the market (investment banks)
- **Dealers/market-makers:** match orders from different sides of the market

21

Types of traders

Traders can be classified:

- **Retail investors** vs **institutional investors**

- **Retail:** regular people, individuals with money. Eaton et al. (2020): retail accounts for about 20% of market activity.

- **Institutional:** pension funds, mutual funds, hedge funds, private equity funds...

- **Informed** vs **uninformed** investors

- Retail investors and pension funds usually have little “private information”.

- Hedge funds and short-selling funds have good analytics and can react fast.

- “Private information” $\neq$ “insider information”! Insider trading is banned in most jurisdictions.

22

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23

Motivating questions

- 1 How are market prices created? How are they influenced by traders?
- 2 How should traders act on their information?
- 3 Do the market rules matter? And which rules are good for whom?
- 4 How do we measure if markets are working well?

24

Course overview

- Institutional details, policy issues, applications (ch.1, 6-10, articles)
- Theories to understand and to analyze policy (ch.3-4, 6-10, articles)
- Some empirical issues (ch.2, 5, articles)

Prerequisites: Finance, micro, games, (math)

Disclaimer: We focus on 'rational' models in this course. Behavioral finance is a complementary and exciting topic.

25

For next class

- 1 Match into groups if you want / message me if you are looking for a group or group members.
- 2 Read the article about GameStop (on Absalon). Think/discuss:
 - What kind of traders are involved on both sides? Based on that fact: why has this story made the news?
 - Would now be a good time to buy GameStop stock? Why/why not?
 - Robinhood (trading app) halted trading in GameStop stocks at some point. Is there a scope for some regulatory intervention to prevent this kind of exclusion of retail traders?
- 3 Read the Nikola story. Is Hindenburg Research an insider? Is it an informed trader?
- 4 Do exercises 1-3 after Chapter 1 (pages 44-45) in the textbook

Financial Markets Microstructure

Lecture 2

Prices and Liquidity
Chapter 2 of FPR

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What did we do last week?

- 1 Introduced **financial markets** broadly speaking and motivate why we wanted to talk about it
- 2 Characterized **market formats**: order-driven markets (auctions) and dealer markets
- 3 Introduced some of the key **concepts and language**: dealer sets bid/ask price, traders submit market/limit orders

Fundamental Value

- We'd like to believe there is some "objective" / "fundamental" value of a stock – at least to some representative agent.
- The question of whether to buy or sell then often amounts to:
"Is the current asset price above or below its fundamental value?"
- To answer, need to understand what **fundamental value** is.
- **Short answer:**

4

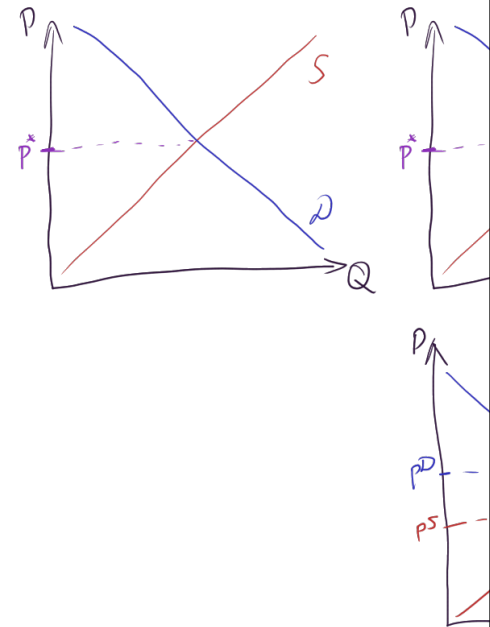
Fundamental Value 2

- Long answer: study **asset pricing**.
 - Field of finance devoted to calculating asset price relative to other assets, assuming perfect markets.
- Question of **market microstructure**: take the '**fundamental value**' as given and analyze how it **translates into prices** in realistic markets.
 - In GameStop case, divergence was due to bubble(?)
 - Another broad reason for divergence: **limited liquidity**
- Dual question: **price discovery**
"How much information about the fundamental value can be extracted from market prices?"

5

What is liquidity?

- In **perfect markets**:
 - There is one price = valuation cutoff
 - Agents who value asset above the cutoff end up with it (keep or buy)
 - Agents who value asset less end up without it (sell or do not buy)
 - This is the **efficient** allocation that we want
- In (financial and many other) **real markets**:
 - Bid/ask prices different from that ideal cutoff/fundamental value
 - (due to limited liquidity)
 - Allocation **inefficient**



7

Still, what exactly is liquidity?

- **Market liquidity** =
- Do not confuse with (related notions of):
 - 1 **Funding liquidity** =
 - 2 **Monetary liquidity** =

8

Why do we care about liquidity?

- Traders: liquidity provides a measure of trading costs, affects how costly it is to implement a given theoretical trading strategy
- Regulators:
 - 1 Efficiency is tricky to measure in financial markets: liquidity provides a proxy
 - 2 Illiquid markets also seem to be more prone to medium-run price deviations from fundamentals
 - 3 Illiquidity *may* be a sign of structural problems in the market
- Relation to **depth**: depth measures how much must be traded to move price by certain amount
 - $\approx$ sensitivity of liquidity to trade size

9

Liquidity measures

Liquidity is not a very well-defined concept, so it's not immediate how to measure it either. We will consider several measures:

- 1 **Spread measures**: quoted spread, effective spread, realized spread
- 2 **Implementation measures**: volume-weighted average price, price impact, implementation shortfall
- 3 **Non-trading measures**: trading volumes
- 4 *Missing data estimators*: Lee-Ready algorithm for trade direction, Roll's measure for quotes

11

Quoted spread

- Let a_t (b_t) be the best ask (bid) price at time t

- **Quoted spread:**

$$S_t = a_t - b_t$$

- Contemporaneous: spread facing trader at time t
- Normalize to get **normalized quoted spread**

$$s_t = \frac{S_t}{m_t},$$

where m_t is the midprice:

$$m_t = \frac{a_t + b_t}{2}.$$

- We can generalize it to consider average spread for trade size q :

$$S_t(q) = \bar{a}_t(q) - \bar{b}_t(q)$$

where $\bar{a}_t(q)$ and $\bar{b}_t(q)$ are average execution prices

13

Effective spread

- Suppose one market order is executed per period, and
 - d_t : trade direction (1: buyer-initiated, -1: seller initiated)
 - p_t : price

- **Effective (half-)spread:**

$$S_t^e = d_t(p_t - m_t),$$

$$s_t^e = \frac{S_t^e}{m_t}$$

- Backward looking: spread faced by previous trader
- Compare actual price with midquote the instant before: measures price impact and captures 'price improvements'

15

Realized spread

- Realized spread:

$$\begin{aligned} S_t^r &= d_t(p_t - m_{t+\Delta}) \\ &= d_t(p_t - m_t) - d_t(m_{t+\Delta} - m_t) \end{aligned}$$

- Spread realized by somebody who holds the asset for Δ periods
- Idea: measure the spread after prices have adjusted to new information
- As a forward-looking measure:
 - $\mathbb{E}_t S_t^r = d_t(p_t - m_t) = S_t^e$ if $\mathbb{E}_t m_{t+\Delta} = m_t$
- As a backward-looking measure:
 - Typically smaller than effective spread: why?

17

Comparing the spreads

- The **quoted spread** and the **effective spread** may be more useful to traders:
 - Quoted spread: what is the quoted trading cost now
 - Effective spread: what was the trading cost faced by the last trader
- These are (imperfect) measures of the cost of executing a market order now
- The **realized spread** is more relevant to a market maker (liquidity provider) or a researcher:
 - It measures the cost of taking a position (long or short) for an amount of time
 - Can also be interpreted as the long-run (informational) impact of trades on prices

19

Estimating direction of trade

- We often only observe quotes and realized prices: not the direction of trade
- Thus, we need to develop methods to classify trades
- Complication: trading may be 'within the quotes': harder to guess direction
- **Lee-Ready algorithm:** (Lee and Ready [1991])

$$d_t = \begin{cases} 1 & \text{if } |p_t - a_t| < |p_t - b_t| \\ & \text{or } p_t = m_t \text{ and } p_t > p_{t-1} \\ -1 & \text{if } |p_t - a_t| > |p_t - b_t| \\ & \text{or } p_t = m_t \text{ and } p_t < p_{t-1} \end{cases}$$

21

Lee-Ready precision

- Odders-White [2000]: large-scale (144 NYSE stocks over 3 months; > 400k transactions) test of Lee-Ready algorithm with NYSE data
- 85% correct
- Most mistakes with:
 - trades at the midpoint
 - small transactions
 - transactions in large-cap / frequently traded stocks

26

Quote data

- We often lack information on quotes to compute the spread
 - Can we estimate the spread knowing only trade prices?
- Roll [1984]: use transaction prices to estimate it
 - 1 Construct a simple model of trading and calculate spread
 - 2 Estimate it
 - 3 Check robustness to simplifying assumptions

28

Roll's model

Suppose the following:

- 1 All trades have the same size. $d = 1$: buy, $d = -1$: sell
- 2 Arriving orders are i.i.d. with $\mathbb{P}(d_t = 1) = \frac{1}{2}$
- 3 Midquote is random walk: $m_t = m_{t-1} + \epsilon_t$, where ϵ_t are i.i.d. shocks
- 4 Market orders are not informative: $\mathbb{E}(d_t \epsilon_t) = \mathbb{E}(d_t \epsilon_{t+1}) = 0$
- 5 Spread $S = a_t - b_t$ is constant.

Then

$$p_t = m_t + \frac{d_t S}{2}.$$

We know p_t but not m_t . How do we estimate S ?

29

Roll's model

- Roll's observed that although ϵ_t and d_t are i.i.d., $\Delta d_t = d_t - d_{t-1}$ is mean-reverting, yielding:

$$\text{Cov}(\Delta p_t, \Delta p_{t-1})$$

- Intuitively: if $\Delta d_t > 0$, this means that we go from a sale to a buy - then the next change must be opposite

30

The estimator

- We can then work out that

$$\text{Cov}(\Delta p_t, \Delta p_{t-1}) = -\frac{S^2}{4},$$

giving us the estimator

$$S_t^R = 2\sqrt{-\text{Cov}(\Delta p_t, \Delta p_{t-1})}.$$

- Recall the assumptions of the model. We (the book) can work out extensions to treat some of them
- In our example: $S_t^R = 0.01$

31

Price impact

- How much do trades affect prices? **Price impact** λ ; $1/\lambda$ captures market *depth*

$$\Delta m_t = \lambda q_t + \epsilon_t.$$

Here q_t is the **order imbalance** in period t . In our example: $\lambda = 0.15$ (q_t in 100,000EUR)

- **Hasbrouck measure** (γ): sensitivity of returns to trading volume (Hasbrouck [2007])

$$|\Delta m_t| = \gamma Vol_t + \epsilon_t.$$

Meaningful for single trades, but if t aggregates many trades, γ is hard to interpret.

In our example: $\gamma = 0.01$ (Vol_t in 100,000EUR)

- **Amihud measure** (I_t): take ratio btw return Δm_t and volume to get *illiquidity ratio*: (Amihud [2002])

$$I_t = \frac{|\Delta m_t|}{Vol_t}.$$

33

On Hasbrouck and Amihud measures

- Neither Hasbrouck, nor Amihud measures make immediate sense when applied to aggregate data – yet this is the most common application.
- Afaik, at least the Hasbrouck measure was born to deal with pre-1983 historical stock data, which only contained aggregated daily prices and volumes, and no intraday data.
- Hasbrouck [2007] shows that γ is mildly correlated with λ under some distributional assumptions
- Bottom line: do not use γ or I if you have data that lets you estimate λ directly.

35

Implementation Measures

We can also look at measures of how costly it is for traders to implement a given trade in reality, as opposed to “paper trading” (looking at trades-you-could-have-done).

Example

[Perold (1988) observed that] from 1965 to 1986, a paper portfolio based on the Value Line ranking system outperformed the market by 20% per year, and the real Value Line fund, which implemented the trades recommended in the newsletter, outperformed the market by only 2.5% per year, emphasizing that the quality of implementation is at least as important as the investment idea itself. [Anand et al., 2012]

37

Volume based measure

- **Volume-Weighted Average Price (VWAP):**

$$VWAP = \sum w_i p_i,$$

where $w_t = |q_i| / \sum_i |q_i|$ is the order weight, q_i is the size of order i

- This equals the amount of dollars traded over the number of shares traded: average price
- Trader can compare the price he got with VWAP to evaluate how good was his deal relative to market.
- This measure may depend excessively on few orders (if they are large) and therefore be subject to manipulation
- For our example, $VWAP = 3.02$

38

Implementation shortfall

- Aim at time 0: to (net) purchase q shares at current (mid)price m_0
 - Suppose by time t , fraction κ_t has been executed, at an average execution price $\bar{p}_t$
 - The realized trading gain is $\kappa_t q(m_t - \bar{p}_t)$
 - An ideal gain from immediate execution without price impact would have been $q(m_t - m_0)$
 - The difference is the **implementation shortfall**:

$$\begin{aligned} IS_t &= q(m_t - m_0) - \kappa_t q(m_t - \bar{p}_t) \\ &= \kappa_t q(\bar{p}_t - m_0) + (1 - \kappa_t)q(m_t - m_0). \end{aligned}$$

- Interpretation: Execution cost plus opportunity cost

39

Implementation shortfall

- Anand et al. [2012] show evidence that top market performers have a consistently negative implementation shortfall.
- “there is more to a trading strategy than just selecting a broker”
 - (e.g., when to trade, how much, how to react to market movements...)

42

Other measures

- Measures such as **trading volume** and **trade frequency** are also used
- **Time-to-trade** for limit orders is another measure, but difficult to use (depends on order size, depends on past traders' intent – some post LOs to provide liq-ty, trading is not the final goal)
- Some measures may contradict each other, e.g.:
 - trading volume and spreads are both positively correlated with information releases (why?)
 - price volatility is low in very liquid – but also very illiquid markets
- Frequency of trading or related measures may be more relevant in 'thin' markets, for instance in emerging economies

43

Conclusion

- We have looked at different manners in which to estimate liquidity
- No method is perfect: depends on trade size, time horizon, trade motivation
- Data shows that liquidity varies both continuously throughout a trading day, and more abruptly around big events
- Next time we will start analyzing *what* drives the spread

44

Exercises for next week

- Recreate the graphs and figures and numbers I presented today using the KrispyKreme dataset. Better: calculate the (average) values of all measures for the whole dataset.
- Solve exercise 8 regarding implementation shortfall, on page 75 in the textbook. Discuss the meaning of m_t in this analysis.

45

References I

- Y. Amihud. Illiquidity and stock returns: cross-section and time-series effects. *Journal of financial markets*, 5(1):31–56, 2002. URL [https://doi.org/10.1016/S1386-4181\(01\)00024-6](https://doi.org/10.1016/S1386-4181(01)00024-6).
- A. Anand, P. Irvine, A. Puckett, and K. Venkataraman. Performance of Institutional Trading Desks: An Analysis of Persistence in Trading Costs. *Review of Financial Studies*, 25(2): 557–598, Feb. 2012. ISSN 0893-9454, 1465-7368. doi: 10.1093/rfs/hhr110. URL <https://academic.oup.com/rfs/article-lookup/doi/10.1093/rfs/hhr110>.
- J. Hasbrouck. *Empirical market microstructure: The institutions, economics, and econometrics of securities trading*. Oxford University Press, 2007.
- C. M. Lee and M. J. Ready. Inferring trade direction from intraday data. *The Journal of Finance*, 46(2):733–746, 1991. URL <https://doi.org/10.1111/j.1540-6261.1991.tb02683.x>.

References II

- E. R. Odders-White. On the occurrence and consequences of inaccurate trade classification. *Journal of Financial Markets*, 3(3):259–286, 2000. URL [https://doi.org/10.1016/S1386-4181\(00\)00006-9](https://doi.org/10.1016/S1386-4181(00)00006-9).
- R. Roll. A simple implicit measure of the effective bid-ask spread in an efficient market. *The Journal of finance*, 39(4):1127–1139, 1984. URL <https://doi.org/10.1111/j.1540-6261.1984.tb03897.x>.

Financial Markets Microstructure

Lecture 4

Informative Order Flow
Chapter 3.1-3.3 of FPR

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What did we do last week?

- 1 Discuss liquidity
- 2 Talk about different measures of:
 - spreads (quoted, effective, realized)
 - depth and price impact
 - execution costs
- 3 Estimators for missing data
 - Lee-Ready algorithm for estimating trade direction
 - Roll's spread estimator requiring only price data

Overview

Before we start, let's look at an overview of what we'll do in the rest of the course

Part 1: Setting up the models

- Dealer models 1; [Glosten-Milgrom](#), fixed trade size; Bayes' Rule; adverse selection
- Dealer models 2; [Kyle](#), variable trade size; market depth; estimating liquidity determinants, inventory risk
- Limit order book; [Glosten](#): static, random market order demand; market design; [Parlour](#): dynamic, endogenous market order demand

3

Overview

Part 2: Applying the models

- Fragmentation: costs and benefits
- Transparency: search costs and order flow transparency
- Value of liquidity
- Liquidity and corporate policy

4

Overview

Part 3: Topics and specialized models

- High-frequency trading ([Biais et al.](#), [Budish et al.](#))
- Public information; optimal disclosure policies; public announcements and trade volumes ([Kondor](#))
- Bubbles; herding; common knowledge ([Smith and Sørensen](#), [Abreu and Brunnermeier](#))

5

Today

- 1 Beginning of a long discussion about determinants of the spread
- 2 Go in-depth with the relation between information and prices
- 3 What is informational efficiency?
- 4 Discuss Glosten and Milgrom's model of information-based trading

6

Determinants of liquidity

- Last week: **illiquidity** $\Rightarrow$ deviation of realized prices from the asset's fundamental value.
- Source of illiquidity: (too few traders present in the market at a given time). Or so I told you...
- If that was the only reason, trivial solutions for efficiency:
 - or
 - This narrows down the spread, but does not close it.
- So **what drives the spread**? (apart from market thinness)
 - To answer that, first talk about **what drives the prices**.

8

Price determinants

- Stock prices move around all the time
- Media reports are full of ex post rationalizations...
- ...but these can be pretty arbitrary $\rightarrow$
- What actually **determines stock prices** and their movement?

9

Price determinants

Prices result from clearing the market. Two options:

- everyone agrees on asset value → no trade (not the case in reality);
- traders have **different valuations** (**why?**) → prices clear demand and supply

In this course: everybody agrees* on assets' future *cash flows conditional on fundamentals*.

⇒ Reasons for trading:

- Traders have different value from the same cash flows
 - **Risk**: Getting the right risk profile
 - (Funding) **Liquidity**: Trader needs liquid funds or has excess funds to invest
- Traders have **private information** about the fundamentals

*IRL, people are very noisy in decision-making and disagree a lot with each other and even with past selves given the same info [Kahneman, Sibony, and Sunstein, 2021]

10

Types of information

- Different types of information (it's a spectrum really)
 - **Public information**: when revealed, asset price may move *without* trade
 - **Private information** possessed by some traders → reveal this information through their trading
- Reminder: we draw a wedge between **legal** and **academic** (← **our**) definitions of **private information**
 - **Legal**: insider info is that which is only available to a restricted number of people. Trading based on insider info is illegal in most jurisdictions.
 - **Academic**: some agents may be better at analyzing publicly available info, thus have better info about fundamentals.

11

How Dr. Alchian learned to build the bomb

We knew they were developing this H-bomb, but we wanted to know, what's in it? What's the fissile material? Well there's thorium, thallium, beryllium, and something else, and we asked Herman Kahn and he said, 'Can't tell you'... I said, 'I'll find out', so I went down to the RAND library and had them get for me the US Government's Dept. of Commerce Yearbook which has items on every industry by product, so I went through and looked up thorium, who makes it, looked up beryllium, who makes it, looked them all up, took me about 10 minutes to do it, and got them. There were about five companies, five of these things, and then I called Dean Witter... they had the names of the companies also making these things, 'Look up for me the price of these companies...' and here were these four or five stocks going like this, and then about, I think it was September, this was now around October, one of them started to go like that, from \$2 to around \$10, the rest were going like this, so I thought 'Well, that's interesting'... I wrote it up and distributed it around the social science group the next day. I got a phone call from the head of RAND calling me in, nice guy, knew him well, he said 'Armen, we've got to suppress this'... I said 'Yes, sir', and I took it and put it away, and that was the first event study.

—from a 2000 interview with A.Alchian, transcribed by Newhard [2014]

13

Types of informational efficiency

There are different kinds of **price efficiency**:

- **Weak form:** Prices reflect historic (price) information
- **Semi-strong form:** Prices reflect all public information
- **Strong form:** Prices reflect all public and private information

Strong is the kind of efficiency we had in mind in the previous lectures. It arises in classical models in Economics (see General Equilibrium theory)

14

Issues with EMH

Issues with EMH:

- **No-trade theorem:** If traders have private information and care only about fundamentals, nobody should ever trade (Milgrom and Stokey [1982])
- **GS informational paradox:** (Grossman and Stiglitz [1980])
 - Suppose markets were informationally efficient, then prices would reflect traders' private information
 - Thus: **no incentive to acquire information** (if this is costly)
- **Volatility:** prices move a lot, such volatility cannot be justified by public news alone
- **Process:** **how** does information get incorporated into prices?
 - Suppose every agent has some private signal about the fundamental, but also knows that the price incorporates all private info of all agents. Then it's optimal to ignore own signal, since it adds nothing to the price signal. But if everyone ignores all signals, price cannot be informative.

So we will not take EMH for granted, but rather see whether it arises in models endogenously.

15

Asset value

- Let's try to figure out how price efficiency looks like in math.
- **Information:** Ω_t captures the market's (public) knowledge at time t : $\Omega_{t+1} = (\Omega_t, I_{t+1})$ (I_{t+1} is the new info + assume perfect memory).
- **Market valuation** ($\neq$ price) of an asset = expectation of underlying **fundamental value** v given information Ω_t :

$$\mu_t = \mathbb{E}[v | \Omega_t].$$

Think of v as the sum of discounted cash flows: $v = \sum_{s=t}^{\infty} \delta^{s-t} c_s$ (uncertain as of t).

16

Informational efficiency (1)

- Informational efficiency then corresponds to: $p_t = \mu_t = \mathbb{E}[v|\Omega_t]$
(Ω_t is “market’s knowledge”, so efficiency understood as semi-strong)
- Valuation only changes if new information arrives: ‘innovation in value’ is a random variable: $\epsilon_{t+1} = \mu_{t+1} - \mu_t$. Then

$$\begin{aligned}\mathbb{E}[\epsilon_{t+1}|\Omega_t] &= \mathbb{E}[\mu_{t+1} - \mu_t|\Omega_t] \\ &= \mathbb{E}[\mu_{t+1}|\Omega_t] - \mathbb{E}[\mu_t|\Omega_t] \\ &= \mathbb{E}[\mathbb{E}[v|\Omega_{t+1}]|\Omega_t] - \mu_t \\ &= \mathbb{E}[v|\Omega_t] - \mu_t \\ &= \mu_t - \mu_t \\ &= 0.\end{aligned}$$

17

Informational efficiency (2)

- Also, $\mathbb{E}[\epsilon_s\epsilon_t] = 0$, $\forall s \neq t$.
- Under efficiency, price innovation is equal to the valuation innovation:

$$p_{t+1} - p_t = \mu_{t+1} - \mu_t = \epsilon_{t+1}$$

- Thus

$$\mathbb{E}[p_{t+1}|\Omega_t] = p_t$$

- When we have informational efficiency, the price is a **martingale**

18

Interim conclusion

- We asked the questions of “why do people trade?” and “how are prices determined?”
- One set of answers: informational reasons for trade; prices aggregate private info to some extent
- Some say prices must be informationally efficient – reflect the market valuation perfectly, so aggregate all available information
- Just shown that efficiency must imply that price is a martingale. It’s a testable implication: if price movements are not random, then prices are not efficient.
- Next: study one model of informational trading, see whether it predicts efficiency.

19

Glosten and Milgrom [1985]

All models are wrong; some models are useful.

– *George Box*

21

GM85: Overview

- The simplest model explaining **spread** through private information.
- Spread is driven by adverse selection
- Dynamic model, periods $t = 1, 2, \dots$;
(though we will be analyzing the stage game for a given period – essentially static)
- Two players in every period:
 - trader and dealer
 - **dealer** long-lived; trader new every period
 - **trader** can be informed or not
- One asset with fundamental value v (unknown), common belief $v \sim F(v)$

22

GM85: Model (1)

Trader: is either a speculator or a noise trader, can submit a market order $d_t \in \{1, -1\}$ to buy or sell one unit of the asset with fundamental value v (or do nothing, $d_t = 0$)

- **Speculator/informed trader** (probability π): has private information about v .
 - We will usually assume speculator simply knows v (not much changes if he only has a noisy private signal about it).
 - Risk neutral, chooses his market order d_t to maximize expected profit $d_t \cdot (v - p_t)$:
- **Noise trader** (probability $1 - \pi$): no pvt info about v ; trades for other reasons (hedging, liquidity).
 - We assume he follows some fixed strategy: buys with probability β_B ; sells w.p. β_S ; abstains w.p. $1 - \beta_B - \beta_S$
 - **Important:** this assumption is for simplicity only; this strategy can be perfectly rational! We just don't model what generates it.
E.g., could say noise traders choose d_t to maximize profit $\mathbb{E}[d_t(v + y_t - p_t)]$, where y is t -trader's idiosyncratic valuation (due to risk, time, liquidity preferences...)

23

GM85: Model (2)

Dealer (market maker)

- Risk neutral
- Willing to trade **exactly one unit** (buy/sell/no trade) each period
- Sets **bid and ask prices** (for a single unit)
- Quote price before seeing trade (limit order)
- Does not know whether trader is speculator or noise trader (but knows π)
- Expected profit from trade is $\mathbb{E}[-d_t(v - p_t)]$
- **Competitive**: prices=expected asset value conditional on information
- Trading is sequential: market orders served one by one

24

GM85: Model (3)

- **Equilibrium:**
 - An equilibrium consists of **bid and ask prices** and **speculator's strategy**
 - They must be such that: (i) prices are competitive (zero profit for MM), (ii) speculator best-responds to prices (maximizes expected gain).

26

Analysis. A: Market making

- Dealer quotes bid and ask prices on *one unit*
 - Can revise prices between each incoming trade
- Quoted ask price a_t only relevant if next incoming trader decides to buy
 - Dealer's payoff in this case is given by $\mathbb{E}[a_t - v | \Omega_{t-1}, Buy] = a_t - \mathbb{E}[v | \Omega_{t-1}, Buy]$
 - Same for bid b_t ; payoff: $\mathbb{E}[v | \Omega_{t-1}, Sell] - b_t$
 - (Note payoffs above rely on risk-neutrality)
- Perfect competition among dealers implies zero expected profit from either trade type $\Rightarrow$ ask price and bid price are

$$a_t = \mathbb{E}[v | \Omega_{t-1}, Buy];$$

$$b_t = \mathbb{E}[v | \Omega_{t-1}, Sell].$$

- Notice that both sides of the equality depend on prices

27

Analysis. B: Informed trading

- Speculator knows v . Given prices a_t and b_t , the expected profits Π are:

$$\Pi(v, a_t, b_t, d_t) = \begin{cases} v - a_t & \text{if } d_t = 1; & (Buy) \\ 0 & \text{if } d_t = 0; & (Abstain) \\ b_t - v & \text{if } d_t = -1. & (Sell) \end{cases}$$

- Speculator's best response to (a_t, b_t) is: (assume $a_t \geq b_t$)
 - Buy when $v > a_t$, i.e. when v is large enough
 - Sell when $v < b_t$, i.e. when v is small enough
 - Abstain if $a_t > v > b_t$

28

Analysis. C: Equilibrium definition

Dealer must make zero profit (competition), traders must trade optimally. This gives us two **equilibrium conditions**.

- Let σ_t denote the speculator's strategy, where $\sigma_t(d_t|v)$ is the probability that the speculator places order d_t if value is v
- **An equilibrium** consists of **prices** (a_t, b_t) and **strategy** σ_t such that:

- 1 the ask and bid prices solve

$$a_t = \mathbb{E}[v|\Omega_{t-1}, \text{Buy}];$$

$$b_t = \mathbb{E}[v|\Omega_{t-1}, \text{Sell}],$$

given σ_t

- 2 for each v , σ_t solves

$$\max_{\sigma_t} \{ \sigma_t(1|v)[v - a_t] + \sigma_t(-1|v)[b_t - v] \},$$

given (a_t, b_t) .

29

To be continued...

- We will finish the derivation of the general case next time.
- But the knowledge so far should be sufficient to solve the example:

GM Example

- **Single period:** Suppose only one period (drop t subscript, drop Ω)
- **Binary outcome:** $v \in \{0, 1\}$, equally likely ex ante: $\mathbb{P}(v = 1) = 0.5$.
- Suppose $0 < b < a < 1$ and noise trader's order obeys $\beta_B = \beta_S = 0.5$.

Questions:

- 1 What is the speculator's trading strategy?
- 2 Can you derive dealer's **prices** a and b , as a function of π ?
 - If not, refresh your knowledge of conditional expectations and try again.
 - If you already read the solution in the book, try to replicate it without looking back at the book.
- 3 Are the resulting prices efficient? (Check all three forms)

30

References I

- L. R. Glosten and P. R. Milgrom. Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics*, 14(1):71–100, Mar. 1985. ISSN 0304-405X. URL [https://doi.org/10.1016/0304-405X\(85\)90044-3](https://doi.org/10.1016/0304-405X(85)90044-3).
- S. J. Grossman and J. E. Stiglitz. On the impossibility of informationally efficient markets. *The American economic review*, 70(3):393–408, 1980. Publisher: JSTOR.
- D. Kahneman, O. Sibony, and C. R. Sunstein. *Noise: a flaw in human judgment*. Hachette UK, 2021.
- P. Milgrom and N. Stokey. Information, trade and common knowledge. *Journal of Economic Theory*, 26(1):17–27, Feb. 1982. ISSN 0022-0531. doi: 10.1016/0022-0531(82)90046-1. URL <https://www.sciencedirect.com/science/article/pii/0022053182900461>.

References II

- J. M. Newhard. The stock market speaks: How Dr. Alchian learned to build the bomb. *Journal of Corporate Finance*, 27:116–132, Aug. 2014. ISSN 0929-1199. URL <https://dx.doi.org/10.1016/j.jcorpfin.2014.05.002>.

Financial Markets Microstructure

Lecture 5

Glosten-Milgrom Model
Chapter 3.3 of FPR

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Spring 2025

What did we do last time?

- 1 Argued that market thinness is not the only source of illiquidity
- 2 Poked holes in the Efficient Market Hypothesis
- 3 Defined price efficiency in many ways
- 4 Began talking about the GM model

Today

- 1 more Glosten-Milgrom!

3

GM85: Overview

- Dynamic model, periods $t = 1, 2, \dots$;
(though we will be analyzing the stage game for a given period – essentially static)
- Two players in every period:
 - trader and dealer
 - **dealer** long-lived; trader new every period
 - **trader** can be informed or not
- One asset with fundamental value v (unknown), common belief $v \sim F(v)$

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GM85: Model (1)

Trader: is either a speculator or a noise trader, can submit a market order $d_t \in \{1, -1\}$ to buy or sell one unit of the asset with fundamental value v (or do nothing, $d_t = 0$)

- **Speculator** (probability π): has private information about v .
 - We will usually assume speculator simply knows v (not much changes if he only has a noisy private signal about it).
 - Risk neutral, chooses his market order d_t to maximize expected profit $d_t \cdot (v - p_t)$:
- **Noise trader** (probability $1 - \pi$): no pvt info about v ; trades for other reasons (hedging, liquidity).
 - We assume he follows some fixed strategy: buys with probability β_B ; sells w.p. β_S ; abstains w.p. $1 - \beta_B - \beta_S$
 - **Important:** this assumption is for simplicity only; this strategy can be perfectly rational! We just don't model what generates it.
E.g., could say noise traders choose d_t to maximize profit $\mathbb{E}[d_t(v + y_t - p_t)]$, where y is t -trader's idiosyncratic valuation (due to risk, time, liquidity preferences...)

6

GM85: Model (2)

Dealer (market maker)

- Risk neutral
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- Sets **bid and ask prices** (for a single unit)
- Quote price before seeing trade (limit order)
- Does not know whether trader is speculator or noise trader (but knows π)
- Expected profit from trade is $\mathbb{E}[-d_t(v - p_t)]$
- **Competitive:** prices=expected asset value conditional on information
- Trading is sequential: market orders served one by one

7

GM85: Model (3)

■ Equilibrium:

- An equilibrium consists of **bid and ask prices** and **speculator's strategy**
- They must be such that: (i) prices are competitive (zero profit for MM), (ii) speculator best-responds to prices (maximizes expected gain).

8

Analysis. A: Market making

- Dealer quotes bid and ask prices on *one unit*
 - Can revise prices between each incoming trade
- Quoted ask price a_t only relevant if next incoming trader decides to buy
 - Dealer's payoff in this case is given by $\mathbb{E}[a_t - v | \Omega_{t-1}, Buy] = a_t - \mathbb{E}[v | \Omega_{t-1}, Buy]$
 - Same for bid b_t ; payoff: $\mathbb{E}[v | \Omega_{t-1}, Sell] - b_t$
 - (Note payoffs above rely on risk-neutrality)
- Perfect competition among dealers implies zero expected profit from either trade type $\Rightarrow$ **ask price** and **bid price** are

$$a_t = \mathbb{E}[v | \Omega_{t-1}, Buy];$$

$$b_t = \mathbb{E}[v | \Omega_{t-1}, Sell].$$

- Notice that both sides of the equality depend on prices

9

Analysis. B: Informed trading

- Speculator knows v . Given prices a_t and b_t , the expected profits Π are:

$$\Pi(v, a_t, b_t, d_t) = \begin{cases} v - a_t & \text{if } d_t = 1; & (Buy) \\ 0 & \text{if } d_t = 0; & (Abstain) \\ b_t - v & \text{if } d_t = -1. & (Sell) \end{cases}$$

- Speculator's best response to (a_t, b_t) is: (assume $a_t \geq b_t$)

- Buy when $v > a_t$, i.e. when v is large enough
- Sell when $v < b_t$, i.e. when v is small enough
- Abstain if $a_t > v > b_t$

10

Analysis. C: Equilibrium definition

Dealer must make zero profit (competition), traders must trade optimally. This gives us two **equilibrium conditions**.

- Let σ_t denote the speculator's strategy, where $\sigma_t(d_t|v)$ is the probability that the speculator places order d_t if value is v
- **An equilibrium** consists of **prices** (a_t, b_t) and **strategy** σ_t such that:

- 1 the ask and bid prices solve

$$\begin{aligned} a_t &= \mathbb{E}[v | \Omega_{t-1}, Buy]; \\ b_t &= \mathbb{E}[v | \Omega_{t-1}, Sell], \end{aligned}$$

given σ_t

- 2 for each v , σ_t solves

$$\max_{\sigma_t} \{ \sigma_t(1|v)[v - a_t] + \sigma_t(-1|v)[b_t - v] \},$$

given (a_t, b_t) .

11

Back to homework

GM Example

- **Single period:** Suppose only one period (drop t subscript, drop Ω)
- **Binary outcome:** $v \in \{0, 1\}$, equally likely ex ante: $\mathbb{P}(v = 1) = 0.5$.
- Suppose $0 < b < a < 1$ and noise trader's order obeys $\beta_B = \beta_S = 0.5$.

Questions:

- 1 What is the speculator's trading strategy?
- 2 Can you derive dealer's prices a and b , as a function of π ?
 - If not, refresh your knowledge of conditional expectations and try again.
 - If you already read the solution in the book, try to replicate it without looking back at the book.
- 3 Are the resulting prices efficient? (Check all three forms)

12

Analysis. D: Solving for equilibrium (1)

- The orders reveal information about v . E.g., a **buy order** is submitted:
 - either by a noise trader (probability $(1 - \pi)\beta_B$) – no new information, $\mu_t = \mu_{t-1} = \mathbb{E}[v|\Omega_{t-1}]$;
 - or by a speculator (probability $\pi\mathbb{P}(v \geq a_t|\Omega_{t-1})$) – then learn that $v \geq a_t$, so $\mu_t = \mathbb{E}[v|\Omega_{t-1}, v \geq a_t]$.
- Then a_t is given by (using Bayes' rule and law of total probability; N=Noise, I=Informed):

meaning that in the end, $a_t \geq \mu_{t-1}$.

13

Analysis. D: Solving for equilibrium (2)

- Similarly for **sell orders**:
 - sell order from a noise trader arrives with [unconditional] probability $(1 - \pi)\beta_S$ – no new information, $\mu_t = \mu_{t-1}$;
 - sell order from a speculator arrives with probability $\pi\mathbb{P}(v \leq b_t | \Omega_{t-1})$ – then learn that $v \leq b_t$, so $\mu_t = \mathbb{E}[v | \Omega_{t-1}, v \leq b_t]$.
- Then b_t is given by:
 - so $b_t \leq \mu_{t-1}$, and we have confirmed that indeed $a_t \geq b_t$.
- These prices a_t, b_t together with the speculator's trading strategy constitute the equilibrium.

14

Analysis. E: Profits

- **Informed traders** earn positive profit (since know v and have an option of doing nothing)
- Dealers assumed competitive, hence zero profit
- **Uninformed traders** incur a loss
 - Although this is because we modeled this as a zero-sum game and explicitly ignored the uninformed traders' potential trading motives.

15

Analysis. F: Price efficiency

- $\mu_t \equiv \mathbb{E}[v|\Omega_t]$ is the expectation of v *after* the time- t trade order is observed (by the dealer and outside observers). Note that in our model:

$$\mu_t = \begin{cases} a_t > \mu_{t-1} & \text{if buy order at } t; \\ b_t < \mu_{t-1} & \text{if sell order at } t. \end{cases}$$

- Meaning market price is efficient: $p_t = \mu_t$
 - in **semi-strong** form (prices anticipate and incorporate all information conveyed through trades),
 - not in the **strong** form (which would be equivalent to $p_t = v$)
- This is because dealers are **competitive** – dealers' market power would ruin efficiency

16

Example (as in the book)

- Let's do an example similar to your homework; calculate the prices more explicitly.
- **Single period:** Suppose only one period (drop t subscript, drop Ω)
- **Binary outcome:** $v \in \{v^H, v^L\}$, with prior $\theta = \mathbb{P}(v^H)$
- **Prior value:** What is the prior value of the asset before trading?

$$\mu = \theta v^H + (1 - \theta) v^L.$$

Skip example

18

Example (2)

- How do we solve the model? Look for equilibrium with trade.
- Suppose $v^L < b < a < v^H$.
- Then speculator buys if $v = v^H$, sells if $v = v^L$.
 - That is, $\sigma(1|v^H) = 1$ and $\sigma(-1|v^L) = 1$
- The procedure is then the following
 - 1 Use the equilibrium conditions from before to calculate prices given the above speculator strategy
 - 2 Check that these prices satisfy $v^L < b < a < v^H$

19

Example (3)

- Let's solve for the ask price. First:

$$\mathbb{P}(Buy|v^H) = (1 - \pi)\beta_B + \pi$$

$$\mathbb{P}(Buy|v^L) = (1 - \pi)\beta_B$$

Then by Bayes' Rule

$$\begin{aligned}\mathbb{P}(v^H|Buy) &= \frac{\mathbb{P}(v^H)\mathbb{P}(Buy|v^H)}{\mathbb{P}(Buy)} \\ &= \frac{\theta[(1 - \pi)\beta_B + \pi]}{(1 - \pi)\beta_B + \pi\theta} \\ &= \theta + \frac{\theta(1 - \theta)\pi}{(1 - \pi)\beta_B + \pi\theta}\end{aligned}$$

20

Example (4)

- The ask price is the expected value, given a buy order:

$$\begin{aligned} a &= \mathbb{P}(v^H | Buy) v^H + [1 - \mathbb{P}(v^H | Buy)] v^L \\ &= \left[\theta + \frac{\theta(1-\theta)\pi}{(1-\pi)\beta_B + \pi\theta} \right] v^H + \left[1 - \left(\theta + \frac{\theta(1-\theta)\pi}{(1-\pi)\beta_B + \pi\theta} \right) \right] v^L \\ &= \mu + \frac{\theta(1-\theta)\pi}{(1-\pi)\beta_B + \pi\theta} (v^H - v^L). \end{aligned}$$

- Doing a similar exercise for b we find

$$b = \mu - \frac{\theta(1-\theta)\pi}{(1-\pi)\beta_S + \pi(1-\theta)} (v^H - v^L)$$

- Finally, we must check that our assumption holds: easy to check that $v^H > a > b > v^L$.
Hence, **this is an equilibrium**

21

Example: Lessons

$$\begin{aligned} a - \mu &= \frac{\theta(1-\theta)\pi}{(1-\pi)\beta_B + \pi\theta} (v^H - v^L) \\ \mu - b &= \frac{\theta(1-\theta)\pi}{(1-\pi)\beta_S + \pi(1-\theta)} (v^H - v^L) \end{aligned}$$

- Add the two expressions to get bid-ask **spread** $S = a - b$
 - S **increases in π** : more informed trading exacerbates adverse selection. Opposite for $(\beta_B + \beta_S)$.
 - If $\beta_B = \beta_S = 1/2$, S is **increasing in $\theta(1-\theta)$** , i.e. spread higher when dealer faces greater initial uncertainty about v . Same for $(v^H - v^L)$.

22

Example: Price discovery

- Return to the multiperiod setting. One unit traded every period, v persistent.
- Trade flow is **informative** – trades have long-lasting effect on prices
- Each order conveys information, dealers learn, and

$$p_t \rightarrow v$$

Dynamics

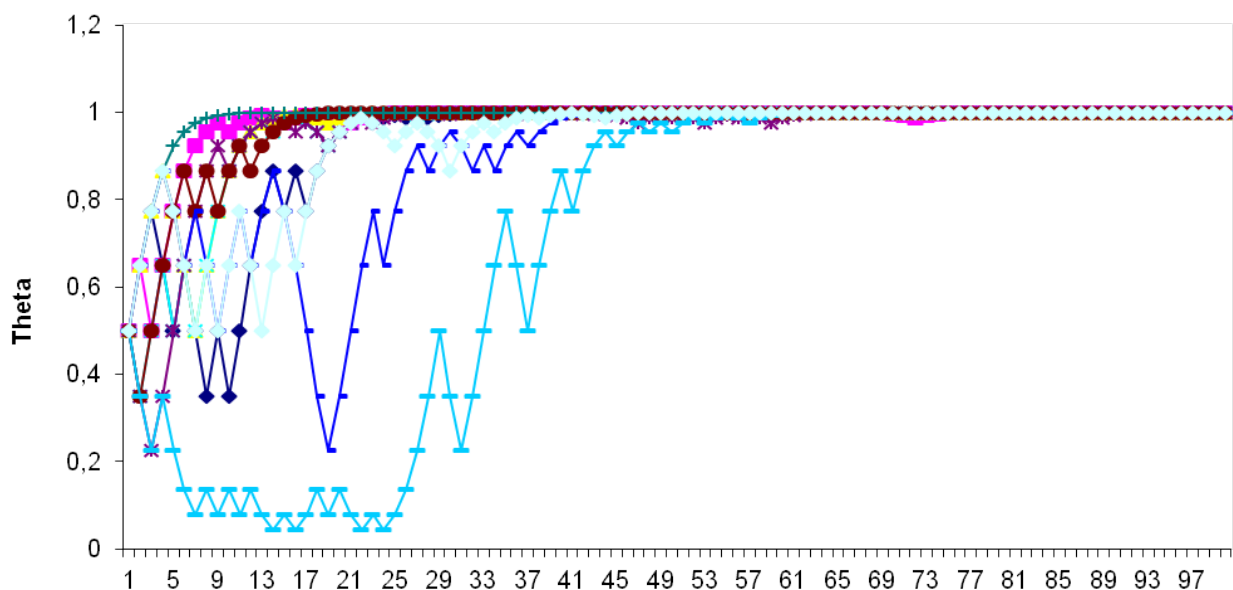
prices **strong-form efficient** in the long run

- Speed of price discovery increasing in π
 - Trade-off between **price discovery** and **liquidity**

23

Example: Simulation

Dealer beliefs: Each curve shows the evolution of dealer's beliefs in each run (10 runs of 100 orders)



24

GM: Summary of the findings

What did we learn from the Glosten and Milgrom model?

1 Information, prices and the spread

- Prices will reflect the information revealed by trades
- The spread is increasing in informational asymmetry (adverse selection) and in uncertainty about asset value

2 Informational efficiency

- Prices are always semi-strong efficient, in the long run also strong-form efficient

3 Noise trading

- Noise trading keeps the market liquid and improves spreads
- Informed speculation increases spreads, but improves price discovery - dilemma for regulators

26

GM: Discussion

(anti?)Features

- **Dealer model:** Prices are set each period, discriminative, normally competitive (zero profits)
- **Non-market clearing:** Only one unit traded - not market clearing (traders may wish to buy/sell more)
- Only **fundamental value** matters, no speculation/resale

Discussion

- **Insights:** Adverse selection as a driver of the spread
- **Shortcomings:** Trade fixed amount, trade once, no resale
- **Advantages:** (Relatively) simple analysis, flexible

27

Homework

■ Reading:

- Read two articles on absalon on how ESMA restricted trading and binary options and SEC restricted trading in certain stocks.
- What is the difference between the underlying assets in the two cases?
- Explain ESMA's decision using GM model.

■ Solving:

- FPR chapter 3, exercise 3 (GM model where speculators are not perfectly informed, but instead receive a signal about the value of the asset)
- GM example with $v \sim U[0, 1]$ (rest as in the problem assigned before today; goal: derive the equilibrium bid and ask prices)

28

Dynamics

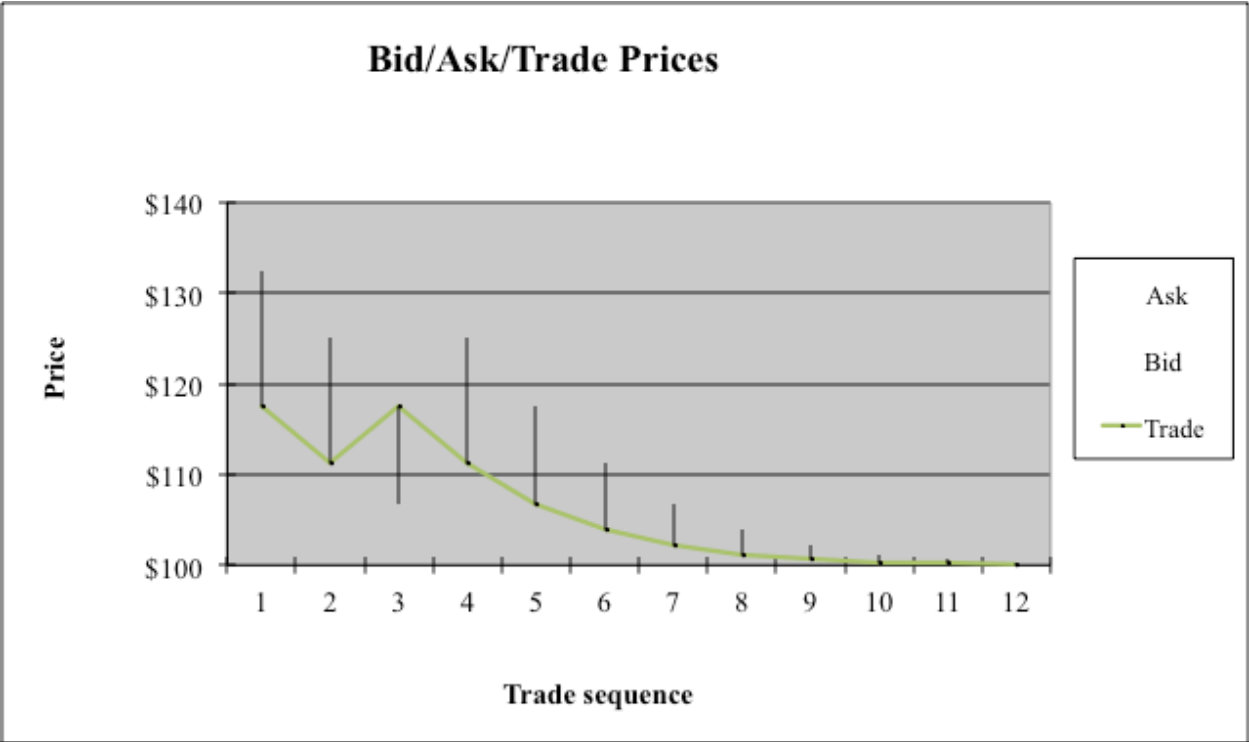
Suppose we are in the simple binary model with the following parameters

- Probability of informed speculators: $\pi = 0.3$
- Probability (ex ante) of high value: $\theta = 0.5$
- $v^H = 150$ and $v_L = 100$

Consider 12 periods, with the following sequence of buys (b) and sells (s)

ssbsssssssss

30



Financial Markets Microstructure

Lecture 6

Liquidity and Price Dynamics
Chapter 3.4-3.7 of FPR

Egor Starkov

Københavns Universitet
Spring 2025

What did we do last week?

- 1 Information and prices
- 2 Efficiency and markets
- 3 Glosten and Milgrom: Workhorse model to analyze adverse selection in markets
 - Analysis of what drives the spread
 - Tradeoff between market liquidity and price discovery
 - The model had reasonably good efficiency properties

Today

- 1 Look at other drivers of the spread
 - Order-processing costs
 - Dealer inventory risk
- 2 We'll look at how their dynamic effect on prices differ

3

What order-processing costs exist?

A liquidity supplier (for instance a dealer) can have a range of different order-processing costs

- Trading fees: charged by exchanges
- Clearing and settlement fees: paid if a central clearinghouse is used to minimize trading risks
- Overheads: back office expenses
- (Dealer rents)

These costs must somehow be compensated by traders, and will therefore enter the spread

5

How do these costs affect the spread?

- Let $\mu_t \equiv \mathbb{E}[v|\Omega_t]$ be the expectation of v *after* the time- t trade order is observed, and let s_t^a and s_t^b denote the 'half-spreads'
- Hence, μ_{t-1} represents what we now when period t starts
- Then in the GM model we can write prices as

$$a_t = \mu_{t-1} + s_t^a$$

$$b_t = \mu_{t-1} - s_t^b$$

- Assume dealer has order cost γ , and charges this directly to trader:

$$a_t = \mu_{t-1} + \gamma + s_t^a$$

$$b_t = \mu_{t-1} - \gamma - s_t^b$$

6

How do these costs affect the spread? (2)

- Hence, the new bid-ask spread is

$$S_t = a_t - b_t = 2\gamma + s_t^a + s_t^b$$

- The spread is now made up of order costs (2γ) and adverse selection costs ($s_t^a + s_t^b$)
- Suppose we want to determine whether spread in a given market is due to adverse selection or order costs
 - The instantaneous effect of order costs is similar to that of adverse selection costs
 - But we shall see that the dynamic effect is different

7

The dynamics of the spread

- As before, let $d_t = 1$ denote a buyer-initiated trade, and $d_t = -1$ a seller-initiated trade
- Also, let $s(d_t)$ be the adverse-selection-related half-spread depending on the trade:
 $s(1) = s_t^a$ and $s(-1) = s_t^b$
- Then the realized price can be written as

$$p_t = \mu_{t-1} + (s(d_t) + \gamma)d_t$$

- Since $\mu_t = \mu_{t-1} + s(d_t)d_t$, then

$$p_t = \underbrace{\mu_t}_{\text{updated valuation}} + \underbrace{\gamma d_t}_{\text{order cost}}$$

8

The dynamics of the spread

Then the effect of time- t trade on prices:

- **short-run:**

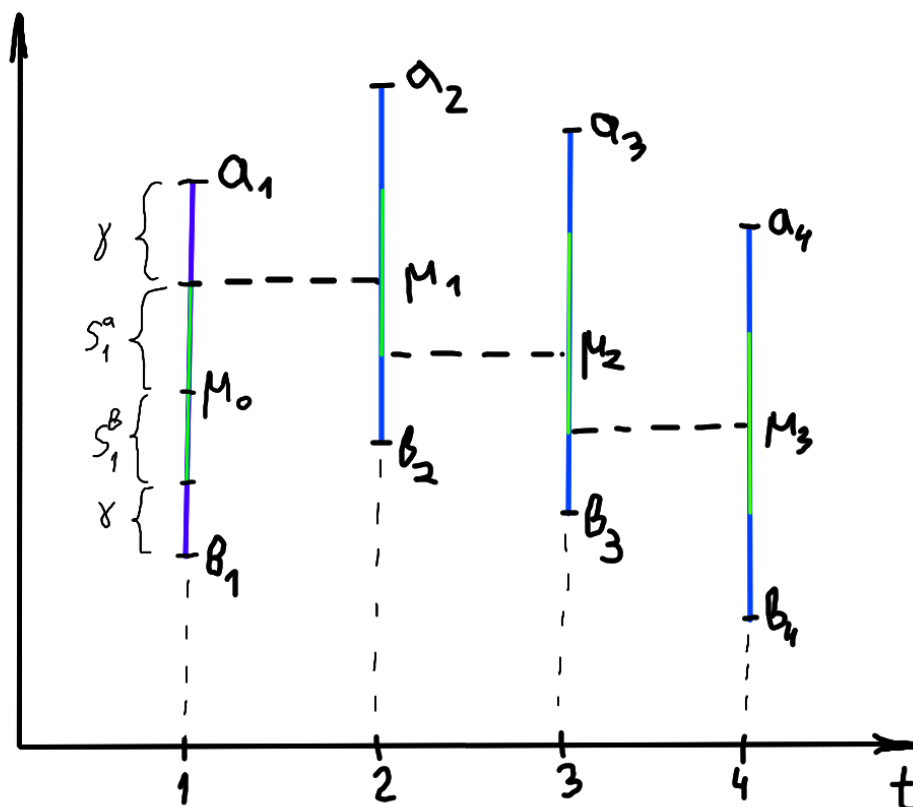
$$p_t - \mu_{t-1} = (s(d_t) + \gamma)d_t$$

- **long-run:**

$$\begin{aligned}\mathbb{E}_t[p_{t+s}] - \mu_{t-1} &= \mathbb{E}_t[\mu_{t+s-1} + (s(d_{t+s}) + \gamma)d_{t+s}] - \mu_{t-1} \\ &\approx \mathbb{E}_t[\mu_{t+s-1}] - \mu_{t-1} \\ &= \mu_t - \mu_{t-1} \\ &= s(d_t)d_t\end{aligned}$$

- so order cost effect on prices is *transient* and is reversed by future trades;
- effect of adverse selection term is *permanent*

9



10

Inventory risk

- Illiquidity can arise due to dealers' asset **inventory cost**
 - Holding inventory is **risky**, so dealers adjust their quotes to unwind any accumulated inventory.
- Textbook illustrates this using Stoll (1978) model
 - But this model is from pre-game theory times and has a strange solution method
 - So we will instead **extend the Glosten-Milgrom model** (extension not in the book)

12

Glosten-Milgrom model: Inventory risk edition

- Suppose for simplicity there are **no speculators** ($\pi = 0$)...
- ...but there are public news about asset value: $\mu_{t+1} = \mu_t + \epsilon_t$ (where $\mu_t = \mathbb{E}[v|\Omega_t]$)
- Noise traders behave as usual
- **Dealer** has some **inventory** z_t of the stock and c_t of cash
- Dealer is **risk averse**
 - For concreteness, assume mean-variance preferences over next-period wealth:

$$U(w_{t+1}) = \mathbb{E}[w_{t+1}] - \frac{\rho}{2} \mathbb{V}(w_{t+1}),$$

where $w_t = z_t \mu_t + c_t$ and $\rho > 0$ measures risk aversion

- (equivalent to CARA expected utility preferences when returns are normal)

13

GM-IRE: Dealer's utility

- As usual, let $\mathbb{E}[\epsilon_t] = 0$, $\mathbb{V}(\epsilon_t) = \sigma^2$, and remember that z_t is the current inventory, and $w_t = z_t \mu_t + c_t$.
- Dealer's utility at the end of period t is

if no orders: $U(w_{t+1}|N) =$

if Buy order: $U(w_{t+1}|B) =$

if Sell order: $U(w_{t+1}|S) =$

14

GM-IRE: Quotes

To derive equilibrium **quotes**, use the **zero-profit condition**

- i.e., ensure that the dealer's expected utility does not change after trading
- (assume all competing dealers have same inventory z_t)

$$U(w_{t+1}|B) = U(w_{t+1}|N) \Rightarrow a_t =$$

$$U(w_{t+1}|S) = U(w_{t+1}|N) \Rightarrow b_t =$$

15

GM-IRE: Quotes (2)

- **Spread** is $S_t = \rho\sigma^2$
 - Positive due to dealer's risk-aversion
 - Increasing in risk-aversion coeff ρ and asset value volatility σ^2
- **Midquote** is $m_t = \mu_t - \rho\sigma^2 z_t$
 - Depends on dealer's inventory z_t . Dealer demands risk premium for taking a position in the asset.
 - Effects depend on risk parameters ρ and σ^2
 - To emphasize: prices here are **not efficient**
 - This inefficiency would in principle also motivate traders to submit the "right" orders – arbitrage! (Not present in this model, since we assumed traders are not strategic and not sensitive to the price.)

16

Summary

- The spread is driven not only by adverse selection: order costs and inventory risk have an effect as well
 - Price efficiency is lost once these new factors come into play
- How can we figure out which of these factors are more/less important? By their long-term effects! Will talk about that soon.
- But before we go there, next time: what affects market *depth*?

17

Homework

We said today that inventory risk is priced when the dealer is risk-averse. Risk-aversion is one explanation, but other factors can also contribute to inventory risk. The two following cases explore this issue:

- A big trader was punted off the Nordic power market after failing to meet margin calls (two articles on [absalon](#)).
 - How does inventory risk manifest in this story?
 - Explain why such inventory risk can be priced even by risk-neutral agents.
- Negative oil futures prices were registered in 2020 (blog post on [absalon](#) or [here](#)).
 - Why did it happen? How do negative prices make sense?
 - How does inventory risk manifest in this story?
 - Explain why such inventory risk can be priced even by risk-neutral agents.

18

Financial Markets Microstructure

Lecture 7

Depth determinants
Chapter 4 of FPR

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Spring 2025

Previously on...

- 1 The spread is not only driven by adverse selection: order costs and inventory risk have an effect as well

Homework from last time

We said today that inventory risk is priced when the dealer is risk-averse. Risk-aversion is one explanation, but other factors can also contribute to inventory risk. The two following cases explore this issue:

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 - How does inventory risk manifest in this story?
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- Negative oil futures prices were registered last year (blog post on [absalon](#) or [here](#)).
 - Why did it happen? How do negative prices make sense?
 - How does inventory risk manifest in this story?
 - Explain why such inventory risk can be priced even by risk-neutral agents.

3

Today

- **Trade size**
 - How does trade size affect prices?
 - I.e., what determines market [depth](#)?
 - (Spoiler: mostly the same factors as with liquidity)
 - Will look at Kyle (1985) model – an alternative to GM that allows flexible trade size

6

Prices and trade size

- How does trade size affect prices?
 - Spread larger for large trades, price moves further from efficient level
 - I.e., market has limited **depth**
- Why?
 - 1 **Adverse selection**
 - 2 **Inventory risk**
 - 3 **Imperfectly competitive dealers**
 - 4 **Order processing costs**

7

Kyle model

- We will look at Kyle (1985) model which links market depth to adverse selection
- It can be extended to accomodate imperfect competition among dealers (see 4.2.4) and inventory risk (4.3)
 - the inventory risk version is broadly similar to the Stoll model that we skipped
 - trading costs are very difficult to incorporate in this model

8

Setup: Broad strokes

- A **call auction**; orders come from a “large” speculator and a population of noise traders; market cleared by a dealer.
- **Speculator/informed trader**: has private information
 - Trades using a ‘large’ speculative market order
 - Strategically moderates order size to reduce price impact
 - ‘Hides’ behind noise traders who submit a random size order
- Representative **market maker (MM)/dealer**
 - Risk neutral and competitive (zero profits)
 - Clears orders in *batches* (as opposed to one-by-one in Glosten & Milgrom)
 - Cannot distinguish speculative orders from noise orders in a batch

9

Setup

- **Asset**: Trade in one risky asset with value $v \sim \mathcal{N}(\mu, \sigma_v^2)$
- **Speculator**: Observes true value v (perfect information)
 - Places market order x
 - If the order clears at price p : gain is $x(v - p)$
 - Does **not know** p when choosing x : maximizes *expected* gain (risk neutral) given $\mathbb{E}[p|x]$
- **Noise trader**: Has random demand $u \sim \mathcal{N}(0, \sigma_u^2)$
- **MM**: Submits a supply schedule of (q, p) combinations:
 - “If the order imbalance is $q = x + u$, I will absorb it at price p ”
 - Observes aggregate flow $q = x + u$, but not x and u
 - Competitive (zero profit): $p = \mathbb{E}[v|q]$
- **Assumption**: u and v are jointly normal and independent

10

Setup: Timing

- To be explicit, the timing is as follows:
 - 1 at the beginning of the period:
 - speculator chooses order size x
 - noise traders submit their order u
 - dealer submits price schedule (q, p)
 - 2 then market price $p(q)$ is determined given total order $q = x + u$
 - 3 at the end of the period payoffs are realized

11

Linear equilibrium: outline

- The equilibrium is described by the speculator's strategy $x(v)$ and the dealer's pricing schedule $p(q)$.
- Look for equilibrium where speculator's strategy is linear: $x = \beta(v - \mu)$
 - Note: β is endogenously determined by the equilibrium, we'll derive it
 - $\beta > 0$ measures speculator *aggression*
- MM's pricing is driven by the zero-profit condition: $p = \mathbb{E}[v|q]$
 - In eqm, MM knows the speculator's strategy ($x = \beta(v - \mu)$)
 - So MM observes $q = x + u = \beta(v - \mu) + u$, and wants to **estimate v**

12

Aside: Hitchhiker's guide to normal updating

Updating normal beliefs with normal signals might seem daunting at first, but here's how it works.

- Suppose there is some uncertain variable x and our prior belief is that $x \sim \mathcal{N}(\mu_x, \sigma_x^2)$
- This prior belief is said to have precision $\tau_x \equiv \frac{1}{\sigma_x^2}$.
- Suppose we get signal y about x with precision τ_ϵ .
I.e., we observe $y = x + \epsilon$, where $\epsilon \sim \mathcal{N}\left(0, \frac{1}{\tau_\epsilon}\right)$ (so $\tau_\epsilon \equiv \frac{1}{\sigma_\epsilon^2}$).
- Then our posterior belief about x has precision $\tau_{x|\epsilon} = \tau_x + \tau_\epsilon$, and the posterior mean weighs the prior mean μ_x and the signal y according to their precisions:

$$x|y \sim \mathcal{N}\left(\frac{\tau_x}{\tau_x + \tau_\epsilon}\mu_x + \frac{\tau_\epsilon}{\tau_x + \tau_\epsilon}y, \frac{1}{\tau_{x|\epsilon}}\right)$$

(you can verify this by directly calculating the conditional pdf)

13

Deriving the distribution of $v|q$

In our case:

- $v \sim \mathcal{N}(\mu, \sigma_v^2)$,
- $q = \beta(v - \mu) + u$, where $u \sim \mathcal{N}(0, \sigma_u^2)$, $u \perp v$.
- So the prior precision (of the belief about v) is $\tau_v \equiv$,
- and the signal is $\tilde{q} \equiv$, which has precision $\tau_{\tilde{q}} =$
- Then

$$\begin{aligned} v|q &\sim \mathcal{N}\left(\frac{\tau_v}{\tau_v + \tau_{\tilde{q}}}\mu + \frac{\tau_{\tilde{q}}}{\tau_v + \tau_{\tilde{q}}}\tilde{q}, \frac{1}{\tau_v + \tau_{\tilde{q}}}\right) \\ &\sim \mathcal{N}\left(\right) \end{aligned}$$

14

Dealer's strategy

Going back to the zero profit condition:

$$\begin{aligned} p &= \mathbb{E}[v|q] \\ \Leftrightarrow p &= \mu + \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2} q \\ \Leftrightarrow p &= \mu + \lambda q, \\ \text{where } \lambda &\equiv \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2} \left(= \frac{\mathbb{C}(v, q)}{\mathbb{V}(q)} \right) \end{aligned}$$

Here λ is the **price impact** coefficient.

Conversely, $1/\lambda$ is a measure of **market depth**

15

Speculator's strategy

Back to the speculator's problem. The speculator takes for granted the MM's pricing rule $p = \mu + \lambda q$

- Speculator's **profit** is $\Pi(x) = x(v - p) = x(v - \mu - \lambda x - \lambda u)$
- **Expected profit** is $\mathbb{E}[\Pi(x)] = x(v - \mu - \lambda x)$, since $\mathbb{E}[u] = 0$
- Speculator chooses x to maximize $\mathbb{E}[\Pi(x)]$. Using the first-order condition:

$$\begin{aligned} v - \mu - 2\lambda x &= 0 \\ \Rightarrow x &= \beta(v - \mu), \\ \text{where } \beta &= 1/(2\lambda) \end{aligned}$$

- Confirmed that it is optimal for the speculator to use a linear strategy!
- Note analogy to monopoly problem:

16

Closing the equilibrium

- Finally, 'match' the coefficients:

$$\frac{1}{2\beta} = \lambda = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2}$$

i.e. $\beta^2\sigma_v^2 + \sigma_u^2 = 2\beta^2\sigma_v^2$ which yields

$$\beta = \frac{\sigma_u}{\sigma_v} \text{ and } \lambda = \frac{\sigma_v}{2\sigma_u}.$$

- Thus: the strategies are optimal given the prices, and the prices optimal given the strategies → **equilibrium**

17

Equilibrium properties

$$\beta = \frac{\sigma_u}{\sigma_v} \text{ and } \lambda = \frac{\sigma_v}{2\sigma_u}.$$

- Speculator is more **aggressive** (β higher) when:

- 1 The informational advantage σ_v is smaller (why?)
- 2 There's more noise σ_u to hide behind (why?)

- **Market depth:**

$$\frac{1}{\lambda} = 2\beta = 2\frac{\sigma_u}{\sigma_v}$$

The market is deeper when there is less insider trading and more noise trading

18

Equilibrium properties

- Insider's a priori (before observing v) **expected gain**:

$$\begin{aligned}\mathbb{E}[x(v - \mu - \lambda x)] &= \mathbb{E}\left[\beta(v - \mu)\left(v - \mu - \frac{v - \mu}{2}\right)\right] \\ &= \beta \frac{\sigma_v^2}{2} = \frac{\sigma_v \sigma_u}{2}\end{aligned}$$

Comment: speculator expects a positive profit (could abstain). Competitive risk-neutral MM earns zero profits. Noise traders lose. Same as in GM.

- Market maker's **posterior variance** of v is

$$\mathbb{V}(v|q) = \frac{1}{1/\sigma_v^2 + \beta^2/\sigma_u^2} = \frac{\sigma_v^2}{2}$$

Exactly half the prior variance: [Insider reveals half his information](#)

19

Kyle's model: summary

- Dealer/market maker model**: Competitive, risk-neutral (zero profit) dealer chooses a supply schedule
- Informed trader**: Observes signal about asset value and places market order
- Market clearing**: [Auction](#), dealer observes only total demand (informed + noise), total demand clears
- Insights**: informed trading is a factor generating limited market depth, insider always reveals half his information
- Advantage**: Richer trading opportunities, trader not price-taker
- Shortcomings**: Still no resale

20

Kyle with inventory risk I

- Now let's look at how market maker's **inventory risk** can lead to limited depth.
- Assume **no informed trading**: $x = 0$.
- Asset value $v \sim \mathcal{N}(\mu, \sigma_v^2)$
- Market maker has mean-variance preferences over their next-period wealth:

$$U(w_{t+1}) = \mathbb{E}[w_{t+1}] - \frac{\rho}{2} \mathbb{V}(w_{t+1}),$$

where w is composed of cash and asset holdings: $w_{t+1} = (z_t - q_t)v + q_t p_t$

- MM's initial asset position is z_t (initial cash is irrelevant, ignore it).

21

Kyle with inventory risk II

- To get the pricing schedule, follow the competitive logic:
 - The market-maker takes some market price p as given, chooses how much $q(p)$ to sell at this price:

$$\max_q \left\{ \underbrace{(z_t - q)\mathbb{E}[v] + qp}_{\mathbb{E}[w_{t+1}]} - \underbrace{\frac{\rho}{2}(z_t - q)^2 \mathbb{V}(v)}_{\mathbb{V}(w_{t+1})} \right\}$$

- FOC: $p - \mu + \rho(z_t - q)\sigma_v^2 = 0 \iff q(p) = z_t + \frac{p - \mu}{\rho\sigma_v^2}$
- For market to clear, need $q(p) = u = q$ (dealer's supply = total traders' market order), so inverting the pricing schedule we get:

$$p(q) = \mu + \rho\sigma_v^2(q - z_t).$$

22

Kyle with inventory risk III

$$p(q) = \mu + \rho\sigma_v^2(q - z_t)$$

Takeaways:

- 1 Depth (dictated by the dealer's willingness to trade at a given price) is limited
- 2 This is despite traders still being completely price-insensitive in this model!
- 3 Price impact depends on asset riskiness σ_v^2 and MM's risk aversion ρ .
- 4 Midquote depends on z_t

So really, all the same stuff as in GM with inventory risk.

The book also looks at versions with many MMs with heterogeneous ρ s, and many imperfectly competitive MMs.

23

Extensions

Other extensions are possible:

1 Dynamics

- In a fully dynamic model, the insider reveals less than half of the information in each period. Why?
- In the limit where trade is continuous over $[0, T]$, then $\mathbb{V}(v|q_0, \dots, q_t) \simeq (T - t) \frac{\sigma_v^2}{T}$: variance decreases linearly in time. [Model of how to split a large trade over time](#)

2 More insiders

- More insiders are more competitive; more aggressive
- The market is more liquid and more information revealed
- In dynamic model with several insiders: rush to trade on common information from the beginning

24

3 Imperfect market maker competition (Cournot style)

- Finite number of market makers, $k = 1, \dots, K$
- Market maker k supplies $y^k = \phi(p - \mu)$
- Market clears at price p with $\sum y^k = q$
- Strategic market maker takes into account effect of orders on profits
- Now: $p = \mu + \lambda q$ where $\lambda = \alpha(K - 1)/(K - 2) > \alpha$.

4 Trading costs

- Trivial in GM. Very difficult here, both technically and conceptually.
- Don't know how many trades there are, don't know the total volume (not q – some noise traders' orders could've cancelled each other out)
- Even taking costs as a linear function of order imbalance $|q|$ makes things difficult

25

Homework

- 1 We will talk about empirical estimation of factors of illiquidity next time (ch.5) and begin talking about LOB markets (without dealers; ch.6)
- 2 Solve ex 3 in ch.4 (p.159): Kyle's model with competition among speculators.

26

Financial Markets Microstructure

Lecture 8

Empirics of illiquidity
Chapter 5 of FPR

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Spring 2025

What did we do last week?

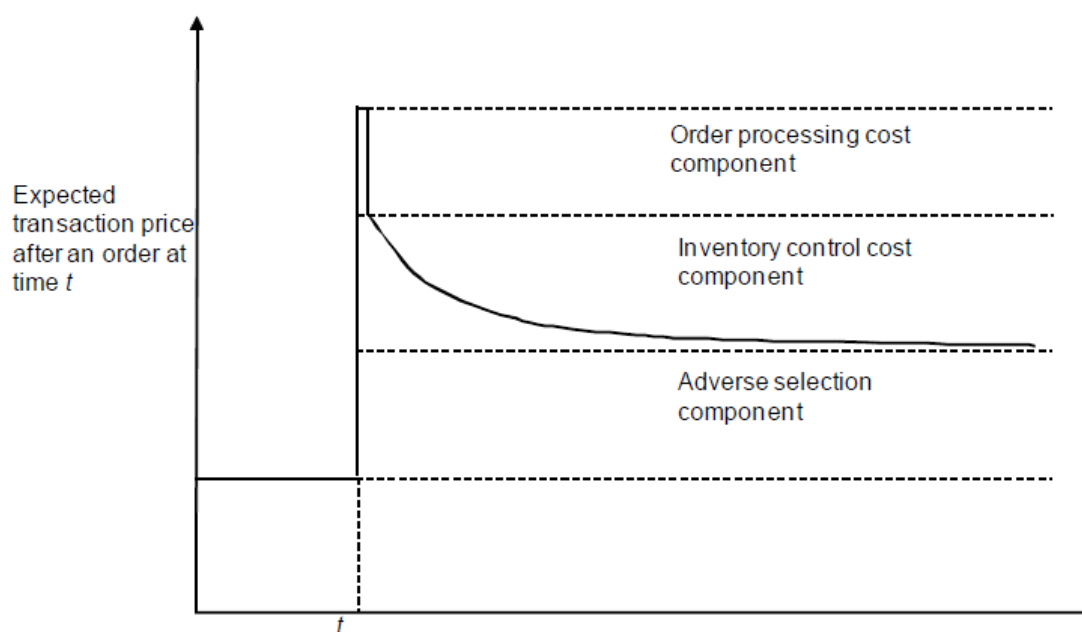
- 1 Depth is driven by mostly same factors as liquidity: adverse selection and inventory risk
 - Although the effect is, in general, less obvious with order costs and imperfectly competitive dealers
- 2 Kyle model demonstrates how adverse selection can drive limited depth
 - Call auction market where dealer has to cover the order imbalance
 - Informed traders face a trade-off: trade more vs trade at better price
 - see textbook for Kyle model with inventory risk.

Today

Empirical estimation of liquidity

- Earlier we looked at how to estimate the spread, but without a theory for what drives it
- Today: talk about estimating drivers of the spread
 - Knowing what drives illiquidity helps understand how to eliminate (or exploit) it
- Note: most estimates mentioned today are pretty old, and I could not find a lot of more recent ones – a fine thesis topic!

3



4

Measured price impact and the theories

- Parameters of interest:
 - λ : price impact related to information
 - β : price impact related to MM risk aversion
 - γ : order-processing cost (including dealer profits!)
- Data:
 - Transaction prices p_t , net market order flow q_t , order sign d_t

5

How to estimate?

- Consider a market with [adverse selection](#) and [order costs](#).
- Suppose prices are ex post efficient (apart from order costs), then:

$$p_t = \mu_t + \gamma d_t$$
$$\Rightarrow \Delta p_t = \mu_t - \mu_{t-1} + \gamma \Delta d_t$$

- and market valuation μ_t evolves due to order flow q_t and other public news ε_t :

$$\mu_t = \mu_{t-1} + \lambda q_t + \varepsilon_t$$

- Then

$$\Delta p_t = \lambda q_t + \gamma \Delta d_t + \varepsilon_t$$

6

?

- ? estimate

$$\Delta p_t = \gamma_0 \Delta d_t + \gamma_1 \Delta q_t + \lambda_0 d_t + \lambda_1 q_t + \epsilon_t \quad (5.7)$$

jointly with trade directions d_t .

- Accumulated order flow affects price level via λ , cyclical order flow gives γ in the style of Roll's measure (2.15)
- On NYSE data from early 80s, they find $\gamma_1 = \lambda_0 = 0$ and estimate $\gamma_0 = .0465$, $\lambda_1 = .0102$ for average stock in sample (so a \$1000 trade pays \$56.7 total in trading costs and permanently shifts the price by \$10.2)

7

Issues

- Apart from a few ad hoc concerns...
 - not much data (20 stocks, 800 transactions each)
 - poor dataset (neither d_t , nor quotes are observed; prices p_t were rounded to \$1/8 back then)
 - questionable specification (why have both λ_0 and λ_1 ?)
- ...there is the issue of ignoring [inventory costs](#).
- In the short run, the effect of MM risk aversion would be similar

$$\Delta p_t = \gamma \Delta d_t + (\lambda + \beta) q_t + \epsilon_t \quad (5.14)$$

- λ and β are not separately identified.

8

Including inventory costs

- ? : order flow is auto-correlated, so part of the order is not 'news' but only moves inventory
- ? : suggest

$$q_t = \phi q_{t-1} + \eta_t \quad (5.17)$$

- Then (5.14) becomes Derivation

$$\Delta p_t = \gamma \Delta d_t + \lambda(q_t - \phi q_{t-1}) + \beta q_t + \epsilon_t \quad (5.21)$$

- They estimate the model for 20 major NYSE stocks (mostly ignoring volumes)

$$\phi = -0.74 \quad \lambda = 9.59\%S \quad \beta = 28.65\%S \quad \gamma = 61.76\%S$$

- Also: greater β for larger transactions

9

Including inventory costs (2)

- ? find that λ is higher in the morning, and $\beta + \gamma$ higher in the afternoon
 - Morning trading reveals all the information accumulated during off-market hours. Evening trading driven by traders' desire to close open positions
 - ? : closing auctions account for 7.3% of daily trading volume, but contribute nothing to price discovery. Closing prices deviate a lot from midquotes, but revert back quickly. Why? Probably because traders rush to close their positions before EoD. But also in part because closing auctions are more closed and are of lower quality, see ?.
- ? had data on a FOREX dealer's inventory, and found β very large
 - Probably also because FX dealers really don't like to hold overnight positions

10

Long-run impact

- Different dynamic approach: ? uses the long-run impact of an order q_{t+1} on prices $p_{t+T} - p_t$ to identify the informational part of the trade
 - The 'impulse response' of prices to orders
 - Opens door to richer time series analysis of transactions data
- Findings: short-run effect of a trade $\approx 2\%$ of the price
- long-run effect of a trade ($T = 5$ trades is "long enough") $\approx 1\%$
 - greater LR impact for stocks with lower capitalization
 - related: ? find that stocks with lower analyst coverage are less liquid

11

Depth impact

- ? do analysis similar to ? with data from Paris Bourse, and taking trade quantities into account
- Find that LR price impact is between 25% and 115% of the spread (depending on estimation method)
- Find that adverse selection costs increase with trade size...
 - aligns with Kyle model
- ...while the converse is true for order processing costs. Possible reasons:
 - costs are per-transaction rather than per-unit
 - large orders come from institutional traders that get better terms of trade? (However: Paris Bourse was a pure-LOB market, no dealers, so this is probably not the reason)

12

Probability of Informed Trading

- ? : use GM-type model to estimate the prob. of informed trading
- On any given trading day, w.p. α there is some information event
 - good or bad
 - observed by informed traders, not by dealer
- Within the day, {informed traders, uninformed sellers, uninformed buyers} arrive as Poisson process with intensity $\{\epsilon_i, \epsilon_s, \epsilon_b\}$
 - Probability of observing n traders of a particular type over the trading day is

$$e^{-\epsilon} \frac{\epsilon^n}{n!}$$

13

Probability of Informed Trading

- Data for the number of buys and sells allow for estimation of these parameters with **maximum likelihood**
- ? estimate the 'probability of informed trading' associated with any given order

$$PIN = \frac{\alpha \epsilon_i}{\epsilon_b + \epsilon_s + \alpha \epsilon_i} \quad (5.27)$$

- In NYSE data from 1983 to 1998:
 - median PIN = 19%
 - for 90% of stocks, PIN is between 10% and 30%
 - greater for small-cap stocks, positively correlated with spread and price volatility
- ? show that PIN is higher when markets are more anonymous

14

Conclusion

Using the insights from previous lectures we can estimate the importance of different components of the spread

- Perhaps surprisingly, order costs are by far the largest cost (but estimated on major stocks)
- Adverse selection is a smaller, yet significant factor
 - is more of an issue in small-cap stocks

15

Homework

- Read the Economist article on the corporate bond market. Discuss the following questions:
 - 1 How does corporate bond market liquidity differ from the stock market liquidity? Why?
 - 2 Why do investors' liquidity expectations matter?
 - 3 How do investors form their expectations of liquidity?
 - 4 Can we *measure investors' expectations* of liquidity?
- Fill in the midterm evaluations if you have any comments (link on absalon)
- Start looking at problem set 1

16

Next week

- Analyze limit order markets: what is the difference to what we have done so far?
- Talk about the role of the 'ticks', the priority rule, the interaction between dealers and LOBs, and a way to interpret limit orders

17

Derivation of (5.21)

Suppose $q_t = \phi q_{t-1} + \eta_t$.

- **Adverse selection.** Notice that the market valuation now evolves as

$$\mu_t = \mu_{t-1} + \lambda[q_t - \mathbb{E}[q_t|\Omega_{t-1}]] + \epsilon_t.$$

(Only non-expected part of order flow carries information.)

- **Price.** The price equation then becomes

$$p_t = m_t + \beta q_t + \lambda[q_t - \mathbb{E}[q_t|\Omega_{t-1}]] + \gamma d_t,$$

where $m_t = \mu_t - \beta z_t$ is the 'mid-price' from the Stoll model (or GM/Kyle models with inventory risk)

Derivation of (5.21)

- Use $q_t = \phi q_{t-1} + \eta_t$ to get $\mathbb{E}[q_t | \Omega_{t-1}] = \phi q_{t-1}$.
- Take first differences of price equation

$$\Delta p_t = \Delta m_t + \lambda[\Delta q_t - \phi \Delta q_{t-1}] + \beta \Delta q_t + \gamma \Delta d_t$$

- Notice that $\Delta z_t = -q_{t-1}$ from market clearing condition. Then

$$\Delta m_t = \Delta \mu_t - \beta \Delta z_t = (\lambda + \beta)q_{t-1} - \lambda \phi q_{t-2} + \epsilon_t.$$

- Substitute in to get

$$\Delta p_t = (\lambda + \beta)q_t - \lambda \phi q_{t-1} + \gamma \Delta d_t + \epsilon_t.$$

[Back](#)

Financial Markets Microstructure

Lecture 9

Limit order book, part 1
Chapter 6.1-6.2 of FPR

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Spring 2025

Last time

- The Kyle model helps us analyze market depth:
 - Hence, it tells us something about how adverse selection causes spread to vary with trade size
 - The model has batch clearing instead of the single-unit market of GM
- We can use insights from theory to estimate the importance of different components of the spread
 - Perhaps surprisingly, order costs are by far the largest cost (but estimated on major stocks)
 - Around 19% of trading is informed
 - Adverse selection is stronger for less liquid/small-cap stocks

Homework

- Read the Economist article on the corporate bond market.
Discuss the following questions:
 - 1 How does corporate bond market liquidity differ from the stock market liquidity? Why?
 - 2 Why do investors' liquidity expectations matter?
 - 3 How do investors form their expectations of liquidity?
 - 4 Can we *measure investors' expectations* of liquidity?

3

LOB Markets: Liquidity Demand

- Most markets are order-driven these days
 - Some exchanges combine LOBs and dealers
- There is also conceptual convenience in starting with dealer models...
- “Dealer” is an abstraction of the “market”:

Trader interacting with the dealer $\approx$ Trader interacting with the market

So for a **liquidity demander** who submits market orders, it does not really matter who to trade against.

8

LOB Markets: Liquidity Providers

But **liquidity provision** works quite differently in LOB markets:

- 1 Traders have a choice between **demanding and supplying liquidity** (market vs limit orders) – so LO should be no worse than MO for trading
- 2 LO give better prices but suffer from execution risk
- 3 Conditional on electing to trade via LO, traders face a **different informational environment** from the dealer (dealer knows more about what's happening in a market)
- 4 Also there's discriminatory pricing: LO execute each at their posted price, while a dealer can clear everything at uniform price.

We look at the two latter issues today; two former next time.

9

LOB Markets: Modelling Issues

A limit order is (t, x, p) : submission time t , quantity x , price p .

Modelling LOBs properly is very difficult, many factors to account for:

- 1 Traders' **action set** is large: submit buy/sell LO/MO, how much, choose price if LO
- 2 **Dynamics** is relevant: choose when to submit, when to cancel/resubmit (since LO might not execute)
- 3 **State** of LOB is rich: trader's choice depends on other LOs in the book (though not all of them might be observable)
- 4 Private information affects not just valuation, but **expectations** about LOB evolution...

So we'll look at very simple models that explore different slices of this problem.

10

? model

- Begin by looking at ? model
 - (Probably) the first model of LOB markets
 - Does not capture all of traders' choices
 - Consider it to be "one step towards LOB markets from Kyle model".
- Questions simple:
 - What drives the prices? Are they efficient?
 - How are prices different from dealer markets?
 - How is LOB depth determined?

12

Continuous model: Limit order book (asset supply)

- **Single asset:** Unknown value v with cdf $G(v)$
- **Limit order book:**
 - Focus on the ask side of the market (limit orders to sell vs market order to buy)
 - Assume a continuum of competitive limit traders, each submits an infinitesimally small order
 - Let $p(q) \equiv$ price of q th unit (increasing in q due to price priority)
 - A market buy order of size q will 'walk up the book' until the final bit of it is cleared at price $p(q)$
 - The entire payment for buying volume q is then $P(q) \equiv \int_0^q p(\tilde{q}) d\tilde{q}$.
 - The average price per unit is $\bar{p}(q) \equiv \frac{P(q)}{q}$ – the (inverse) **supply curve**
- Bliz quiz: in the Kyle model, did the dealer announce **marginal price** $p(q)$ or **average price** $\bar{p}(q)$ schedule?

13

Continuous model: Market orders (asset demand)

- **Single market[-order] trader** per period (as in GM/Kyle)
- Abstract view: assume some total utility $U(x|v)$ from buying x units, increasing in v , increasing and concave in x , and $\frac{\partial^2 U(x|v)}{\partial x \partial v} > 0$.
- Trader then **demands quantity x** that solves

$$\max_x \{U(x|v) - P(x)\} \quad \Rightarrow \quad U'(x|v) = P'(x) = p(x)$$

- If trader was perfectly informed, then $U(x|v) = v \cdot x$
- Assume again some superposition of informed and uninformed traders, $U(x|v)$ is average over types
- Abusing notation, $U'(x|v)$ is average/representative trader's marginal valuation for x th unit of the asset
- The cross-derivative implies that $\mathbb{E}[v|U'(x|v) = p]$ for a given x is strictly increasing in p
 - the fact that the trader stopped at a higher marginal price p suggests higher v

14

Continuous model: Market makers

- **Limit[-order] traders** are competitive, post limit orders
 - Think of a continuum of traders, each posts a limit order for one (infinitesimally small) unit of the asset
- Limit price $p(q)$ quoted for the q -th unit is relevant (=is traded against) iff

$$x \geq q \iff U'(x) \geq p(q)$$

- Hence, if supply is competitive then in equilibrium,

$$p(q) = \mathbb{E}[v|x \geq q]$$

(as before - price of trade equals the expected value of the traded asset. The conditioning event is the interesting part)

15

Continuous model: Equilibrium

The assumed correlation of v and x implies that

$$p(q) = \mathbb{E}[v|x \geq q] > \mathbb{E}[v|x = q] \quad (1)$$

(to compare, in the Kyle model we had **what pricing schedule?** $\bar{p}(q) = \mathbb{E}[v|x = q]$)

Implications

- Thus, market makers (ex post) profit on the sale of the last units
- At small realized trades, $q \simeq 0$, MM always profit
- Even with continuous prices, there is a non-zero **inside spread** between ask and bid prices as the order size goes to zero (contrast with Kyle)
- After a trade of size q , new expected asset value is below $p(q)$.
New asset valuation is $\mu = \mathbb{E}[v|x = q] < \mathbb{E}[v|x \geq q] = p(q)$. Often new limit orders will be posted below $p(q)$ – price reversal.

16

Conclusion

- First look into LOB markets using ?.
- Limit traders act in the same capacity as the dealer did before
 - but face different **informational environment**
 - so act differently
 - which leads to different market outcomes
- Competitive limit traders may get positive or negative profit ex post, depending on the order they trade against, but get zero profit on average.

17

Next time

- What happens if we restrict prices to discrete ticks?
- Many aspects of market design (tick size, priority rules) are all double-edged swords, swing them carefully
- (maybe) Dynamic analysis: traders can choose between limit and market orders

Financial Markets Microstructure

Lecture 10

Limit order book, part 2

Chapter 6.2-6.3 of FPR

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Last time

- **Glosten model**: see how the behavior of competitive liquidity providers in a LOB is different from dealers' behavior.
- Pricing rule: **marginal price** of q th unit (on the ask side) is

$$p(q) = \mathbb{E}[v | x \geq q].$$

- Reminder: in a dealer market, average price when q units are traded is

$$\bar{p}(q) = \mathbb{E}[v | x = q].$$

Ticks

- The version of the Glosten model we've seen last time outlines some basic differences between LOB and dealer markets
 - Adverse selection affects prices differently
- It neglects the **discreteness of prices**:
 - Often prices are discrete and must lie at a tick – tick size is the increment b/w prices
 - E.g. at NYSE it was \$1/8 for stocks with prices over one dollar until June 1997, when, under regulatory pressure, it was reduced to \$1/16 and finally, in 2000, to one cent.
- Q for today: how does this discreteness (and tick size in particular) affect market outcomes?
 - Prioritized limit orders become profitable when there are ticks (since no 'marginal undercutting')

4

Discrete Glosten model: Setup

- **Asset**: Continue with single asset with value $v \sim G$
- **Market order** x correlated with v (reminder: notation different from the book)
 - Unconditional c.d.f. $F(x)$
 - Again, focus on the ask side of the book, $x > 0$
- **Discrete price grid**
 - A_1 is lowest price tick above μ
 - $A_k - A_{k-1} > 0$ is the tick size
- **Limit orders**
 - *Time priority*: first posted, first executed
 - *Display cost*: C per unit (paid regardless of whether order executes)
 - Let q_k denote the *cumulative* volume supplied (depth) at prices up to A_k

5

Discrete model: Equilibrium

- **Competition:** Limit orders are supplied at each tick until the **last order earns zero profit**
- **Zero-profit condition:**

$$\mathbb{P}(x \geq q_k) \cdot [A_k - \mathbb{E}[v|x \geq q_k]] - C = 0,$$

solved by

$$A_k = \underbrace{\mathbb{E}[v|x \geq q_k]}_{\text{Adverse selection}} + \underbrace{\frac{C}{\mathbb{P}(x \geq q_k)}}_{\text{Execution risk}}$$

(Though we actually want to solve for endogenous depth q_k given exogenous price ticks A_k)

6

Discrete Glosten model: comments

- The pricing rule seems to be exactly the same as in the continuous model...
- ...but this is only for the marginal units!

Either way, let's now look at a few examples to practice applying this pricing rule!

7

Example 1: Setup

- **Asset.** Let g be the marginal distribution of G and

$$g(v) = \begin{cases} 1/2 & \text{if } v = v^H; \\ 1/2 & \text{if } v = v^L, \end{cases}$$

with $v^H = \mu + \sigma$ and $v^L = \mu - \sigma$.

- **Traders.** Single trader, who uses a market order.

- Prob. π : risk-neutral speculator (**S**) who knows v
- Prob $1 - \pi$: noise trader (**N**) who buys/sells with equal probability, and uses large (x_L) or small ($x_S < x_L$) order with equal probability:

$$\mathbb{P}(x = x_S | \mathbf{N}) = \mathbb{P}(x = x_L | \mathbf{N}) = \mathbb{P}(x = -x_S | \mathbf{N}) = \mathbb{P}(x = -x_L | \mathbf{N}) = 1/4$$

- **No display cost.** Let $C = 0$
- **Continuous prices.**

8

Example 1: Equilibrium

- **Equilibrium:** Look for eq. with $q_1 = x_S$ and $q_2 = x_L$ for some ask prices $\mu < A_1 < A_2 < v^H$. In this prb: q_1, q_2 given, we look for A_1, A_2 .
- **Speculator:**
 - If $x > x_L$, speculator reveals himself – never optimal
 - Since $v^H > A_1, A_2 > \mu$, if $v = v^H$ then speculator buys x_L units; (and if $v = v^L$ then sells x_L).
Why is it not optimal to shade the order?
 - Shading order (strategically restricting trade size) is not optimal for the speculator because buying more does not worsen the price of the previous units, unlike in dealer mkt
- **Price.** In equilibrium, price must equal $\mathbb{E}[v | x \geq q_k]$:

$$A_1 = \mathbb{E}[v | x \geq x_S] = \mu + \pi\sigma,$$

$$A_2 = \mathbb{E}[v | x \geq x_L] = \mu + \frac{2\pi}{1 + \pi}\sigma$$

- Obvious that indeed, $\mu < A_1 < A_2 < v^H$. Thus: equilibrium.

9

Example 1: Comment

- Not the best example for discreteness:
 - The example does *not* assume fixed ticks...
 - But they arise endogenously in equilibrium...
 - Due to discreteness of noise traders' strategy
 - (A very artificial assumption)
- But focus on adverse selection leading to limited depth:
 - Price increases in order size
 - Not due to informed trader having stronger info, as in Kyle model
 - But due to noise traders' order becoming (relatively) less likely

10

Example 2: Setup

- **Market orders:** Exponential distribution, $f(x) = \frac{\theta}{2} e^{-\theta|x|}$
- **Asset.** Assume 'price impact' equation $\mathbb{E}[v|x] = \mu + \lambda x$, where $\lambda > 0$ is a constant measuring informativeness of order flow
 - Thus, we are taking a short-cut and modeling adverse selection in a 'reduced form': rather than modeling the informed traders, we model their price impact
- There is some order submission cost C
- **Goal:** find the equation connecting A_k and q_k (given arbitrary tick grid $A_1, \dots, A_k, \dots$)

11

Example 2: Equilibrium

- Focus on **ask side**: $q_k > 0$. For $x \geq q_k$:

$$\begin{aligned} f(x|x \geq q_k) &= \frac{f(x)}{\mathbb{P}(x \geq q_k)} \\ &= \frac{f(x)}{\int_{q_k}^{\infty} f(x) dx} \\ &= \frac{\frac{\theta}{2} \cdot e^{-\theta x}}{e^{-\theta q_k}/2} \quad (|x| = x \text{ since } x \geq q_k > 0) \\ &= \theta \cdot e^{-\theta(x-q_k)} \\ &= e^{\theta q_k} [\theta \cdot e^{-\theta x}] \end{aligned}$$

12

Example 2: Equilibrium (2)

- The expected value at tick k becomes

$$\begin{aligned} \mathbb{E}[v|x \geq q_k] &= \mu + \lambda \mathbb{E}[x|x \geq q_k] \\ &= \mu + \lambda \int_{q_k}^{\infty} x \cdot f(x|x \geq q_k) dx \\ &= \mu + \lambda e^{\theta q_k} \int_{q_k}^{\infty} x \cdot \theta \cdot e^{-\theta x} dx \\ &= \mu + \lambda e^{\theta q_k} \left\{ [-x \cdot e^{-\theta x}]_{q_k}^{\infty} - \int_{q_k}^{\infty} -e^{-\theta x} dx \right\} \quad (\text{int. by parts}) \\ &= \mu + \lambda e^{\theta q_k} \left\{ [0 + q_k \cdot e^{-\theta q_k}] - \frac{1}{\theta} [0 - e^{-\theta q_k}] \right\} \\ &= \mu + \lambda \left(\frac{1}{\theta} + q_k \right) \end{aligned}$$

13

Example 2: Equilibrium (3)

- Hence, the ask price at tick k can be found by solving

$$\underbrace{\mathbb{P}(x \geq q_k)}_{e^{-\theta q_k}/2} [A_k - \underbrace{\mathbb{E}[v|x \geq q_k]}_{\mu + \lambda(\frac{1}{\theta} + q_k)}] - C = 0,$$

which gives

$$A_k = \mu + \lambda \left(\frac{1}{\theta} + q_k \right) + \frac{2C}{e^{-\theta q_k}}.$$

- (Again: we actually need the opposite – find q_k for a given tick A_k – but it is hard to get a closed-form expression for that.)

14

Glosten: Empirical evidence

- ? estimates Glosten model (in a form similar to example 2 above) via GMM, using intraday snapshots of LOB from Stockholm Stock Exchange and data on market orders
- Estimates the info content of market orders vs actual pricing schedules, so effectively the $\mathbb{E}[v|x \geq q]$ inferred from pricing schedule and the actual $\mathbb{E}[v|x = q]$ from the price dynamics.
- Zero profit condition is tested and rejected: LOB not deep enough to drive average expected profits to zero
- Also, estimated order execution costs are negative for the best bid and ask – i.e., those limit traders have some intrinsic preference for trading (although these days many exchanges do offer negative execution fees to limit traders to incentivize liq-ty provision)

15

Glosten model: Conclusion

- Limit traders act in the same capacity as the dealer did before
 - but face different **informational environment**
 - so act differently
 - which leads to different market outcomes
- With discrete ticks and time priority, even competitive limit traders can get positive expected profits

16

Market design

- There are many dimensions in which legislation or exchange rules can regulate trade
- Today's phrase of the day: "**unintended consequences**"
 - Attempts to mitigate a particular inefficiency may have far-fetching consequences
 - We will look at a few examples

18

Tick size

- Assume **time priority** is the second order after price priority
 - I.e., first limit order posted at tick executes first
- Profit of the limit trader at price A_k is:
 - Zero for the marginal (last) limit order at A_k
 - Strictly positive for inframarginal orders because (1) order executes with higher probability and (2) info content of mkt order is weaker
- Q: what happens if we **change the tick size**?

19

Tick size

- Driving away limit traders intuitively also has dynamic repercussions
- ? explored the NYSE 1997 case (tick size from \$1/8 to \$1/16)
 - Trading costs decreased for small orders
 - Unclear for large orders
 - Aligns with our predictions (smaller spread, smaller depth)

20

Priority rules

- With **pro-rata** allocation (limit orders at given tick executed proportionally to their size), as opposed to **time priority**:
 - The expected profit of all orders at price A_k must be zero (as opposed to strictly positive)
 - So execution probabilities must be lower for all orders
- Lower profits lead to the same consequences as with reducing tick size
 - Less liquidity provision in the long run
 - Lower LOB resiliency (slower replenishment)
- Pro-rata allocation rule used in the electronic futures markets for the leading short-term interest rate and for the two-year U.S. Treasuries.

21

Hybrid market

Hybrid market

- Suppose a **dealer** can compete with the limit order book, as follows
- The dealer may observe trade size x before serving the order (and can fulfill the order before it is matched against LOB)
- Can profit by pricing at $\mathbb{E}[v|x = q]$ rather than an average of $\mathbb{E}[v|x \geq y]$ for $y \leq q$ which is used by the LOB
- Especially profitable on small trades
- But the existence of such a dealer invalidates our analysis of the LOB
 - Profitable limit orders are being picked off
 - So limit traders would gain negative profits if they follow the old strategy $\Rightarrow$ incentive to change their strategy (or quit the market)
 - Liquidity demanders might also change their trading behavior

22

Example 1 with hybrid market

- Example 1 from before continued. Assume an uninformed dealer receives order x . Can either send order to LOB or execute himself (at better price)
- Focus on ask side. Let A_k^H be the hybrid ask price. When dealer observes $x = x_S$, he knows trader is noise trader and thus $\mathbb{E}[v|N] = \mu$.
 - Can execute order at just below A_1^H and earn profit $A_1^H - \mu$.
- Hence, only large orders $x = x_L$ are sent to LOB. LOB traders will expect this, and will price as if any order arriving to the LOB is large:

$$\mathbb{E}[v|x \geq x_S] = \mathbb{E}[v|x \geq x_L] = \mu + \frac{2\pi}{1+\pi}\sigma,$$

and thus $A_1^H = A_2^H = \mu + \frac{2\pi}{1+\pi}\sigma$.

- $A_1^H > A_1$ and $A_2^H = A_2$: hybrid market less liquid than normal market

23

Hybrid market: conclusions 1

- To be fair: adding a dealer to LOB market...
 - decreases liquidity in “good times”, when there would’ve been a thick LOB
 - but can help in bad times: if LOB is empty then adding a dealer has no adverse effects and will actually increase liquidity
 - So in the end, adding a dealer is like a [liquidity insurance](#) for the market
- Banks and other systemic internalizers with their dark pools can act as dealers too, picking off the good orders and forwarding the rest to the market.
 - Recent regulation heavily restricted banks and platforms in how much they can internalize their clients’ trades.
- More analysis of hybrid markets with risk-averse traders: see ?

24

Hybrid market: conclusions 2

Analysis of the example above relied on a bunch of implicit **assumptions** (which are not necessarily true):

- Assumed the dealer had time priority over (could undercut all of) the LOB. If MO-traders can trade against the LOB before the dealer can act, the conclusions are different.
- Assumed the dealer is a monopolist – competitive dealers would yield different predictions.
- Assumed the market order revealed enough information. If MO-traders split their orders (trade one unit at a time), dealers no longer have any advantage. (?)

25

Market Design: conclusion

- Regulation aimed at improving market liquidity can backfire by distorting agents' incentives

26

Next week

- Dynamic LOB analysis: traders can choose between limit and market orders

27

Homework

- Thinking in the framework of the discrete model: suppose tick size is actually zero; quotes can be placed in a continuous price space. Suppose that there is price priority. What then is the role of time priority, so that first-come quotes at identical prices are served first?
- Solve exercise 1 after ch.6 (pages 232-233) in the textbook. Note that you need to use Bayes' rule to assess the conditional distribution over v given a market order of size x (and work through slightly different notation)

28

Financial Markets Microstructure

Lecture 11

Limit order book (part 3)
Chapter 6.3-6.4 of FPR

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Last time

- First look into LOB markets using ? (continuous and discrete versions).
- Limit traders act in the same capacity as the dealer did before
 - but face different incentives
 - so act differently
 - which leads to different market outcomes.
- First dive into market design: the effects of tick sizes, priority rules, and dealer interventions

Today

- Dynamic analysis of LOB markets:
 - How do traders choose between limit and market orders?

3

Tick size and time priority

Suppose that there is no tick; quotes can be placed in a continuous price space. Suppose that there is price priority. What is then the role of time priority, so that first-come quotes at identical prices are served first?

5

Dynamic analysis

- ? shows how price schedule is formed under adverse selection
 - But this analysis is **static** – traders have no choice between limit/market orders = making/taking liquidity
- Proper **dynamic** analysis is quite difficult
- Choice between **MO/LO** depends on:
 - 1 **Price differential** (=bid-ask spread)
 - 2 **Execution probability** of a limit order
 - ...which depends on the current state of LOB
 - But also on next trader's choice between MO/LO
 - ...which depends on execution probability ...
 - 3 **Adverse selection** faced by limit orders
 - ...which depends on *who* will trade against my limit order
 - ...which depends on next trader's choice between market/limit order depending on their pvt info ...

8

Literature

- We will look at a very simple version of the ? model
 - fixed ask/bid prices, endogenous choice between MO and LO
 - The book for some reason attributes it to ?, which is a quite different model of LOB markets. Old class materials may also call this “the Parlour model”
- Other models:
 - ? and ?: models with heterogeneous discount factors; patient traders choose LO, impatient choose MO
 - ?: combine all of the above in a single tractable model
 - Worth noting: limit orders are effectively free call/put options, so could be priced as such (?)

9

Foucault Model: setup

- **Exogenous prices.** **Exogenous** bid and ask prices $A > v > B$
- **Strategic traders.** One trader arrives per period, chooses b/w limit or market order (one unit)
 - LO valid for one period, then automatically cancelled. Choice depends on prob. of LO being executed, i.e. 'hit' by a MO from the next trader
 - Valuation: $v + y$, where $y \sim U[-Y, Y]$ is idiosyncratic valuation, private and independent across traders. v is known and common to all. Note: book assumes a binary distr-n $y \in \{-Y, Y\}$.
- **Noise traders.** Numerous and patient. Always submit limit orders at A (LSell) and/or B (LBuy). Prioritised *after* the strategic trader.
- **Goal:** derive the traders' optimal strategy

10

Profits

- Let $P_M^B(P_M^S)$ be the (endogenous) probability that the next trader submits a market buy (sell) order

Market sell: $B - (v + y)$

Limit sell: $(A - v - y)P_M^B$

Limit buy: $(v + y - B)P_M^S$

Market buy: $v + y - A$

- Limit order more attractive when more likely to execute: IRL (not in this model), this leads to automatic tendency for the limit order book to be replenished (resiliency)

11

Analysis

- Look for stationary eqm where A, B, P_M^B, P_M^S are constant
- From the trader profits, we see that there must exist $\bar{y}, \underline{y}$ and $\hat{y}$ such that the best response (optimal order) of the trader is

$$BR = \begin{cases} MSell & \text{if } -Y \leq y \leq \underline{y} \\ LSell & \text{if } \underline{y} \leq y \leq \hat{y} \\ LBuy & \text{if } \hat{y} \leq y \leq \bar{y} \\ MBuy & \text{if } \bar{y} \leq y \leq Y \end{cases}$$

Traders with greater urgency/need to trade (extreme y) use market orders; “patient” traders use limit orders.

12

Equilibrium

- From trader's BR:

$$P_M^S = \mathbb{P}(-Y \leq y \leq \underline{y}) = \frac{\underline{y} + Y}{2Y}$$

$$P_M^B = \mathbb{P}(\bar{y} \leq y \leq Y) = \frac{Y - \bar{y}}{2Y}$$

- At $\underline{y}$, indifferent btw $MSell$ and $LSell$. At $\hat{y}$, indifferent btw $LSell$ and $LBuy$. At $\bar{y}$, indifferent btw $LBuy$ and $MBuy$. Thus:

$$B - (v + \underline{y}) = (A - v - \underline{y}) \frac{Y - \bar{y}}{2Y} \quad (1)$$

$$(A - v - \hat{y}) \frac{Y - \bar{y}}{2Y} = (v + \hat{y} - B) \frac{\underline{y} + Y}{2Y} \quad (2)$$

$$(v + \bar{y} - B) \frac{\underline{y} + Y}{2Y} = v + \bar{y} - A \quad (3)$$

13

Equilibrium (2)

- Solving (1)-(3) we obtain the thresholds.

If we set $v = 0$, $Y = 2$, $A = 1$ and $B = -1$ they become

$$\underline{y} = \frac{1}{2}(3 - \sqrt{33}) \simeq 1.4$$

$$\hat{y} = 0$$

$$\bar{y} = \frac{1}{2}(\sqrt{33} - 3) \simeq -1.4$$

- Thus, in equilibrium the probability of a market buy/sell order next period is $\frac{2-1.4}{2.2} = \frac{-1.4+2}{2.2} = 0.15$
- Given a large spread ($A - B = 2$), limit order traders are willing to accept a low execution probability (15%) in order to obtain better price

14

Dynamic analysis: Conclusion

- Trade occurs when seller and buyer meet and agree on terms
 - Exchange brings sellers and buyers together
 - Impatient traders search more actively (use market orders)
 - Patient traders are more passive, offer liquidity (use limit orders)
- Ordinary traders can use limit orders:
 - 1 to compete with market makers (and profit from providing liquidity as a self-contained activity)
 - 2 or to reduce their trading costs by accepting some non-execution risk. E.g., ? : avg trading cost of a large institutional trader is $\sim 0.015\%$ (a couple of orders of magnitude smaller than quoted spreads), arguably due to using LOs to minimize price impact.
- Limit order book is dynamic: influence the choice of order
- In reality, many features to think about when placing orders

15

Dynamics with adverse selection

- Our analysis above focuses on non-execution risk, ignoring **adverse selection**
 - which can arise due to privately informed investors picking off the limit orders
 - or even **public news** arriving over time – then all future traders are more informed!
 - see ch. 6.4.3 & 6.6 of the textbook or [the extra slides](#)
- There is not too much interaction between the two, but there's an interesting dynamics of spread w.r.t. volatility of μ_t ($= v_t$ in the book)
- Read this at home.

17

Public information: issues and solutions

- **Monitoring costs:** if limit traders do not monitor the news constantly, their orders will get picked off. Monitoring reduces adverse selection, but is costly.
- **Pegged limit orders:** tied to another security or another market, automatically repriced when that price changes. Another insurance device, available on selected exchanges.
- **Algorithmic trading:** allows to automatically monitor relevant variables and reprice limit orders as needed. But at the same time increases the speed at which orders can be picked off, so cuts both ways.

18

Informed limit trading?

- But of course, the best way to avoid being picked off is to be already informed.
- Even the full version of Foucault model only deals with public info – there are no privately informed traders.
- But data shows that LOs are informative (???)
- Even more: informed traders might be using more LOs than MOs (?)
- ? have a LOB model in which informed traders use both LOs and MOs, depending on how full the book is.

19

LOB: conclusion

- A lot of stuff going on in LOBs, difficult to analyze, but we tried.
- See ? for a slightly more detailed review of literature on LOBs.

20

Exercise for next week

- Solve exercise 5 after chapter 6 (page 235) in the textbook on the effect of fees charged for limit orders and market orders in Foucault model

21

Dynamic model with adverse selection

- Consider now the Foucault model with **public news**: the market valuation evolves as $\mu_t = \mu_{t-1} + \epsilon_t$ where $\epsilon_t \in \{-\sigma, \sigma\}$ with equal probabilities.
- μ_t is observed in period t (but LOs from $t - 1$ cannot be cancelled or repriced). The period- t trader has valuation $\mu_t + y_t$
- Again, we look for a stationary equilibrium. In particular:
 - Suppose that $A_t - \mu_{t-1} = \mu_{t-1} - B_t = S$ is constant.
 - Suppose there exist $\underline{y}(\epsilon_t), \bar{y}(\epsilon_t), \hat{y}(\epsilon_t)$ that describe the trader's strategy in *each* period as a function of the value innovation ϵ_t
 - Thus there exist time-invariant P_M^S and P_M^B that characterize the probability of a next-period market sell/buy order

Dynamic model with adverse selection (2)

- Intuitively, $\underline{y}(\epsilon_t), \bar{y}(\epsilon_t), \hat{y}(\epsilon_t)$ are decreasing in ϵ_t :
 - higher ϵ_t means more favorable quotes for MB (A_{t-1} is relatively low),
 - less favorable for MS (B_{t-1} is relatively high)
- The interesting trade-off here is generated by σ : when σ increases,
 - the worse is the risk of being picked-off (so limit orders are less appealing),
 - but the lower is the execution risk (so limit orders are more appealing)

Dynamic model with adverse selection (3)

- To solve the model: the indifference conditions at $\underline{y}, \hat{y}, \bar{y}$ are

$$\begin{aligned} B_t - (\mu_t + \underline{y}(\epsilon_t)) &= (A_{t+1} - \mathbb{E}_t[\mu_{t+1}|MB] - \underline{y}(\epsilon_t))P_M^B \\ (A_{t+1} - \mathbb{E}_t[\mu_{t+1}|MB] - \hat{y}(\epsilon_t))P_M^B &= (\mathbb{E}_t[\mu_{t+1}|MS] + \hat{y}(\epsilon_t) - B_{t+1})P_M^S \\ (\mathbb{E}_t[\mu_{t+1}|MS] + \bar{y}(\epsilon_t) - B_{t+1})P_M^S &= \mu_t + \bar{y}(\epsilon_t) - A_t \end{aligned}$$

- Focus on the first and last equation. Use the definition of S :

$$\begin{aligned} -S - \epsilon_t - \underline{y}(\epsilon_t) &= (S - \mathbb{E}_t[\epsilon_{t+1}|MB] - \underline{y}(\epsilon_t))P_M^B \\ (S + \mathbb{E}_t[\epsilon_{t+1}|MS] + \bar{y}(\epsilon_t))P_M^S &= -S + \epsilon_t + \bar{y}(\epsilon_t) \end{aligned}$$

- Solve for $\underline{y}(\epsilon_t)$ and $\bar{y}(\epsilon_t)$. Determine S from $A_t = \mathbb{E}_t[\mu_t|MB]$ and $B_t = \mathbb{E}_t[\mu_t|MS]$. Notice adv. sel. only affects limit orders. [Back](#)

Financial Markets Microstructure

Lecture 12

Market Fragmentation
Chapter 7 of FPR

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Last week(s)

- Models of order-driven markets
- (Glosten's model): limit traders provide liquidity in a way similar but not same to dealers
- (Foucault's model): needier traders more likely to use MOs. Also LOB is resilient / self-replenishing
- Market design: measures directed at improving liquidity can backfire by distorting incentives

Today

- Analyze fragmented markets: i.e. multiple markets selling the same asset
- Look at fragmentation costs and benefits
- Compare these across batch markets and LOB markets
- This will give us an opportunity to revisit some of the models we have looked at previously
- Finally, we will look at regulation

3

History lesson

- A listing in the US used to confer near-exclusive trading rights
 - For instance, if a stock were NYSE-listed, almost all of the trading would occur on the NYSE
 - Until 1999, most stock options were only traded on the exchange where they were listed
- Many European countries used to require that stocks of their companies are only traded on local exchanges
- All of that has changed in the past 20 years
 - Today, many (at least high-cap) stocks are [cross-listed](#) on many exchanges
 - Even if a stock is not listed on a given exchange, it can be [admitted for trading](#) (factsheet from LSE on absalon clarifies the difference between the two)
 - Even listed stocks/assets can be often traded outside the regulated exchanges (dark pools)

5

Fragmentation and consolidation

- Refer to markets with multiple venues trading the same stock as **fragmented** (as opposed to consolidated)
- Regulation often tries to create ‘**virtual consolidation**’: make fragmented markets act *as if* they were consolidated
 - A central concept in the US is the *National Best Bid and Offer* (NBBO): the highest bid and lowest ask price in (any exchange within) the market at a given time
 - US regulation requires that trade always takes place at the NBBO (brokers should trade at best prices; exchanges often must forward orders to other exchanges if a better quote is available there)
- EU’s **ban** of countries’ “**concentration rules**” aims for a different kind of consolidation – one where all stocks can be traded on same exchange rather than spread around local exchanges.

6

Fragmentation and priority violations

- LOB has different kinds of priority: price, visibility, time
- In a fragmented market, no overall coordination mechanism, so all of these rules can be violated:
 - Violation of **visibility priority**. An undisplayed order at a price of 100 might be executed on exchange A even though there are quantities visible at 100 on exchange B
 - Violation of **time priority**. A limit order to buy at a price of 100 that was entered at 10:00 AM on exchange A might be filled before an order to buy at 100 that was entered at 9:30 AM on Exchange B
 - Violation of **price priority**. A limit order to buy at a price 100 on exchange A might be executed even though there is, at the same time, a limit order to buy at a price of 101 on exchange B. Known as a *trade through*

7

Other consequences

8

Platform competition reduces trading costs

- Competition among exchanges is a significant driver of trading fees and commissions
- Sometimes just the threat of entry is enough
- E.g. case of Dutch stocks:

- before 2003, most of Dutch stocks were traded on Euronext's NSC
- in May 2003, Deutsche Börse and LSE separately announced they wanted to enter this market
- Euronext's **order entry fee** was 0.3 EUR in Jan 2004...
- ...halved to 0.15 for limit orders on Apr 4, 2004...
- ...suspended for market orders on May 24, 2004 (LSE's EuroSETS launch day)...
- ...and finally waived completely for market and limit orders on Jan 31, 2005 together with reducing execution fees.
- (EuroSETS never really gained much traction)

9

Brokers' agency

- Traders rarely place their orders directly – they usually go to a broker (bank)
- When market is fragmented, brokers should ideally search all markets for the best price
 - (Not a trivial problem, given the existence of dark pools, crossing networks of unknown depth, and hidden orders and/or iceberg orders on standard exchanges)
- But **cost** of this search is incurred by **broker**, while **gains** are transmitted to **trader**.
 - So there is an agency problem
 - Exacerbated by payments brokers can receive from exchanges for routing all or some orders through this exchange
- Regulation (US) aims to solve this via
 - “best execution rules” imposed on brokers
 - no-trade-through/order-protection rules which say order must be routed automatically to trade at NBBO

10

Kyle model: refresher

To talk about liquidity and price impact, let us look at the effects of fragmentation within **Kyle model**.

Recall the baseline model from past lectures:

- Risky asset value $v \sim N(\mu, \sigma_v^2)$
- **Insider** observes v
 - Places market order x ; maximizes $\mathbb{E}[x(v - p)|x]$
- **Noise trader** demand $u \sim N(0, \sigma_u^2)$
- **Market maker** observes aggregate order flow $q = x + u$
 - Competitively prices asset at $p = \mathbb{E}[v|q]$ (Recall the difference here to what we did in the LOB model)

12

Kyle model: refresher (2)

- The insider uses a linear strategy $x = \beta(v - \mu)$
- Market makers observe $q = x + u = \beta(v - \mu) + u$. Then

$$p = \mathbb{E}[v|q] = \mu + \lambda q$$

- Solving the MM's problem we get price-impact parameter

$$\lambda = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2}$$

- The insider takes for granted the pricing rule $p = \mu + \lambda q$
 - Expected gain is $x(v - \mu - \lambda x)$ since $\mathbb{E}[u] = 0$
- Solving trader's problem gives $x = \beta(v - \mu)$ where $\beta = 1/(2\lambda)$

13

Kyle model: refresher (3)

- 'Match the coefficients' $\frac{1}{2\beta} = \lambda = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2}$ to get the equilibrium

$$\frac{1}{\lambda} = 2\beta = 2\frac{\sigma_u}{\sigma_v}$$

- Insider equilibrium trading:

$$x = \frac{\sigma_u}{\sigma_v}(v - \mu)$$

- MM makes zero profit: insider's gain is noise traders' loss

$$\underbrace{\mathbb{E}[x(v - p)]}_{\text{Insider gain}} = \underbrace{\mathbb{E}[u(\lambda u)]}_{\text{Noise loss}} = \lambda\sigma_u^2 = \frac{\sigma_v\sigma_u}{2}$$

14

Kyle model with a fragmented market

Now fragment the market in this Kyle model. Suppose:

- 1 Two noise trader groups: $u = u_1 + u_2$, u_i independent and $u_i \sim N(0, \sigma_{ui}^2)$
- 2 Noise traders trade in separate markets; competitive dealers/MMs present in each market (can only see the order in their market); cannot move between markets
- 3 One speculator trades in both markets (or two speculators trade each in their market – does not really matter).

15

Fragmented Kyle model: prices

- **Prices.** Since on each market $i = 1, 2$ we must have $p_i = \mathbb{E}[v|q_i]$,

$$\begin{aligned} p_i &= \mu + \lambda_i q_i = \mu + \lambda_i (x_i + u_i) \\ &= \mu + \lambda_i (\beta_i (v - \mu) + u_i) \\ &= \mu + \frac{\sigma_v}{2\sigma_{ui}} \left(\frac{\sigma_{ui}}{\sigma_v} (v - \mu) + u_i \right) \\ &= \mu + \frac{1}{2} (v - \mu) + \frac{\sigma_v}{2\sigma_{ui}} u_i \end{aligned}$$

- On average, price is the same in both markets. But in very short run prices may differ across markets

16

Fragmented Kyle model: volumes

- **Trading.** Informed trading is given by $x_i = \beta_i(v - \mu)$ with $\beta_i = \frac{\sigma_{ui}}{\sigma_v}$:

$$x_1 + x_2 = (v - \mu) \frac{\sigma_{u1} + \sigma_{u2}}{\sigma_v} > (v - \mu) \frac{\sigma_u}{\sigma_v} = x$$

- **More informed trading** in total under fragmentation than in consolidated case
- To see this, note:

$$\begin{aligned}\mathbb{V}(u) &= \sigma_u^2 = \sigma_{u1}^2 + \sigma_{u2}^2 = \mathbb{V}(u_1 + u_2) \\ &\Rightarrow \sigma_u^2 < (\sigma_{u1} + \sigma_{u2})^2 \\ &\Rightarrow \sigma_u < \sigma_{u1} + \sigma_{u2}\end{aligned}$$

17

Fragmented Kyle model: profits

- **Adverse selection costs.** Measure by loss of noise traders:

- The expected loss of group i is $\sigma_v \sigma_{ui}/2$: Total loss is

$$\frac{\sigma_v(\sigma_{u1} + \sigma_{u2})}{2} > \frac{\sigma_v \sigma_u}{2}$$

- **Fragmentation gives greater adverse selection loss**
- Of course, also greater profits for informed traders
- In the above, we compare one speculator (per market) under fragmentation to one speculator in the consolidated market.
If we have different speculators in the two markets who would compete in the consolidated mkt, the effect is even stronger.

18

Fragmented Kyle model: depth

- **Market depth.** Depth in each market lower than in consolidated market:

$$\lambda = \frac{\sigma_v}{2\sigma_u} < \min\{\lambda_1, \lambda_2\}$$

- What about **aggregate** depth? Say you want to trade X total.
 - When splitting optimally across two markets $X = x_1 + x_2$, you equalize expected marginal prices: $\mathbb{E}p_1 = \mu + \lambda_1 x_1 = \mu + \lambda_2 x_2 = \mathbb{E}p_2$ (solve a cost minimization problem to confirm this).
 - So $x_1 = \frac{\lambda_2}{\lambda_1 + \lambda_2} X$ and $x_2 = \frac{\lambda_1}{\lambda_1 + \lambda_2} X$ and trading moves prices in each market to $p_i = \mu + \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} X$ compared to $p = \mu + \lambda X$ in consolidated market. Can verify that $\frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} < \lambda$
 - **Fragmented market is deeper in aggregate** (that's why the speculation is more aggressive)

19

Fragmented Kyle model: price discovery

- **Price discovery:** what is the residual variance $\mathbb{V}(v|q)$?
We used the fact that everything is normal and $\mathbb{E}[q] = 0$ to obtain (see slides on Kyle)

$$v|q \sim \mathcal{N}\left(\mu + \frac{\mathbb{C}(v, q)}{\mathbb{V}(q)} q, \frac{\sigma_v^2}{2}\right)$$

- This still works in vector form with $\vec{q} = (q_1, q_2)$
- ...or with $\vec{p} = (p_1, p_2)$, since $p_i = \mu + \lambda_i q_i$
- Doing the magic, get $\mathbb{E}[v|\vec{p}] = \mu + \frac{2}{3}(p_1 - \mu) + \frac{2}{3}(p_2 - \mu)$ and $\mathbb{V}(v|\vec{p}) = \frac{\sigma_v^2}{3}$
- **Better price discovery in fragmented market** once we combine all the information.
- Because have more signals from which to learn about v
 - although again comparing "one speculator in two mkts" vs "one speculator in one mkt". Two competing insiders would yield **same price discovery** in consolidated market.

20

Fragmented Kyle model: liquidity provision

- In the analysis above, traders were chain-bound to their markets
- If many speculators could choose a market to participate in, a jointly optimal thing to do is to join different markets to avoid competing with each other.
 - But note that it is a prisoners' dilemma for them – it is always *individually* optimal to participate in all markets
- What if noise traders can choose?

21

Conclusion: fragmented Kyle model

Adverse selection costs

- Fragmentation is bad for noise traders (welfare)
- If noise traders coordinate, it can be stable that they gather around a less efficient platform
- Fragmentation *may* create extra depth

Aside

- Note that “fragmentation” may also mean time
- Above observations may explain why trade volume is often concentrated at specific times of day (early and late)
- Also an argument for batch trading versus continuous trading

22

Glosten model

Competing *limit order books* may also provide better aggregate depth

- To show this, use LOB model of section 6.2 with display cost $C > 0$, tick size $\Delta > 0$, **no adverse selection** (asset value is μ , only the non-execution risk is relevant)
- Recall: incoming order x , limit sell orders posted at cumulative quantity q and price A satisfy the zero-profit condition

$$0 = \mathbb{P}(x \geq q)[A - \mu] - C,$$

solved by

$$A = \mu + \frac{C}{\mathbb{P}(x \geq q)}. \quad (6.7)$$

24

Glosten model (2)

Now we make the following assumptions:

- **Fragmentation.** Limit orders are supplied in two markets, I and E (“incumbent” market and “entrant” market); display cost C same in both
 - At single available ask price A , denote cumulative limit sell orders by q^I and q^E
- **Market orders.**
 - Market order is *Buy* with chance $1/2$, and of size $x \sim F(x)$
 - With probability $1 - \gamma$, the entire order goes to I (never continues to E even if unfilled)
 - With probability γ , the incoming order is split:
 - With prob. $1/2$, order first goes to I , whatever remains goes to E
 - With prob. $1/2$, order first goes to E , whatever remains goes to I
- **Trade probability.** Equilibrium condition: zero profit for the marginal trader on each market. We thus need to calculate $\mathbb{P}(x_i \geq q^i)$ for each market i .

25

Glosten model: trading probabilities

- The execution probabilities for the marginal traders are as follows, where $\bar{q} = q^I + q^E$:

$$\mathbb{P}(x_I \geq q^I) = \frac{1}{2} \left[\underbrace{\left(1 - \gamma + \frac{\gamma}{2}\right) (1 - F(q^I))}_{I \text{ is executed first}} + \underbrace{\frac{\gamma}{2} (1 - F(\bar{q}))}_{I \text{ is executed second}} \right] \quad (7.11)$$

$$\mathbb{P}(x_E \geq q^E) = \frac{1}{2} \left[\underbrace{\frac{\gamma}{2} (1 - F(q^E))}_{E \text{ is executed first}} + \underbrace{\frac{\gamma}{2} (1 - F(\bar{q}))}_{E \text{ is executed second}} \right] \quad (7.12)$$

26

Glosten model: equilibrium

- If both markets active ($q^I > 0$ and $q^E > 0$) then the zero-profit condition holds for both
- Then the probabilities in (7.11) and (7.12) must be identical, implying

$$\left(1 - \gamma + \frac{\gamma}{2}\right) (1 - F(q^I)) = \frac{\gamma}{2} (1 - F(q^E))$$

- When $\gamma < 1$, this implies $F(q^I) > F(q^E)$, so $q^I > q^E$
- Why?

27

Glosten model: comparing depth

We can relate q^I , q^E and depth q^C in a consolidated mkt as follows. (Recall $\bar{q} = q^I + q^E$).

- The pricing equation also holds with a single platform. This implies $\mathbb{P}(x_I \geq q^I) = \mathbb{P}(x \geq q^C)$
- By (7.11): $\mathbb{P}(x_I \geq q^I) = \frac{1}{2} \left[\text{weighted avg. of } 1 - F(q^I) \text{ and } 1 - F(\bar{q}) \right]$
- But with a single platform: $\mathbb{P}(x \geq q^C) = \frac{1}{2} [1 - F(q^C)]$
- Hence (since zero-profit cond implies these probabilities are equal):

$$q^I < q^C < q^I + q^E = \bar{q}.$$

- **Fragmented** LOB market is **deeper** than consolidated LOB market

28

Glosten model: conclusion

- Previous slide: fragmentation has positive effect on market depth
- Why? **Fragmentation allows for more competition**. Fragmentation essentially allows limit orders to *partially* sidestep time priority:
 - When market is consolidated, first limit orders always get executed first
 - But in fragmented market, you can post limit order on another market at same price, and (maybe) get executed first
- As we argued when analyzing the pro-rata rule, removing time priority may lead to more orders in the short run, but there might also be general eqm effects (lower liq provider profit $\rightarrow$ fewer liq providers in the mkt $\rightarrow$ less depth and lower resiliency)
- Section 7.4.3: there is a critical value of γ , below which $q^E = 0$
 - Once again, a barrier to entry for new exchanges

29

Conclusion

- Fragmentation is ubiquitous
- It is costly for uninformed traders, who would prefer to coordinate on a single market
- Other costs may include less risk sharing and less competition among traders (see book)
- Some benefits are possible (larger depth), depending on setting and trading format

30

Exercises

- Read the Reuters article on dark markets (on absalon)
- Solve exercise 3 on page 276 (ch.7) on brokers receiving payments for order flow

31

Financial Markets Microstructure

Lecture 13

Market Transparency
Chapter 8 of FPR

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Previously on FMM

- **Fragmentation** is ubiquitous
- It is costly for uninformed traders, who would prefer to coordinate on a single market
- Other costs may include less risk sharing and less competition among dealers (see book)
- Some benefits are possible (larger depth), depending on setting and trading format

Today: Market transparency

- Financial markets are among the more transparent ones
 - Historical price and transaction data often available
- But there are a ways to go
 - Often you do not know the price at which your trade will be executed.
- Today: discuss how transparency affects market outcomes
- Related to last week's discussions
- Different kinds of transparency have different effects

3

Market transparency: introduction

- Market transparency can refer to different information
 - **Pre-trade information:** quotes and state of LOB
 - **In-trade information:** trader identity
 - **Post-trade information:** realized trades and prices
- Exchanges profit from selling this type of data
 - Different traders end up with different information sets
 - Some types of traders may benefit from a lack of transparency

4

Market transparency: regulation

Transparency also **regulated**

- In both Europe and the US: rules to assure pre-trade information
- Also, firms must disclose relevant information
- The US has a centralized system for collecting post-trade information, but not Europe

5

General ideas

- 1 In an opaque market, search costs confer monopoly powers to dealers
 - 2 Transparency may foster competition, but also collusion
 - 3 Risk-sharing may be better when markets are opaque
- Before we begin: remember how we discussed that “private” info is out there, you just need to find it and put it together?
 - Same thing with transparency – “opaque” information is not necessarily “inaccessible”, but can just be “not accessible enough”
 - An (extreme) example of poorly informed trading

6

Quote transparency

- In some markets LOB and dealer quotes are visible (possibly at a cost)
- In some other (esp. illiquid) markets trader must search for quotes
 - or approach dealers in search of price improvements
- How do **search costs** affect market outcomes?
- (This was the problem 3 in PS1)

8

Search costs

Idea based on ?'s **chain store paradox**

- Imagine a product market with consumers and firms
- Firms set prices not initially seen to consumers
- Suppose consumers are searching stores sequentially to find the best price, searching costs c per store
- Look for an equilibrium in which all stores set same price p
- Each store has market power: can charge customer up to $p + c$ if desired
- **Equilibrium:** stores set p at monopoly level

9

Search costs

- The situation is the same in a financial market with search cost
 - It doesn't pay to be the cheapest dealer if you can't advertise the price
 - Can always increase price and still be preferred due to search cost
 - So incentives to exploit the price, no incentives to improve the price
- Model conclusion does not depend on size of search cost (ignore what textbook says about it)
 - Although irl frictions probably increase in search cost – fancier models capture this
- **Welfare** implications of search costs:
 - **Dealers** have market power $\Rightarrow$ **higher profits**
 - All **traders are worse off**, the less sophisticated ones more so (if we maintain the assumption their trades are less elastic)
- Empirical evidence from US municipal bond markets does show very high trading costs, especially for retail-sized trades (??)

10

Quote transparency

- Let's look at another dimension of quote transparency
- While price of the first unit is often observable...
 - US protects NBBO orders for each stock
 - Exchanges or dealers may only quote best bid&ask
- ...**depth** can be more difficult to gauge
- If depth is volatile (which it is), may trade at the “wrong time”

11

Uncertainty and price sensitivity

- Consider a Kyle model with random depth $1/\lambda$.
- **Transparent market:** insider demand is inversely related to price sensitivity λ : $x^T \sim \frac{1}{\lambda}$
- **Opaque market:** traders face uncertainty, so their demand is inversely related to expected price sensitivity: $x^O \sim \frac{1}{\mathbb{E}(\lambda)}$
- Convex function. Use Jensen's inequality:

$$\mathbb{E}\left(\frac{1}{\lambda}\right) > \frac{1}{\mathbb{E}(\lambda)} \quad \Longleftrightarrow \quad \mathbb{E}[x^T] > x^O$$

- **More (informed) trading in transparent market**
 - Risk of high λ (shallow market) provides stronger incentive to reduce x than the incentive to increase x from the chance of low λ .

12

Order flow transparency

- In some markets (OTC, FX) an order may be filled simultaneously by different liquidity providers
- What does it matter if they can or cannot observe the whole order flow?
 - We saw one answer already (Glosten vs Kyle)
 - Will now look at another way to model this

13

Order flow: Model

- Consider a simple variation on a Glosten-Milgrom model
- **Value:** high v^H or low v^L with equal probability
 - Mean: $\mu = (v^H + v^L)/2$
- **Dealers:** set quotes, competitive, risk neutral
- **Traders:** two unit market orders arrive
 - With prob. π : both are from informed trader(s) (there was an info event)
 - With prob. $1 - \pi$: both from liquidity traders; one seller, one buyer
- **Idea:** higher order flow correlation when traders are informed. Intuition:
 - Informed traders: if all learn that the asset value is, say, high, then should all want to buy
 - Liquidity traders: suppose pension fund decides it wants a less risky portfolio. (Probably) uncorrelated with other liquidity traders' decisions. For simplicity we strengthen that to perfect negative correlation.

14

Order flow: Equilibrium

- **Opaque:** dealers quote without seeing the entire market order flow
 - As in chapter 3, $a^O = \mu + \pi(v^H - \mu)$ and $b^O = \mu - \pi(\mu - v^L)$
- **Transparent:** dealers condition quotes on both orders
 - Two buyers: must be informed, $a^T = v^H$
 - Two sellers: must be informed, $b^T = v^L$
 - One of each: trade at μ
- Transparent (T) versus opaque (O) market:
 - T better than O for the uninformed: avoid adverse selection premium
 - Better price discovery in T than in O : private information revealed
 - The informed prefer O : get better prices

15

Post-trade transparency

- If orders arrive sequentially, what effect does information about **past orders** have?
- **Value**: high v^H or low v^L with equal probabilities
 - Mean: $\mu = (v^H + v^L)/2$
- **Dealers**: set quotes, competitive, risk neutral
- **Traders**: two traders arrive, submit unit market orders
 - With prob. π : both are informed
 - With prob. $(1 - \pi)/2$: both liquidity traders; first seller, then buyer
 - With prob. $(1 - \pi)/2$: both liquidity traders; first buyer, then seller
- **Transparent market**: All dealers observe the first order d_1
 - Set $a_1 = \mu + \pi(v^H - \mu)$ and $a_{2,d_1} = \mathbb{E}[v|d_1, \text{buy}]$

17

Post-trade transparency: Period 2

Opaque market: One dealer gains informational advantage. Focus on **ask side**

- **Period 2**. Denote the dealer who observed period-1 trade by **I**, and the other dealer by **U**.
 - For simplicity, suppose **I** sets price after observing **U**'s quote
 - **Dealer I**: Suppose **I** saw the first trade, and second trade is a buy, and **U**-dealer quotes a_2^U :
 - If the first trade was a sell, **I** expects $\mathbb{E}[v] = \mu$, so set price at $a_{2s}^I = a_2^U - \epsilon$.
 - If it was a buy, **I** knows $v = v^H$, so quote $a_{2b}^I \geq v^H$
 - **I** picks off period-2 buy order if d_1 was a sell; otherwise leaves it to **U**
 - **Dealer U**: How to quote if you didn't see the first trade and second trade is buy?
 - **U** knows that they only get to trade if $v = v^H$ (otherwise picked off by **I**)
 - Thus, uninformed dealers need to quote $a_2^U = v^H$
 - In the end, quotes are $a_2^U = v^H$, $a_{2s}^I = v^H - \epsilon$, $a_{2b}^I = v^H$.
 - (We'd get the same if **I** and **U** set quotes simultaneously)

18

Post-trade transparency: Period 1

- **Period 1.** The sequential information advantage uncovered in the previous slide can make dealers bid keenly for the first order (Forex dealers often said to quote negative spread to large traders)
 - In second period, I 's profit is $(1 - \pi)(v^H - v^L)/2$. U 's profit is zero
 - Competition leads the first period half-spread to be reduced by this amount, to $(2\pi - 1)(v^H - v^L)/2$ (dealers undercut each other to obtain information contained in first order)
 - The uninformed's aggregate trading cost is $\pi(v^H - v^L)$ - double the cost under transparency. Why is this?
- Would dealers commit to transparency?
 - No, there is always an individual incentive to hide your orders (section 8.4.2)
 - May explain the rise of less transparent trading venues

19

Post-trade transparency: Collusion

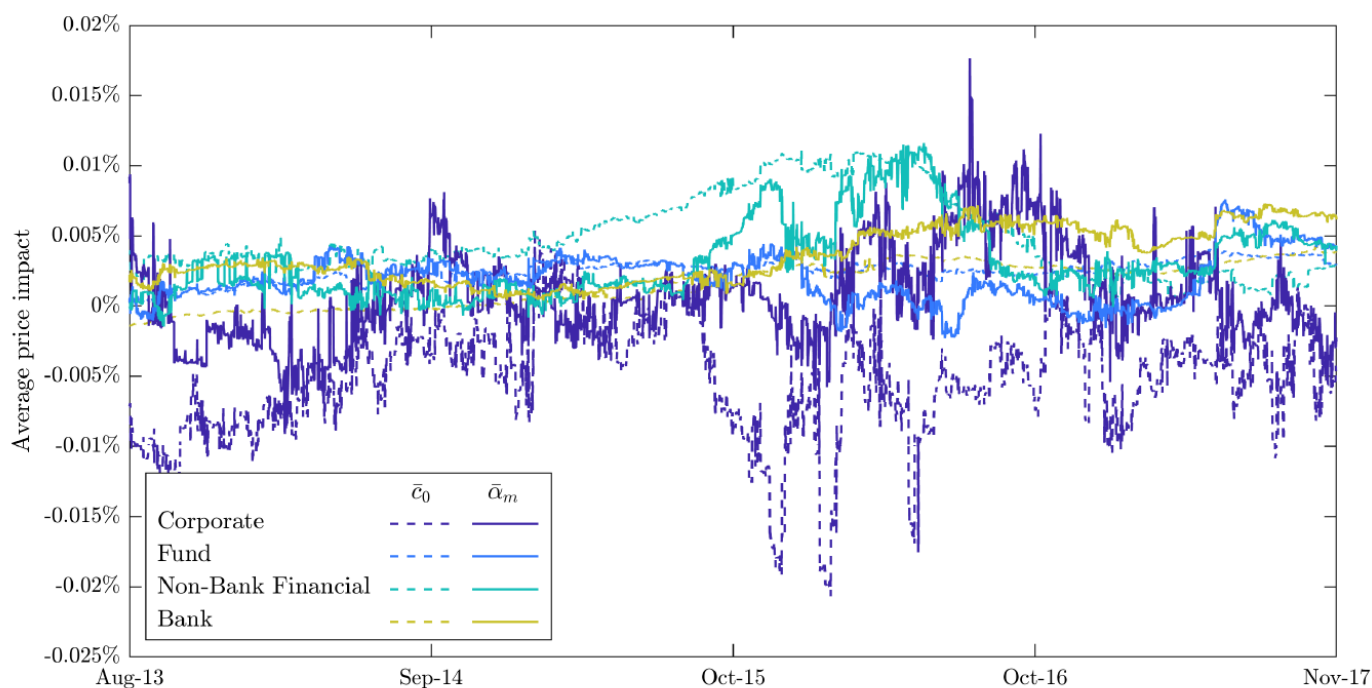
- If dealers are not perfectly competitive, they can try to **collude** to increase their profits
- Cartels are sustained via a threat of **punishment** in case anyone **deviates**
- Prerequisite for collusion: ability to detect deviations
 - Transparency improves this ability
 - So may help collusion

20

In-trade information

- Transparency may relate not only to quote and order data, but also to [trader identity](#).
 - LOB is usually anonymous and thus opaque
 - dealer interactions can be personal
- If trader's identity is visible, it may affect the prices they get
 - Institutional investors rarely engage in informed trading, so will get good price;
 - Insiders will get bad prices.
 - See the figure for FX market on the next slide (from ?)
- If identity is limited to some identifier in the system, trader can still build a reputation through history of actions

22



The cross-sectional average contemporary ($\bar{c}_0^r$) and permanent ($\bar{\alpha}_m$) price impact

23

where

- **fund** includes pension funds, hedge funds, and sovereign wealth funds;
- **nonbank** financial refers to insurance companies, brokers, and clearing houses;
- **corporate** comprises any non-financial organization
- but ~ 80% transactions are bank-bank

24

In-trade information

- There may be ways to signal or credibly **disclose** the fact that your **trade is uninformative**
 - E.g. can advertise trade a few days in advance – “sunshine trading”
- In this case they *will* be used because uninformed traders will want to separate – **transparency will prevail**
- Same may happen due to **cream-skimming**
 - Large banks can execute trades in their own dark pools instead of forwarding to the market
 - They would pick off profitable trades and forward the rest
 - The market would account for this and set wider spreads for trades that make it to the market
 - Reducing cream-skimming has been one focus of MiFID II regulation.

25

In-trade information

- In all of the above, transparency leads to reallocation of welfare from insiders to the uninformed.
 - That's why regulators push for transparency and the market resists
 - You can also argue that transparency would reduce informed trading and reduce price discovery
- Hirshleifer noted that some risk-sharing trades are better conducted before information arrives
 - Think of health insurance
 - Possible to share risks before we know who suffers illness
 - Too late to share risks after the illness is known; market break-down

26

Conclusion

- **Transparency** mostly reallocates welfare across market participants
 - Uninformed traders benefit, so T **helps liquidity**
 - Insiders may lose, so T **worsens price discovery**
 - Dealers may win or lose
- But transparency may also impede risk sharing, and have adverse effects when it is asymmetrically distributed
- Opaqueness can be good in limit books
 - Hidden limit orders help uninformed traders hedge their positions where making these orders visible would by itself create adverse price movements

27

Exercise for next week

- Read the article on MiFID II (on Absalon). Discuss the following questions:
 - What did MiFID II change in regards to market transparency? (There are many aspects to this.)
How will these changes affect market outcomes?
- Read the article on LSE acquiring Refinitiv. What implications can this have for market transparency (e.g. on LSE's own trading platform)?
- Do ex.2 after ch.8 (p.303) on price discovery

Financial Markets Microstructure

Lecture 14

Value of Liquidity
Chapter 9 of FPR

Egor Starkov

Københavns Universitet
Spring 2025

Previously on FMM

- **Transparency** mostly reallocates welfare across market participants
 - Uninformed traders benefit from T, so T helps liquidity
 - Insiders may lose, so T worsens price discovery
 - Dealers/exchanges may win or lose
- But transparency may also impede risk sharing, foster collusion, and have adverse effects when it is asymmetrically distributed

Today: value of liquidity

- So far we looked at how illiquidity makes asset's trade price deviate from its fundamental value
- But illiquidity may itself affect the asset *value*
- Case study – [U.S. Treasury Notes and Bills](#): (Amihud and Mendelson [1991])
 - notes are long-term (2-10y), bills are short-term (< 12m) US govt loans
 - differ only in terms – so soon-to-mature notes are equivalent to bills
 - but notes trade at a discount relative to bills (i.e., offer higher returns) (as of 1991)
 - why? Notes are less liquid (larger spread and brokerage fees). Why less liquid though?

3

Value of liquidity

- Why does liquidity affect asset value?
- Intuitively, an illiquid asset is costlier to transact
 - Traders take into account transactions costs
 - Require a return that compensates for the cost
 - Liquidity premium: less liquid assets trade at lower prices
- Liquidity need not be constant over time
 - If illiquidity rises, asset price falls
 - If future liquidity is random, this is a risk factor
 - Liquidity risk may be priced

4

Liquidity premium (Amihud and Mendelson [1986])

- Before, traders cared only about fundamental value. In this model they care about **resale value**.
- Consider an asset with *constant* relative spread, $s = (a_t - b_t)/m_t$, but fluctuating midprice m_t

- Note that

$$a_t = m_t \left(1 + \frac{s}{2}\right) \text{ and } b_t = m_t \left(1 - \frac{s}{2}\right)$$

- Consider a trader who plans to:
 - 1 buy at t , at the respective ask price a_t ,
 - 2 hold the asset for h periods, and sell at b_{t+h} .

To simplify, suppose the asset pays no dividends.

6

Deriving the premium: risk-adjusted return

- Let r denote the risk-adjusted **real return** per period **required by the market**. What? Then

$$a_t = \frac{\mathbb{E}(b_{t+h})}{(1+r)^h}, \quad \Rightarrow m_t = \frac{\mathbb{E}(m_{t+h})}{(1+r)^h} \times \frac{1 - \frac{s}{2}}{1 + \frac{s}{2}}$$

- If we estimate the required return r using mid-quotes, there is a bias due to illiquidity.
- Let R be the **nominal return** rate, estimated from the midquotes:

$$m_t = \frac{\mathbb{E}(m_{t+h})}{(1+R)^h}$$

The observed R is different from r !

7

Deriving the premium: approximation

- Thus, we have

$$(1 + R)^h = (1 + r)^h \times \frac{1 + \frac{s}{2}}{1 - \frac{s}{2}} \quad (9.5)$$

Thus: (see next slide for derivation)

$$R \simeq r + \frac{s}{h}$$

- Essentially, the asset's return needs to be higher by s/h in order to compensate for the liquidity cost
 - The difference $R - r$ is a **liquidity premium**
 - Take h as representative trader's holding period for asset

8

Appendix on the approximation

- To get the approximation of the previous slide, we must use the approximation $\ln(1 + x) \simeq x$ for small x
- Recall further that $\ln x^h = h \ln x$
- So taking logs of both sides of $(1 + R)^h = (1 + r)^h \times \frac{1 + \frac{s}{2}}{1 - \frac{s}{2}}$, we get

$$h \ln(1 + R) = h \ln(1 + r) + \ln \left(1 + \frac{s}{2} \right) - \ln \left(1 - \frac{s}{2} \right)$$

and assuming r , R , and s are small we apply the approximation

$$hR \simeq hr + \frac{s}{2} - \left(-\frac{s}{2} \right)$$

- Rearranging the terms, we get $R \simeq r + \frac{s}{h}$.

9

Discussion

- Empirical evidence confirms *positive* liquidity premium for stocks, bonds
- For a more general model and empirics, see Bongaerts, De Jong, and Driessen [2011]

10

Clientelle effects

- We obtained $R = r + s/h$ in our toy model
- In reality investors differ in h , expected holding period
- Consider a toy extension of our toy model, with:
 - Two types of investors with $h_1 < h_2$
 - Two assets with $s_1 < s_2$

12

Clientelle effects

- Suppose in eqm h_1 -investors trade in s_1 -asset and h_2 -investors trade in s_2 -asset
- For this to be an eqm, need $R_1 - s_1/h_1 \geq R_2 - s_2/h_1$ and $R_2 - s_2/h_2 \geq R_1 - s_1/h_2$
- The two conditions are equivalent to

$$\frac{1}{h_2} \leq \frac{R_2 - R_1}{s_2 - s_1} \leq \frac{1}{h_1} \quad (9.10)$$

- There exist R_1, R_2 (and r) which solve this so all ok
 - There would not be a solution if we assumed the opposite kind of separation
 - We also cannot have both groups indifferent between both assets (would need two equalities in 9.10)

13

Clientelle effects: discussion

- Some investors specialize in illiquid assets / hope to earn the liquidity premium
 - Should in equilibrium be those who trade less frequently
- (would this explain the case of Treasury Bills vs Bonds?)
- Note: more adverse selection implies larger spread, hence attracts traders with large h
- We assumed that h are fixed, but all the same logic applies if h is random (e.g., traders randomly get liquidity shocks).
- Clientelle effects would then apply whenever different groups of traders have different *distributions* of h .

14

Liquidity risk

- IRL, spread s randomly fluctuates over time
- Further, liquidity of any given asset may be arbitrarily correlated with that of other assets or the whole market
- These are risk factors which can also be priced
- Use the Liquidity CAPM model of Acharya and Pedersen [2005]

16

reminder: regular CAPM

- The standard CAPM postulates that return r_j on asset j is governed by the risk-free rate r_f and a risk premium, which depends on the correlation of r_j with the market return r_M :

$$\mathbb{E}[r_j] = r_f + \beta_j [\mathbb{E}[r_M] - r_f]$$

$$\text{with } \beta_j = \frac{\mathbb{C}(r_j, r_M)}{\mathbb{V}(r_M)}$$

- In particular, only *systematic* risk enters asset price
- *Idiosyncratic* risk of the asset can be diversified away

17

Liquidity CAPM

- Investors care for net return $r = R - s$ where s now denotes the liquidity premium
 - Let f denote risk-free, M the market
- Plugging these into the CAPM equation, we get

$$\mathbb{E}[R_j - s_j] = r_f + \lambda_M \beta_j$$

where $\lambda_M = \mathbb{E}[R_M - s_M] - r_f$ is the market risk premium and

$$\beta_j = \frac{\mathbb{C}(R_j - s_j, R_M - s_M)}{\mathbb{V}(R_M - s_M)}$$

18

- Expand $\mathbb{C}(R_j - s_j, R_M - s_M)$:

$$= \mathbb{C}(R_j, R_M) + \mathbb{C}(s_j, s_M) - \mathbb{C}(R_j, s_M) - \mathbb{C}(s_j, R_M)$$

to get $\beta_j = \beta_{1j} + \beta_j^{liq} = \beta_{1j} + \beta_{2j} - \beta_{3j} - \beta_{4j}$ with

$\beta_{1j} = \frac{\mathbb{C}(R_j, R_M)}{\mathbb{V}(R_M - s_M)}$	: ordinary β
$\beta_{2j} = \frac{\mathbb{C}(s_j, s_M)}{\mathbb{V}(R_M - s_M)}$	: hedge liquidity with liquidity
$\beta_{3j} = \frac{\mathbb{C}(R_j, s_M)}{\mathbb{V}(R_M - s_M)}$	: hedge liquidity with returns
$\beta_{4j} = \frac{\mathbb{C}(s_j, R_M)}{\mathbb{V}(R_M - s_M)}$	: hedge returns with liquidity

19

Liquidity risk: alternative model

- Earlier in the course we derived the spread given the asset return/value (Glosten-Milgrom)
- Earlier today we did the opposite: derived the required return keeping the spread fixed (Amihud-Mendelson)
- Duffie, Gârleanu, and Pedersen [2005, 2007] do both simultaneously! In their model:
 - Traders are randomly hit with liquidity shocks, so want to buy/sell asset over time depending on their current situation
 - But may not always find a trade – can get stuck with asset when shocked/without asset when not shocked
 - This liquidity risk enters the equilibrium asset price
 - (And the spread is driven by dealers' market power)
- Click [here](#) if you want to see the model (you do not need to know it for the exam)

20

Arbitrage

- Main tenet of economics and finance: **no arbitrage**
 - Assets that generate same cash flows must cost the same
 - If arbitrage is possible, it is immediately exploited and then there is no more arbitrage.
- Depending on semantics:
 - **arbitrage in nominal prices** cannot be realized due to limited liquidity...
 - (also, arbitrage itself is costly, due to leverage and short-selling constraints)
 - ...so there is **no real arbitrage**

22

Conclusion

- Empirical research finds evidence on both a **liquidity premium** and a **liquidity risk premium** on stocks
- Further, overall market liquidity may vanish at crisis times when asset prices drop rapidly
 - Important risk for investors, especially speculators
 - Financial institutions are required to hold robustly liquid assets
 - In general, risky positions require costly collateral, margins

23

Exercise

Ex.1 from ch.9 (p.347)

24

References I

- V. V. Acharya and L. H. Pedersen. Asset pricing with liquidity risk. *Journal of Financial Economics*, 77(2):375–410, Aug. 2005. ISSN 0304-405X. URL <https://doi.org/10.1016/j.jfineco.2004.06.007>.
- Y. Amihud and H. Mendelson. Asset pricing and the bid-ask spread. *Journal of Financial Economics*, 17(2):223–249, Dec. 1986. ISSN 0304-405X. URL [https://doi.org/10.1016/0304-405X\(86\)90065-6](https://doi.org/10.1016/0304-405X(86)90065-6).
- Y. Amihud and H. Mendelson. Liquidity, Maturity, and the Yields on U.S. Treasury Securities. *The Journal of Finance*, 46(4):1411–1425, 1991. ISSN 1540-6261. URL <https://doi.org/10.1111/j.1540-6261.1991.tb04623.x>. eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1540-6261.1991.tb04623.x>.

References II

- D. Bongaerts, F. De Jong, and J. Driessen. Derivative Pricing with Liquidity Risk: Theory and Evidence from the Credit Default Swap Market. *The Journal of Finance*, 66(1):203–240, 2011. ISSN 1540-6261. URL <https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.2010.01630.x>. eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1540-6261.2010.01630.x>.
- D. Duffie, N. Gârleanu, and L. H. Pedersen. Over-the-counter markets. *Econometrica*, 73(6): 1815–1847, 2005. URL <https://doi.org/10.1111/j.1468-0262.2005.00639.x>.
- D. Duffie, N. Gârleanu, and L. H. Pedersen. Valuation in Over-the-Counter Markets. *The Review of Financial Studies*, 20(6):1865–1900, Nov. 2007. ISSN 0893-9454. doi: 10.1093/rfs/hhm037. URL <https://academic.oup.com/rfs/article/20/6/1865/1575887>. Publisher: Oxford Academic.

Duffie, Gârleanu, and Pedersen [2005, 2007] Model

- One asset:
 - Pays dividends to its holders each period
 - Traders can hold **one** or **zero units** of the asset
 - The alternative is a bank which pays interest r
 - Assume the asset is supplied to fraction $q < 1/2$ of the population
- Unit mass (continuum) of traders, each has either high or low value for the dividend:
 $j = h, l$
 - If $j = h$, value today's dividend at 1 (high value traders); if $j = l$ then value dividend at $1 - c$ (low value traders), where $c \in (0, 1)$
 - Every period, fraction ψ of traders switch (from h to l or vice versa)
 - (In the long run, half the traders have high value)
 - Switchers would like to trade the asset (h to $l \rightarrow$ want to sell; l to $h \rightarrow$ want to buy)
 - Those willing to trade search for a dealer, find one with probability $\phi < 1$

DGP Model (2)

- Exact **timing** within a period:
 - 1 investor receives dividend payoff (\$1 or \$(1 - c))
 - 2 valuation changes w.p. ψ
 - 3 if needed, trader looks for a dealer; a match happens w.p. ϕ
- Dealers have some **bargaining power** because they are hard to find
 - Rejecting a dealer's quote means the trader has to wait one period for a chance to meet another dealer – costly (and risky) for a trader
 - Use standard notation: dealers quote ask a and bid b ; spread is $S = a - b$; midprice is $\mu = (a + b)/2$
- Look for a stationary equilibrium with a and b constant over time, and all traders with trading needs (“ h without an asset” and “ l with an asset”) looking for a dealer, and agreeing to trade at a and b respectively iff it is optimal for them (may mix).

Solving the DGP model

- Since $q < 1/2$, not all buyers get to buy (buyers must be indifferent $\iff$ dealers have all market power)
- Denote $\bar{a}$ = max price that a buyer will pay, and $\bar{b}$ = min price that a seller will accept
- **Buy side:** limited supply $\rightarrow$ mixed equilibrium. Set $a = \bar{a}$ and set buy probability conditional on finding dealer (can choose this since buyers are indifferent) to

$$p^B = \phi \cdot \frac{\pi_S}{\pi_B},$$

π_S : fraction willing to sell; π_B : fraction willing to buy

- **Sell side:** no restraint, sell with prob. 1, bargain with the dealer over surplus:

$$b = z\bar{b} + (1 - z)\mu.$$

Solving the DGP model (2)

- Method: identify $\bar{a}$ and $\bar{b}$, and then solve for a and b
- Denote the present discounted cash-flow value (as of beginning of the period) of an asset owner with private valuation j by V_j^o and that of a non-owner by V_j^{no}
- Since non-owner buys if $V_h^o - a \geq V_h^{no}$ and owner sells if $V_l^{no} + b \geq V_l^o$, we get

$$\bar{a} = V_h^o - V_h^{no}, \tag{1}$$

$$\bar{b} = V_l^o - V_l^{no} \tag{2}$$

- So $a = V_h^o - V_h^{no}$ and $b = z(V_l^o - V_l^{no}) + (1 - z)\mu$.
- Finally, we must calculate the value functions

Solving the DGP model (3)

- We will just look at two functions, the rest are similar

$$V_h^o = \frac{1}{1+r} + \frac{(1-\psi)V_h^o}{1+r} + \frac{\psi(1-\phi)V_l^o}{1+r} + \frac{\psi\phi(V_l^{no} + b)}{1+r}$$
$$V_h^{no} = \frac{\psi V_l^{no}}{1+r} + \frac{(1-\psi)(1-p^B)V_h^{no}}{1+r} + \frac{(1-\psi)p^B(V_h^o - \bar{a})}{1+r}$$

- Calculate V_l^o and V_l^{no} . Plug back into (1)-(2) to solve for $\bar{a}$ and $\bar{b}$

DGP: Results (1)

- Ask price is then

$$a = \frac{1}{r} - \frac{2\psi}{r(1+z)} \left(1 - \phi \frac{1-z}{2} \right) S,$$

where S is the spread

$$S = a - b = \frac{(1+z)c}{2(r+2\psi) + (1-2\psi)\phi(1-z)}$$

- If $\psi > 0$ the ask price is less than the $1/r$ which would arise if it were always (efficiently) held by high-value traders
- This is due to liquidity costs: buyers anticipate that they will sell in the future and incur the spread cost

DGP: Results (2)

- The midquote is

$$\mu = \frac{1}{r} \left(1 - \frac{c}{2} \right) - \frac{c}{2r} \cdot \frac{\phi(1-z)}{2r + 4\psi - (2\psi - 1)\phi(1-z)}$$

- Increasing in ϕ : larger ϕ = smaller prob of not finding a trade ϕ = smaller illiquidity risk
= more valuable asset

Financial Markets Microstructure

Lecture 15

Liquidity and Corporate Policy
FPR Chapter 10

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Spring 2025

Previously on FMM

Value of liquidity

- Empirical finding: liquidity and liquidity risk affects asset value (not just the price)
- Two explanations
 - **Speculative view**: buy low/sell high speculator makes 'roundtrips' in the asset, and therefore pays the spread
 - **Portfolio view**: investors are hit by liquidity shocks and must engage in costly adjustments
- We looked at this in different frameworks
 - **Asset pricing theory**: $p_t = \frac{p_{t+h}}{(1+r)^h}$
 - **CAPM**: compensation for undiversifiable risk

Today

Liquidity and corporate policy

- In looking at secondary markets, we never spoke about how firms behave
 - Just assumed some fundamental value
- But firms both look at financial markets when making decisions
- and can affect the market through their actions
- **Corporate finance:** liquidity affects opportunities to raise capital
- **Corporate governance:** liquidity affects the influence of shareholders on management
- **Information feedback:** managers use stock prices to evaluate managerial decisions

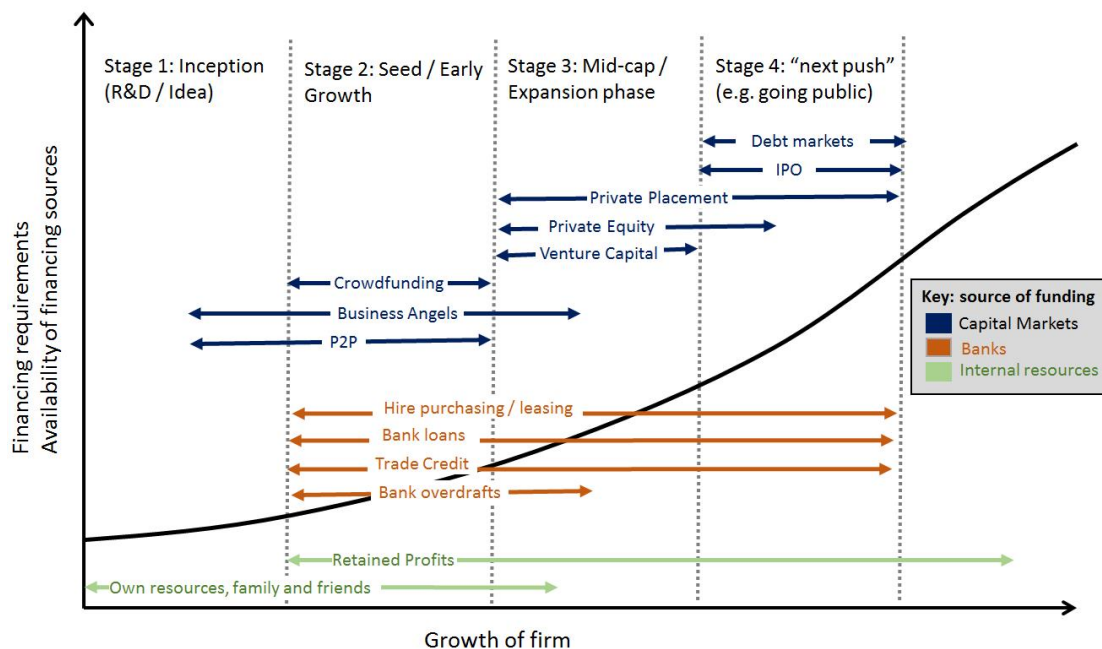
3

Access to capital

- Firms need financing to invest in profitable activities (section 10.2)
- More liquid markets $\Rightarrow$ smaller cost of capital $\Rightarrow$ easier to fund what needs to be funded
- Side channel: easier to progress through the life stages of a firm
 - E.g. early investment often comes from angels/venture capital
 - but they exit once the company has grown enough
 - more risk averse investors enter then etc

5

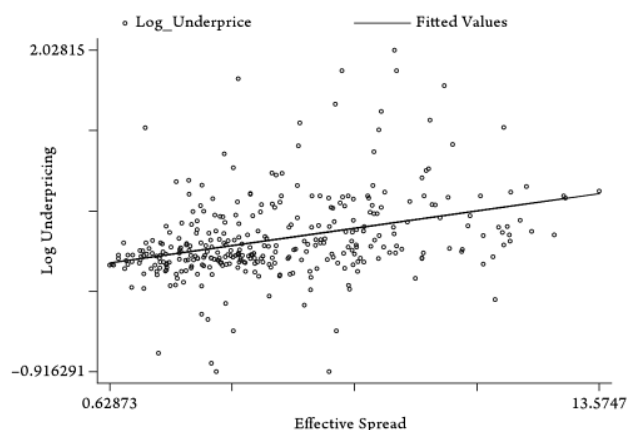
Access to capital



6

Access to capital

- Initial public offering (IPO): first time a firm gets listed on an exchange
 - Initially allocated via a form of auction (bookbuilding)
 - **Underpricing**: initial allocation is on average priced below the exchange's opening price on the following day
 - Many reasons, but liquidity (due to asymm info) seems to be a factor: (Ellul and Pagano [2006])



7

Shareholders and governance

- Incentives of owners and managers are often misaligned
 - managers must put in the effort, but its fruit goes to company owners
 - so **managers must be managed**
- But what are **investors' motives** when buying stocks? Do they want to engage in such oversight?
 - Short-term profit/speculation?
 - Improving governance for sake of long-term profit?

9

Shareholders and governance

- Particular concern with the governance of widely held corporations (Berle and Means [1932]), many small shareholders
 - Who would actively represent shareholder interests?
 - There may be a need for concentrated ownership (10.3)
- If a manager makes bad decisions, a *large* shareholder may (at some private cost) seek to **improve the governance**
 - Alternative – sell shares (the Wall Street Walk, vote with your feet)
- If the market is less liquid, potentially less attractive to sell
 - Could be good for corporate governance, more long-sighted behavior
- [Economist & Bloomberg articles on activist investors in Apple and Intel]

10

Shareholders and governance

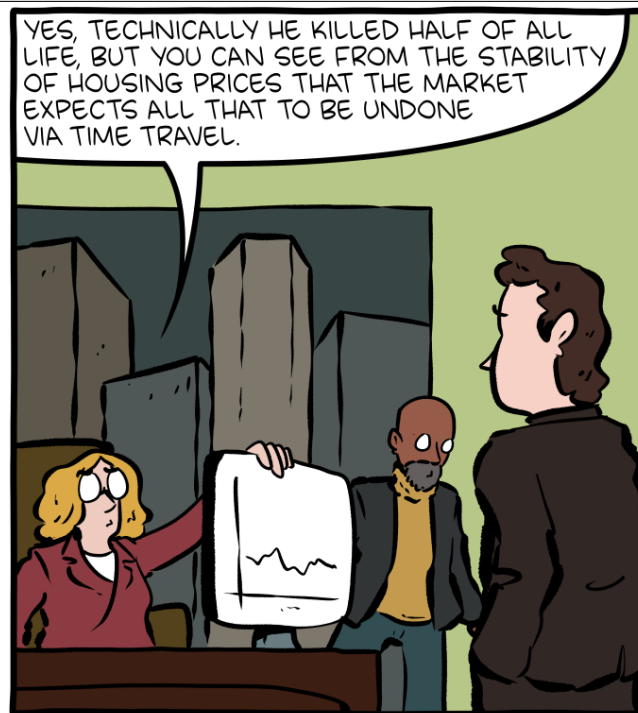
- In illiquid stocks the round-trip cost is large
 - Exit is costly – good for activism
 - But less attractive for an activist investor to buy a block of shares
- So illiquidity is bad in that it doesn't incentivize the centralization of ownership, but once this is achieved, is good for activism
- To make the best of this, US regulators allow opaque building of blocks, but require transparent trading by blockholders

11

Information

- If the market is better informed than the firm on some aspects, the firm can extract this info:
 - 1 Announce a decision
 - 2 Gauge stock price reaction
 - 3 Decide whether to follow through on the decision
- Example: back in 2000, Coca-Cola retracted its \$16bn acquisition offer to Quaker Oats after an 8% dip in stock prices.
- Feedback between stock prices and firm decisions opens up scope for manipulation; see Goldstein and Guembel [2008].
 - Kyle-like model, firm announces a decision and watches the stock market
 - Uninformed speculator sells → firm assumes it could be due to bad news and reverts the decision → stock price drops → speculator closes the position at a profit

13



The finance sector knew Thanos would be defeated before anyone else.

14

Incentivizing Managers

- Investors depend on the firm to produce cash flows
 - Potential conflict of interest, agency: the field of Corporate Governance
- To reduce the agency problem, in general, executive compensation is made to vary with the share price (10.4.2)
 - The share price is a contractible number which forecasts future company value
 - Again, most helpful if the share price is very informative

16

Incentivizing Managers: model

- The book considers a simple agency model
 - **Players:** Manager, Shareholders, (Stock market)
 - **Value:** $V \in \{V^H, V^L\}$, with $\mathbb{P}(V = V^H) = \theta$
 - **Effort:** $\theta = \bar{\theta}$ if manager exerts effort (cost c), otherwise $\theta = \underline{\theta}$
 - **Reservation wage:** manager has zero reservation wage
 - **Limited liability:** salary is non-negative: $w \geq 0$
 - **Stock price:** market observes effort and trades stock at expected value
 - **Contracts:** effort is *not* contractible. But value and stock price is
- First-Best contract (if effort were contractible): $w = c$ if $\theta = \bar{\theta}$ and $w = 0$ otherwise
- Consider contract conditional on either value or stock price.

17

Incentivizing Managers: results

- **Value:** Let w^k be wage conditional on value V^k . Then:

Incentive constraint: $\bar{\theta}(w^H - w^L) - c \geq \underline{\theta}(w^H - w^L)$. **Optimal contract:**

$$\begin{cases} w^L = 0; \\ w^H = c/(\bar{\theta} - \underline{\theta}). \end{cases}$$

- **Stock price:** Price: $\bar{P}$ if $\theta = \bar{\theta}$ and $\underline{P}$ if $\theta = \underline{\theta}$. Wages: $\bar{w}$ and $\underline{w}$. Then

Incentive constraint: $\bar{w} - c \geq \underline{w}$. **Optimal contract:** $\begin{cases} \underline{w} = 0; \\ \bar{w} = c. \end{cases}$

- Stock price-incentivized contract is cheaper: uses more information
- See [Contract Theory](#) course for more

18

Incentivizing Managers: issue

- Tying compensation to stock prices can backfire due to **career concerns**
 - Issue arises if managers care about their perceived skill
- CEO may forego risky – but attractive – investment opportunities for the fear of appearing incompetent
- Or the opposite may happen: take on too much risk if benefits for reputation are convex

19

Instruments

- How can the firm influence the liquidity of its stocks?
- (1) IPO/listing ⇒ double listing
 - 1 cost: increased transparency
 - 2 must obey state and platform regulation
- (2) Hire a dedicated market maker in own stocks
 - 1 popular in EU: MMs post aggressive limit orders
 - 2 such MMs would not have the informational advantage of a dealer in a hybrid market ⇒ smaller effect on rest of market
- (3) Choose optimal capital structure
 - 1 stocks and bonds may have different liquidity
 - 2 **Corporate finance** studies all the factors that feed into the “debt vs capital” decision

21

Conclusion

Corporate governance has a lot of connection to company's financial market performance

- access to capital affected by liquidity
- liquidity and corporate control are somewhat antithetical
- firm can use stock price as market's feedback on its decisions or as benchmark of CEO performance
- firms have some ways in which they can improve the liquidity of their stocks

22

References I

- A. A. Berle and G. G. C. Means. *The modern corporation and private property*. Transaction publishers, 1932.
- A. Ellul and M. Pagano. IPO Underpricing and After-Market Liquidity. *The Review of Financial Studies*, 19(2):381–421, July 2006. ISSN 0893-9454. URL <https://doi.org/10.1093/rfs/hhj018>. Publisher: Oxford Academic.
- I. Goldstein and A. Guembel. Manipulation and the Allocational Role of Prices. *Review of Economic Studies*, 75(1):133–164, Jan. 2008. ISSN 0034-6527, 1467-937X. URL <https://doi.org/10.1111/j.1467-937X.2007.00467.x>.

Financial Markets Microstructure

Lecture 16

Algo-trading, High-Frequency Trading, and Blockchain

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Spring 2025

Previously on FMM

Corporate governance has a lot of connection to company's financial market performance

- access to capital affected by liq-ty
- liq-ty and corporate control are somewhat antithetical
- firm can use stock price as market's feedback on its decisions or as benchmark of CEO performance
- firms have some ways in which they can improve the liquidity of their stocks

Today on FMM...

- Algo-trading!
- High-frequency trading!
- Cryptocurrencies!
- and more...

3

Digital Markets

*“...It should come as no surprise then that the financial system exhibits a Moore's Law of its own – from 1929 to 2009 the total **market capitalization** of the US stock market has **doubled every decade**. The total **trading volume** of stocks in the Dow Jones Industrial Average **doubled every 7.5 years** during this period, but in the most recent decade, the **pace has accelerated**: now the doubling occurs every 2.9 years, growing almost as fast as the semiconductor industry.”*

Kirilenko and Lo [2013]

4

Digital Markets

- The digital revolution of the past few decades has reshaped financial markets as much as (if not more than) any other aspect of our lives
- The quote above mentions the “[extensive margin](#)” akin to the Moore’s Law
- But the “[intensive margin](#)” is also at work
 - Index funds, automated arbitrage, automated execution & market-making only made possible by computers
- In addition to Moore’s Law, [Murphy’s Law](#) does not fail either
 - If something can go wrong it will, and the scope for failures is as big as ever these days. See Kirilenko and Lo [2013] (pp.60-67) for five [stories](#).

5

More on algo-trading

- Algorithms allow for a lot of stuff:
 - [HFT](#) (later today)
 - better [hedging](#) though some automated hedges
 - but also for better [execution](#) via order-splitting.
- Beason and Wahal [2019] give some (actually a lot of) info on how algorithms work for large [institutional investors](#) (a typical counterpart to HFT nowadays)
 - I also put on absalon a 2020 SEC report on algo-trading that gives a bird’s-eye overview
- “Parent” orders are split (by algorithms) into many “child” orders that are routed to markets

7

Institutional algo-trading

- The average parent order attempts to trade \$287,000 over 84 minutes, equivalent to 4.80 percent of volume over the duration of the order.
 - avg: 63.1 runs per parent (avg 10m total duration), 8.8 children per run
- Of the 300 million child orders, less than 0.40 percent are market orders.
 - By comparison, retail investors usage of market orders is over 50 percent
 - $\approx 80\%$ are limit orders, $\approx 20\%$ are PEG orders – dark limit orders that are dynamically “pegged” to the NBBO
 - Of the limit orders, 24% are marketable, 65% are passive, rest inside the spread
- Many orders are unfilled (even marketable)
 - Conditional on filled, median time-to-trade=5ms
 - Even unfilled orders have price impact

8

High-frequency trading: introduction

- **HFT**: Refers to computerized, algorithmic trading at high pace: fastest participants take advantage of opportunities before others
- **Speed is key**: for instance, in 2010, a USD 300 million cable was laid between Chicago and New Jersey (Nasdaq)
- **Ubiquitous**: estimated to account for more than 50% of volume in the US and more than 25% in Europe
- **Recent phenomenon(?)**: the effect on markets is still not well understood. Few empirical studies and fewer theoretical models
- **Today**: Look at two models of HFT

10

Biais, Foucault, and Moinas [2015]

- Simple model of fast trading and investment in speed
- Look at equilibrium behavior and welfare implications
- Endogenize the choice of whether to be fast or slow
 - optimal decision depends on size of trader

11

Model: basics

- **Institution:** continuum of profit-maximizing financial institutions indexed by i , zero endowment, trade one unit
- **Time:** $\tau \in \{0, 1, 2\}$
- **Asset value:** $u_i = v + y_i$, where v is the fundamental value and y the institution's private value
 - **Fundamental value:** $v \in \{\mu - \epsilon, \mu + \epsilon\}$, equal probability, realized at $\tau = 2$
 - **Private value:** $y_i \in \{\delta, -\delta\}$, equal probability and i.i.d. across investors, observed at $\tau = 1$
- **Trading:** Occurs at $\tau = 1$ after private values are learned

12

Model: high-frequency trading

- **HFT:** Fraction α of the institutions invest at $\tau = 0$ to become HFT
- **Information:** Let's call HFT *fast* institutions (viz. *slow* institutions)
 - Fast institutions have better information: learn v at $\tau = 1$ whereas slow institutions learn v at $\tau = 2$
 - Fast institutions find a trading opportunity with probability one, slow institutions with probability $\rho < 1$
- **Timing** (within period $\tau = 1$):
 - 1 Each institution i observes y_i , and if fast, observes v
 - 2 Each institution i finds a trading opportunity or not. If yes, chooses whether to buy/sell/abstain (one-unit trades only): $d_i \in \{-1, 0, 1\}$.
 - 3 Liquidity providers execute order d_i at price $\mathbb{E}(v|d_i)$ (implicit assumption of market maker competition + no aggregate order flow transparency)

13

Information

- **Fundamental value:** Good news/bad news:
 - **Good news** refer to high value: $v = \mu + \epsilon$.
 - **Bad news** refer to low value: $v = \mu - \epsilon$.
- **Fast institutions (FI)** have the following types
 - *GH*: Good news, high private valuation
 - *GL*: Good news, low private valuation
 - *BH*: Bad news, high private valuation
 - *BL*: Bad news, low private valuation
- **Slow institutions (SI)**, on the other hand, are either
 - *H*: high private valuation
 - *L*: low private valuation

14

Equilibrium analysis

- **No fast trading:** If $\alpha = 0$ all orders execute at μ

- **Active fast trading:** Now suppose $\alpha > 0$. Let

β_j^F : prob. that fast institution type j buys

β_j^S : prob. that slow institution type j buys (cond. on trading opp-ty)

- **High value:** Fast GH types have highest possible valuation: $\beta_{GH}^F = 1$
- **Low value:** Fast BL types have lowest possible valuation: $\beta_{BL}^G = 0$
- **Buy side:** Let $a = \mathbb{E}[v|buy]$. Use above observation and Bayes' Rule to get

$$a = \mu + \frac{\alpha \frac{1+\beta_{GL}^F - \beta_{BH}^F}{4}}{(1-\alpha)\rho \frac{\beta_H^S + \beta_L^S}{2} + \alpha \frac{1+\beta_{GL}^F + \beta_{BH}^F}{4}} \epsilon.$$

15

Multiple equilibria

- **Multiple equilibria:** Often markets have several equilibria.
- **Self-fulfilling expectations:** This is caused by the endogenous price:
 - If you think buyers will have high valuations $\rightarrow$ set high a
 - If a is high, only traders with high valuations will buy

- **Assumption:** $\frac{\epsilon}{2} < \delta < \epsilon$: both v and y matter; v more so. Then

$$V_{GH}^F > V_H^S > V_{GL}^F > \mu > V_{BH}^F > V_L^S > V_{BL}^F,$$

where V_j^i is the value of a type- i institution with type- j information.

- **Equilibrium types:** We focus here on the pure strategy equilibria. There will be three types of equilibria: P1, P2 and P3.

16

- **P1:** $\mu \leq a < \mu + \epsilon - \delta$. Fast institutions with good news always buy, slow institutions buy if private value high:

$$\beta_{GL}^F = \beta_H^S = 1 \text{ and } \beta_{BH}^F = \beta_L^S = 0$$

- **P2:** $\mu + \epsilon - \delta < a < \mu + \delta$. Fast institutions with good news and high value buy, but don't trade if information is conflicting. Slow institutions buy if private value high:

$$\beta_{GL}^F = \beta_L^S = \beta_{BH}^F = 0 \text{ and } \beta_H^S = 1$$

- **P3:** $a = \mu + \epsilon$. Fast institutions with good news and high value buy, other types of institutions don't trade (*crowding out*):

$$\beta_{GL}^F = \beta_H^S = \beta_{BH}^F = \beta_L^S = 0$$

- Note: as usual, we look on one side of mkt so "sell" same as "abstain" in betas above

17

When do we have multiple equilibria?

- **P3 equilibrium:** Always exists. If dealers believe only fast institutions trade $\rightarrow$ high a . But then only optimal for fast institutions to trade.

- **Proposition:** **P1 equilibrium** exists if $\alpha < \alpha_{P1} \equiv \frac{\rho(\epsilon-\delta)}{\rho(\epsilon-\delta)+\delta}$. Proof.

1 Suppose institutions expect $a = \mu + \frac{\alpha}{\alpha+(1-\alpha)\rho}\epsilon$

2 Notice $\mu - \epsilon + \delta < \mu < a$: FI never buy with bad news ($\beta_{BH}^F = 0$ optimal)

3 $\alpha < \alpha_{P1} \Rightarrow a < \mu + \epsilon - \delta$: good news imply expected FI gains from buying, regardless of private valuation ($\beta_{GL}^F = 1$ is optimal).

4 Notice $\mu + \delta > \mu + \epsilon/2 > \mu + \epsilon - \delta > a$: SI with high valuation always buys ($\beta_H^S = 1$ is optimal).

5 Conditional on this, $\mathbb{E}[v|buy] = \mu + \frac{\alpha}{\alpha+(1-\alpha)\rho}\epsilon$ (a is optimal price). □

- Similarly, can find values of α s.t. **P2 equilibrium** exists

- P1 is Pareto dominant for $\alpha < \alpha_{P1}$.

18

Institution gain (Fast Institutions)

- FI gain in P1 equilibrium is (focus on buy side):

$$\begin{aligned}\mathbb{E}[u - a|buy, FI] &= \mathbb{E}[u|buy, FI] - \left[\mu + \frac{\alpha}{\alpha + (1 - \alpha)\rho} \epsilon \right] \\ &= \mathbb{E}[u|v = \mu + \epsilon] - \left[\mu + \frac{\alpha}{\alpha + (1 - \alpha)\rho} \epsilon \right] \\ &= \mu + \epsilon - \left[\mu + \frac{\alpha}{\alpha + (1 - \alpha)\rho} \epsilon \right] \\ &= \frac{(1 - \alpha)\rho}{\alpha + (1 - \alpha)\rho} \epsilon \equiv \pi_F(\alpha)\end{aligned}$$

- Notice $\pi'_F(\alpha) < 0$.

19

Institution gain (Slow Institutions)

- SI gain in P1 equilibrium is (focus on buy side):

$$\begin{aligned}\rho \mathbb{E}[u - a|buy, SI] &= \rho \left(\mathbb{E}[u|buy, SI] - \left[\mu + \frac{\alpha}{\alpha + (1 - \alpha)\rho} \epsilon \right] \right) \\ &= \rho \left(\mathbb{E}[u|y_i = \delta] - \left[\mu + \frac{\alpha}{\alpha + (1 - \alpha)\rho} \epsilon \right] \right) \\ &= \rho \left(\mu + \delta - \left[\mu + \frac{\alpha}{\alpha + (1 - \alpha)\rho} \epsilon \right] \right) \\ &= \rho \left(\delta - \frac{\alpha}{\alpha + (1 - \alpha)\rho} \epsilon \right) \equiv \pi_S(\alpha)\end{aligned}$$

- Notice $\pi'_S(\alpha) < 0$.

20

Institution gain

- Both $\pi_F(\alpha)$ and $\pi_S(\alpha)$ are decreasing in α : all trader types lose when proportion of fast traders increases
- This result holds generally (when we focus on the Pareto-dominant equilibria)
 - Fast institutions lose because there is more price impact of trades (more adverse selection)
 - Higher price impact dissuades slow institutions from trading (crowding out)
- So more HFT is always 'bad' for existing traders, but beneficial for institutions that switch to become HFT
 - Note: $\pi_F(\alpha) - \pi_S(\alpha) > 0$ is independent of α
- Shkilko and Sokolov [2020]: periods when HFT is disrupted are characterized by less adverse selection, lower trading costs

21

Endogenous acquisition of technology

- **Cost:** At $\tau = 0$, a trader can become a fast institution at cost C
- **Markets:** There is a size- N continuum of markets (this will simplify the maths). An institution of type n can participate in $n \leq N$ markets
- **Participation:** Type is distributed according to "pdf" $h(n)$ on $[0, N]$ with

$$h(n) = \frac{N}{n}$$

- **Optimal investment:** Invest in becoming fast institution if

$$\begin{aligned} \pi_F(\alpha) \cdot n - C &\geq \pi_S(\alpha) \cdot n \\ \Leftrightarrow n &\geq \frac{C}{\pi_F(\alpha) - \pi_S(\alpha)} \equiv n(\alpha) \end{aligned}$$

22

Endogenous acquisition of technology (2)

- Notice that $n \cdot h(n) = N$: the total number of investments made by type- n institutions is N for all n
- Thus: equal amount of n and n' investors *within each market*.
- In other words, n is uniformly distributed within each market: $n \sim U[0, N]$ such that we get the following fixed-point problem

$$\alpha = \mathbb{P}(n \geq n(\alpha)) = \frac{N - n(\alpha)}{N}$$

23

Endogenous acquisition of technology (3)

- Authors find equilibrium with $\partial\alpha/\partial C < 0$.
- Welfare result:

If $\rho > 1/2$ then welfare-maximizing value of α is 0.

- Hence: in 'well-functioning' markets, equilibrium has too much HFT
- Because HFT effects are:
 - *more trading opportunities* – personal and social benefit
 - *pvt info about v* – personal benefit, social cost (worse prices for everybody)

24

BFM Conclusion

- Fast trading exacerbates adverse selection, but is individually appealing
- If the markets are already reasonably good at matching traders with opportunities, fast trading may be strictly bad for welfare

25

Budish, Cramton, and Shim [2015]

- **Claim:** there is an *arms race* in HFT (perpetual wasteful investment in gaining advantage) and this is a result of bad market design
- **Solution:** must go to the root and construct better markets rather than imposing taxes etc.
- **Proposal:** Authors propose to replace the *continuous auction* with *frequent batch auctions*
 - frequent = every 0.1s
- **Paper:** Claims are backed up with a great deal of data and a (very!) simple model

27

Correlations and arbitrage

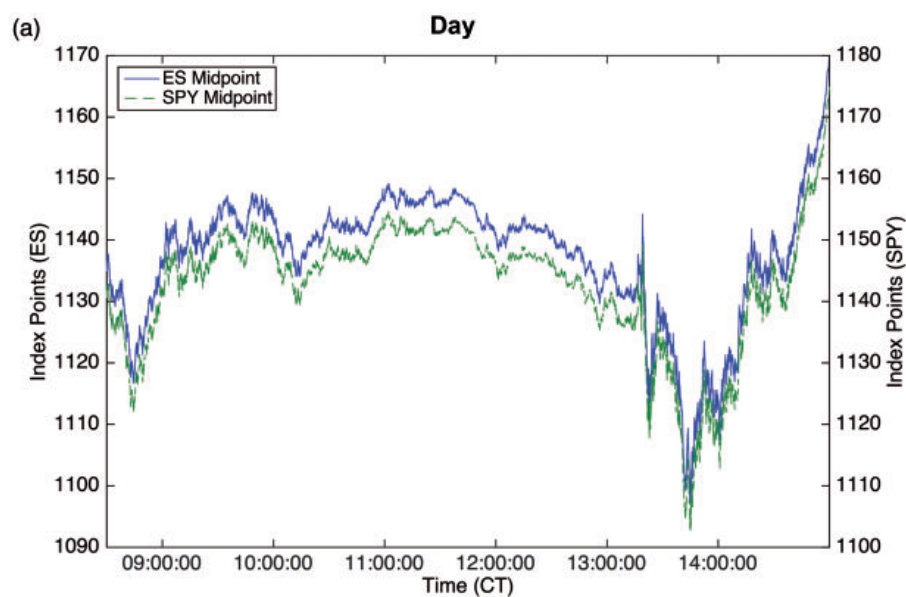
The authors make three points

- 1 **Vanishing correlations:** For short enough latency (time intervals), correlations between almost identical assets break down
- 2 **Arbitrage:** This leads to arbitrage possibilities
- 3 **Perpetual situation:** These arbitrage possibilities do not vanish over time, suggesting that competition does not make them disappear

28

Budish, Cramton, and Shim [2015]

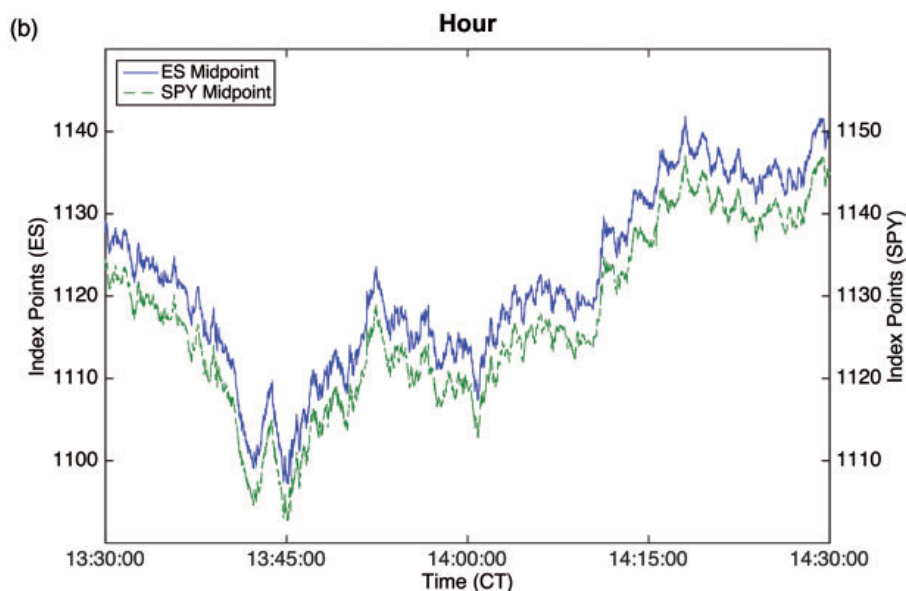
ES and SPY are the two largest instruments tracking S&P500. In theory perfectly correlated. Panel (a) shows a trading day.



28

Budish, Cramton, and Shim [2015]

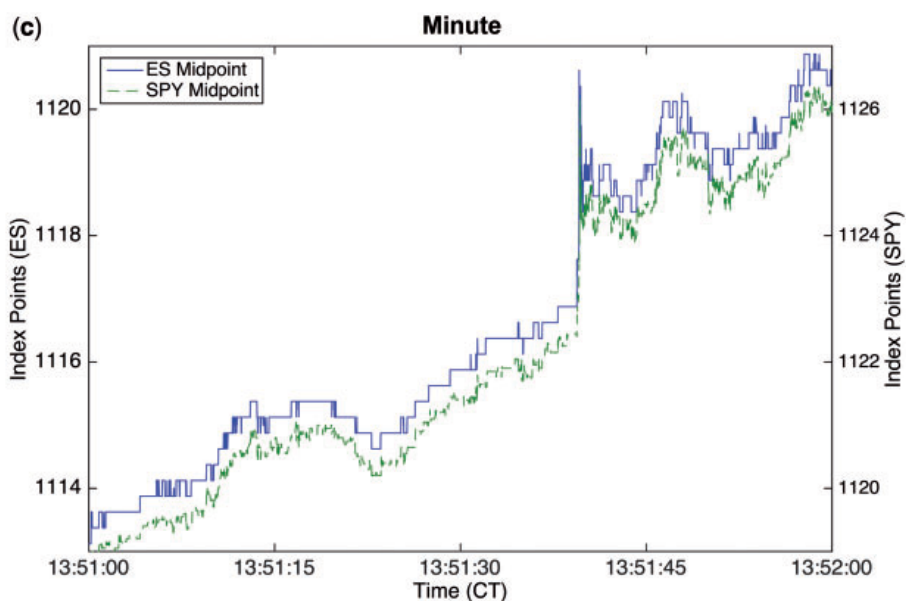
ES and SPY are the two largest instruments tracking S&P500. In theory perfectly correlated. Panel (b) shows a trading hour.



28

Budish, Cramton, and Shim [2015]

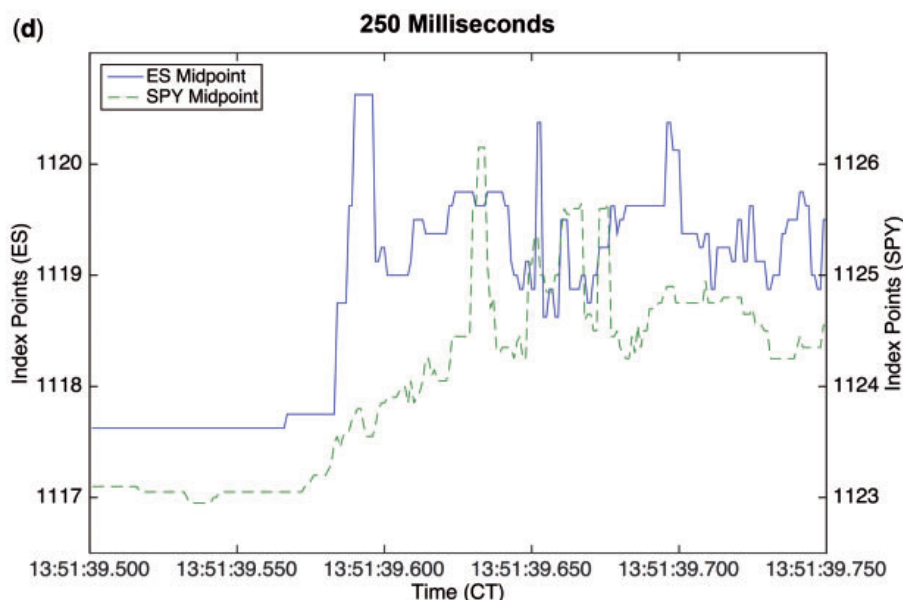
ES and SPY are the two largest instruments tracking S&P500. In theory perfectly correlated. Panel (c) shows a trading minute.



28

Budish, Cramton, and Shim [2015]

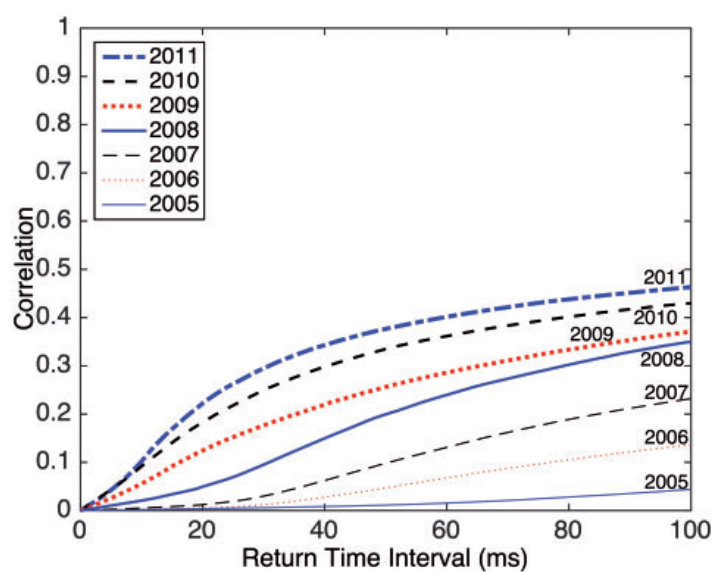
ES and SPY are the two largest instruments tracking S&P500. In theory perfectly correlated. Panel (d) shows a high-frequency breakdown.



28

Budish, Cramton, and Shim [2015]

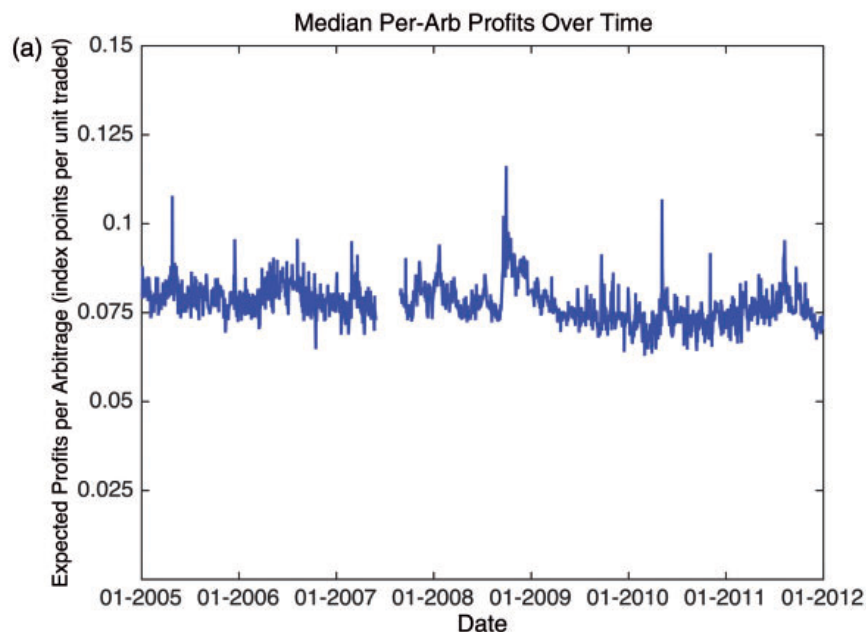
The figure below shows the correlation between ET and SPY by time interval for different years.



28

Budish, Cramton, and Shim [2015]

The figure below shows median arbitrage profits over time. Very stable, total $\sim \$75\text{m/yr}$



29

Model of a continuous market

Security

- **Value:** There is a signal y , which is perfectly correlated with the value x . Signal y follows compound Poisson distribution.
- **Comp. Poisson:** Jumps arrive at rate λ_{jump} and have size $J \sim F_{jump}$.

Players

- **Noise traders:** arrive according to Poisson process (λ_{invest})
 - Want to buy/sell one unit
 - Incur cost of delay, so use marketable limit orders $\simeq$ market orders
- **HFTs:** There are N HFT firms who use market or limit order

Order processing

- If multiple orders/messages at same time, uniform random draw to determine first to be processed

30

Equilibrium

Equilibrium properties: Focus on equilibrium with the following properties

- **Endogenous market maker:** 1 HFT *endogenously* takes the role of liquidity provider. Refer to this as the market maker (MM).
- **Adverse selection:** $N - 1$ HFTs act as *stale quote snipers*

Market maker

- **Quotes:** Suppose signal is y . Set $a = y + \frac{s}{2}$ and $b = y - \frac{s}{2}$ where s is the spread.
- **News:** If news arrive and the new signal is y' , send message to cancel quotes a and b and post new ones: $a' = y' + \frac{s}{2}$ and $b' = y' - \frac{s}{2}$. Noise traders are slower at receiving news.

Snipers

- Trade if $|y' - y| > \frac{s}{2}$.

31

Equilibrium (2)

- **Market maker profits:** The MM **flow profits** (per dt period, normalized by dt) are

$$\lambda_{invest} \cdot \frac{s}{2} - \lambda_{jump} \cdot \mathbb{P}\left(J > \frac{s}{2}\right) \cdot \mathbb{E}\left[J - \frac{s}{2} \mid J > \frac{s}{2}\right] \cdot \frac{N-1}{N}.$$

- **Sniper profits:** The profits to stale-quote snipers are

$$\lambda_{jump} \cdot \mathbb{P}\left(J > \frac{s}{2}\right) \cdot \mathbb{E}\left[J - \frac{s}{2} \mid J > \frac{s}{2}\right] \cdot \frac{1}{N}.$$

- **Equilibrium condition:** Make HFT indifferent btw MM and sniper:

$$\lambda_{invest} \cdot \frac{s}{2} = \lambda_{jump} \cdot \mathbb{P}\left(J > \frac{s}{2}\right) \cdot \mathbb{E}\left[J - \frac{s}{2} \mid J > \frac{s}{2}\right].$$

- **Lack of competition:** Spread s does not depend on N .

32

Continuous auction market versus batch

- **Conclusion:** There will be a **positive bid-ask spread** in continuous market (even as $N \rightarrow \infty$), despite **no asymmetric information** (kind of)
- **Market failure:** Authors argue that this failure is built into the market via processing mechanism
- **Proposed solution:** Frequent batch auction
 - Auction every τ moments. Fast institutions have latency δ_{fast} and slow institutions latency δ_{slow} . Three intervals, depending on when public signal arrives:
 - 1 $[0, \tau - \delta_{slow}]$: all institutions trade, no AS
 - 2 $[\tau - \delta_{slow}, \tau - \delta_{fast}]$: only fast institutions trade, AS
 - 3 $[\tau - \delta_{fast}, \tau]$: no institution trade (inefficient)
- **Outcome:** Before, fast trader always has advantage; now only $\frac{\delta}{\tau}$ of the time, where $\delta = \delta_{slow} - \delta_{fast}$. If $\delta = 100$ microseconds and $\tau = 100$ milliseconds. Then $\frac{\delta}{\tau} = \frac{1}{1000}$. Large reduction in HFT importance.

33

HFT Conclusion

- Effects of speed are similar to those of informed trading
- By design, continuous trading generates arbitrage opportunities
- Firms overinvest in speed in attempts to reap these arbitrage profits
- Risk of being sniped contributes to the spread
- While HFTs can serve as liquidity providers, they do not actually contribute to narrowing the spread
- Use better market design (batch auctions) to improve this

34

Blockchain and cryptocurrencies

- Our discussion would be incomplete without mentioning **blockchain** and **cryptocurrencies**, the biggest trend of 2017
 - blockchain is a “distributed ledger” technology
 - crypto uses blockchain to record transactions in some tokens
- In addition to below, you can find some economic discussion of crypto in Nica, Piotrowska, and Schenk-Hoppé [2017] and Halaburda, Haeringer, Gans, and Gandal [2020]

36

How should it work?

- Cryptos (bitcoin, ethereum) are like distributed payment systems
- You can translate that to a financial market:
 - Say coins serve as shares of some company
 - Or there is a decentralized exchange that records stock ownership transactions in a blockchain

37

Why the hype?

- **Decentralization**: no exchange to profit from traders $\Rightarrow$ lower order costs
 - Even when the market is dominated by exchanges, you do not need to use them to trade (in principle)
- **Transparency**: transaction history is visible, order flow is visible, counterparty's trading history visible
 - note: there is very little anonymity, contrary to what some say!
- **Smart contracts**: algo trading by design

38

Why not?

- **Limited processing capacity**: block size and frequency is $\approx$ fixed
 - Visa: 150m tx/day; Bitcoin: 300k tx/day (15.02.21)
- **Order costs** and **execution risk**: you have to bid for your transaction to be accepted into a block.
 - This is on top of execution risk from other sources (for limit orders)
 - Average order costs fluctuate over time 
 - There are concerns that miners inflate fees (Lehar and Parlour [2020])
- **Delay**: blocks are only processed rarely (one per 10 min avg for bitcoin) 
- **Clearing and settlement**: without a trusted mediator, counterparty and security risks intensify 
- **No transparency requirements**: it is more difficult to enforce disclosure of financial info by firms/coin issuers

39

Revealed preference

- But in the end, the final users (traders) don't care about the fancy technology in the backend
- They choose whichever is (1) cheaper and (2) more convenient to use
- So web3 converged to the same centralized system we had before:
 - crypto is traded via a few centralized exchanges (Binance, Coinbase, ...)
 - NFTs barely exist(ed?) outside OpenSea and Rarible
- Signal founder has a nice post about it.

Financial Markets Microstructure

Lecture 19

Markets and public information

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Spring 2025

Previously on FMM

- **High-Frequency Trading** generates informational asymmetries between traders
- If the markets are already reasonably good at matching traders with opportunities, fast trading may be strictly bad for welfare
- While HFTs can provide liquidity, more HFTs does not necessarily improve liquidity
- So it might be optimal to eliminate the speed game, e.g. by moving away from continuous markets to frequent batch auctions

News

- Incomplete information has been pivotal in many of the models we have considered
- But we really focused on **asymmetric information**
 - When **private news** arrive to insiders, they **trade**, everyone else learns from trades
- What about **symmetric uncertainty**? When **public** news arrive: ...?
 - Glosten-Milgrom/Glosten/Foucault: all orders are repriced, but **no trade should** take place
 - in reality, a lot of trade after public announcements
 - (Budish-Cramton-Shim offer one explanation: HFTs rush to snipe stale quotes when news arrive)
- Let's look at how information is aggregated and how this depends on public info!

3

Context: Hellwig [1980]

- A model of how disagreement leads to trading. Forget about liquidity takers/makers.
- Suppose there is a continuum of ex ante symmetric traders $i \in [0, 1]$ with CARA prefs:

$$U(W_i) = -e^{-\gamma W_i}, \quad W_i = (v - p)x_i$$

- One asset, fundamental value $v \sim \mathcal{N}(0, 1/\tau_v)$
- Every trader gets a private signal $\eta_i = v + \epsilon_i$ with $\epsilon_i \sim \mathcal{N}(0, 1/\tau_\epsilon)$ and submits a price-contingent demand schedule $x_i(p)$
- Aggregate supply $u \sim \mathcal{N}(0, \sigma_u^2)$ (provided by noise traders)
- Price p is set to clear the market (by some non-trading market-maker/“the invisible hand”/...) $\rightarrow p$ is informative about average signal

5

Hellwig: eqm

- Conjecture a linear eqm, where price is a linear function of v and u (and aggregate signal $\bar{\eta} = \int \eta_i di$, which is irrelevant given v and u)
- $v|p, \eta_i$ is normal with some mean and variance $\Rightarrow$ so is W_i (without noise trades, $v|p$ is degenerate)
- Then trader i 's problem is:

$$\begin{aligned} & \max_{x_i} \left\{ \mathbb{E} \left[-e^{-\gamma W_i} | \eta_i, p \right] \right\} \\ (W_i \text{ normal}) & \iff \max_{x_i} \left\{ \mathbb{E} [W_i | \eta_i, p] - \frac{\gamma}{2} \mathbb{V} [W_i | \eta_i, p] \right\} \\ & \Rightarrow x_i(\eta_i, p) = \frac{\mathbb{E} [v | \eta_i, p] - p}{\gamma \mathbb{V} [v | \eta_i, p]}, \quad p = \mathbb{E} \left[\mathbb{E} [v | \eta_i, p] \mid p \right] - \gamma \mathbb{V} [v | \eta_i, p] u \end{aligned}$$

6

Hellwig: conclusion

- $\mathbb{E} [v | \eta_i, p] = a\eta_i + bp$ for $a, b > 0$
- Traders **respond to private signals**: high $\eta_i \Rightarrow$ buy, low $\eta_i \Rightarrow$ sell
- This is because i believes that when $\eta_i > p$, this might be because high supply u depressed price, hence a purchase is justified (traders **disagree about the fundamental** value v)
- What if the traders care not about v , but the resale value instead?

7

Brown and Jennings [1989]

- Now consider the same economy but in dynamics: there are two periods $t = 1, 2$, two populations of traders with CARA preferences:
 - “early traders” $i \in [0, 1]$;
 - “late traders” $j \in [0, 1]$;
 - early traders need to offload their asset holdings to late traders:

$$W_i = (p_2 - p_1)x_i,$$

$$W_j = (v - p_2)x_j$$

- Private signals $\eta_i = v + \epsilon_i$, $\eta_j = v + \epsilon_j$
- Asset aggregate supply u_1, u_2 i.i.d. normal in the two periods

9

Brown-Jennings: eqm

- In $t = 2$, same as Hellwig:

$$x_j(\eta_j, p_2, p_1) = \frac{\mathbb{E}[v|\eta_j, p_2, p_1] - p_2}{\gamma \mathbb{V}[v|\eta_j, p_2, p_1]}, \quad p_2 = \mathbb{E}\left[\mathbb{E}[v|\eta_j, p_2, p_1] \mid p_2, p_1\right] - \gamma \mathbb{V}[v|\eta_j, p_2, p_1] u_2$$

- In $t = 1$, instead of $(v|\eta_i, p_1)$, traders now care about $(p_2|\eta_i, p_1)$:

$$x_i(\eta_i, p_1) = \frac{\mathbb{E}[p_2|\eta_i, p_1] - p_1}{\gamma \mathbb{V}[p_2|\eta_i, p_1]}$$

i.e., their demand depends on $\mathbb{E}[\mathbb{E}[v|\eta_j, p_2, p_1]|\eta_i, p_1]$ – their expectation of later investors' expectation of v .

10

Brown-Jennings: conclusions

- **Second-order beliefs** are important for trading volumes in dynamic settings.
- In this specific model, early traders have no reason to believe that late traders' estimate differs from their own:

$$\mathbb{E}[\mathbb{E}[v|\eta_j, p_2, p_1]|\eta_i, p_1] = \mathbb{E}[v|\eta_i, p_1],$$

hence early traders only trade (amongst themselves) for the same reason as before – they **disagree about the resale value**.

11

Brown-Jennings: public signals

- Revealing a **public signal** ν about v at $t = 1$ would make traders **agree more** (they would put less weight on η_i), hence there would be **less trade** at $t = 1$; the less trade, the more informative ν is.
- But in reality, there is a lot of trading when public news are revealed (Bailey, Karolyi, and Salva [2006]). **Potential explanations** include:
 - Announcements are made up of public **and private** signals (in the presence of HFTs, public news $\approx$ HFTs' private news)
 - Or traders have **heterogeneous priors** and can therefore 'agree to disagree'; announcements then can amplify or mitigate these initial disagreements and so generate trade.
- Turns out, there's another explanation: if you craft a more elaborate information structure, you can generate disagreement from public news!

12

Kondor [2012]: Example

- Two groups of traders again, I and J
- Fundamental value has two components: $v = v_I + v_J$
- I-trader signal: $\eta_i = v_I + \epsilon^i$
- J-trader signal: $\eta_j = v_J + \epsilon^j$
- Public signal: $\nu = v + \epsilon^p$
- Suppose $v_I, v_J, \epsilon^i, \epsilon^j, \epsilon^p$ are independent and normal with zero mean

Main features: public signal about all of v , private signals about *different aspects* of the fundamentals, v_I and v_J .

14

Example: Before public announcement

Benchmark: No public news (ν not observed)

- Traders' beliefs about v are

$$\mathbb{E}(v|\eta_j) = \mathbb{E}(v_J|\eta_j) + \mathbb{E}(v_I|\eta_j) = a_J\eta_j + 0, a_J > 0$$

$$\mathbb{E}(v|\eta_i) = \mathbb{E}(v_J|\eta_i) + \mathbb{E}(v_I|\eta_i) = a_I\eta_i + 0, a_I > 0$$

- I-trader's second-order belief is

$$\mathbb{E}(\mathbb{E}(v|\eta_j)|\eta_i) = \mathbb{E}(a_J\eta_j|\eta_i) = 0,$$

i.e., $\mathbb{E}(\mathbb{E}(v|\eta_j)|\eta_i) = \mathbb{E}(v)$ because η_i and η_j are independent

15

Example: With public announcement

Given public signal ν

- Traders' beliefs about ν are

$$\mathbb{E}(\nu|\eta_j, \nu) = \mathbb{E}(\nu_J|\eta_j, \nu) + \mathbb{E}(\nu_I|\eta_j, \nu) = (b_J\eta_j + c_J\nu) + x_J\nu,$$

$$\mathbb{E}(\nu|\eta_i, \nu) = \mathbb{E}(\nu_J|\eta_i, \nu) + \mathbb{E}(\nu_I|\eta_i, \nu) = (b_I\eta_i + c_I\nu) + x_I\nu,$$

where $b_k, c_k, x_k > 0$ and $b_k < a_k$.

First-order beliefs of i -traders **converge** due to public signal

- But I -agent's second-order belief is

$$\begin{aligned}\mathbb{E}(\mathbb{E}(\nu|\eta_j, \nu)|\eta_i, \nu) &= b_J\mathbb{E}(\eta_j|\eta_i, \nu) + (c_J + x_J)\nu, \\ &= b_J(e\nu - f\eta_i) + (c_J + x_J)\nu\end{aligned}$$

where $e, f > 0$.

16

Example: Conclusion

- $\mathbb{E}(\mathbb{E}(\nu|\eta_j, \nu)|\eta_i, \nu)$ is **decreasing in η_i** and the weight on η_i *increases* with the precision of ν
- i.e., second-order beliefs **diverge** among i -traders: the more precise ν is, the less I -traders agree about the resale value of the asset $\Rightarrow$ more trade among i !
- This disagreement generates trade after public signals.
- The remainder of the slides presents the Kondor's trading model and derivations in *slightly* more detail.

17

Kondor [2012]: Full(er) Model

■ Timing:

- 1 I-traders observe their information and trade
- 2 I-traders liquidate all their positions and sell to J-traders
- 3 v (distributed as before) is realized and J-traders consume asset

■ Traders: Price takers, $i, j \sim U(0, 1)$, demand $x_i(p_t)$, util $u(W_i) = -e^{-\gamma W_i}$ and wealth

$$W_I = (p_2 - p_1)x_I; \quad W_J = (v - p_2)x_J.$$

■ Supply: Time- t asset supply u_t (from noise traders):

$$u_1 \sim \mathcal{N}(0, 1/\delta_1^2), \quad u_2 \sim \mathcal{N}(0, 1/\delta_2^2) \quad (u_2 \equiv u_1 + \Delta u_2)$$

19

Analysis: Trader maximization

■ Random supply: Implies that prices are not perfectly informative.

■ I traders: Solve

$$\max_{x_I} \mathbb{E} \left[-e^{-\gamma W_I} | \eta_i, \nu, p_1 \right]$$

■ CARA utility and normal distributions $\Rightarrow$ can rewrite I traders' problem as

$$\max_{x_I} \left\{ \mathbb{E} [W_I | \eta_i, \nu, p_1] - \frac{\gamma}{2} \mathbb{V} [W_I | \eta_i, \nu, p_1] \right\}$$

■ J traders: Solve

$$\max_{x_J} \left\{ \mathbb{E} [W_J | \eta_j, \nu, p_1, p_2] - \frac{\gamma}{2} \mathbb{V} [W_J | \eta_j, \nu, p_1, p_2] \right\}$$

20

Analysis: Trader maximization (2)

- Taking the FOC and solving for the demands we get

$$x_{1,i}^* = \frac{\tau_{p_2}^2}{\gamma} (\mathbb{E}[p_2 | \eta_i, \nu, p_1] - p_1), \quad (1)$$

$$x_{2,j}^* = \frac{\tau_v^2}{\gamma} (\mathbb{E}[v | \eta_j, \nu, p_1, p_2] - p_2), \quad (2)$$

where $\tau_{p_2}^2 = 1/\mathbb{V}(p_2 | \eta_i, \nu, p_1)$ and $\tau_v^2 = 1/\mathbb{V}(v | \eta_j, \nu, p_1, p_2)$

- In order to calculate expectations, need to make a conjecture about prices

21

Analysis: Linear prices and price signals

- **Equilibrium:** Look for equilibrium with linear price function/demand

$$p_1 = \frac{1}{e_1} [a_1 v_I + c_1 \nu - u_1] \quad (3)$$

$$p_2 = \frac{1}{e_2} [b_2 v_J + c_2 \nu + \eta_2 \mu_1 - u_2], \quad (4)$$

for some $a_1, b_2, c_1, c_2, e_1, e_2, \eta_2$, where μ_1 is the **price signal** of v_I

- **Price signal:** This tells us the information contained in prices:

$$\mu_1 \equiv \mathbb{E}[v_I | p_1, \nu] = \frac{e_1 p_1 - c_1 \nu}{a_1} = v_I - \frac{1}{a_1} u_1; \quad (5)$$

$$\mu_2 \equiv \mathbb{E}[v_J | p_1, p_2, \nu] = \frac{e_2 p_2 - c_2 \nu - \eta_2 \mu_1}{b_2} = v_J - \frac{1}{b_2} u_2. \quad (6)$$

22

Analysis: Reformulating in terms of price signals

- **Rewrite expectation.** All variables jointly normal $\rightarrow$ linear expressions

$$\mathbb{E}[p_2|\eta_i, \nu, p_1] = a_1^e \eta_i + b_1^e \nu + c_1^e \mu_1; \quad (7)$$

$$\mathbb{E}[\nu|\eta_j, \nu, p_1, p_2] = a_2^e \eta_j + b_2^e \nu + c_2^e \mu_1 + d_2^e \mu_2. \quad (8)$$

- **Rewrite FOC.** Plugging (7) and (8) into (1) and (2) we get

$$x_{1,i}^* = \frac{\tau_{p_2}^2}{\gamma} (a_1^e \eta_i + b_1^e \nu + c_1^e \mu_1 - p_1); \quad (9)$$

$$x_{2,j}^* = \frac{\tau_v^2}{\gamma} (a_2^e \eta_j + b_2^e \nu + c_2^e \mu_1 + d_2^e \mu_2 - p_2). \quad (10)$$

- **Market clearing:** $u_1 = \int_0^1 x_{1,i}^* di$ and $u_2 = \int_0^1 x_{2,j}^* dj$ determine p_1 and p_2 resp.

23

Analysis: Equilibrium

- **Matching coefficients:** From market clearing, can show that p_t is linear function as conjectured

- **Equilibrium demand:** Matching up all the coefficients, we can then show that

$$x_{1,i}^* = a_1 \eta_i + c_1 \nu - e_1 p_1; \quad (11)$$

$$x_{2,j}^* = b_2 \eta_j + c_2 \nu + \eta_2 \mu_1 - e_2 p_2. \quad (12)$$

- Demand is increasing in private signal (η_i/η_j), in public signal (ν), in price signal (μ_1), and decreasing in price (p_t) (recall that traders are price takers)

24

Results: Demand period 2

- Let's look at what drives the agents' demands. Rewrite period-2 demand as

$$x_{2,j}^* = b_2(\eta_j - \mu_2) \quad (13)$$

- Notice that η_j is j 's private signal and μ_2 is a noisy signal of all the other agents' signals
- Thus, if j believes to have received a better signal than everybody else, he will buy, otherwise sell
- This is a standard story: J -traders trade due to a **difference in opinion** – they think the asset is worth more/less than others (as in Hellwig/Brown-Jennings)

25

Results: Demand period 1

- Market clearing in period 2 together with (2) implies

$$p_2 = \int_0^1 \mathbb{E}[v | \eta_j, \nu, \mu_1, \mu_2] dj - \frac{\gamma}{\tau_v^2} u_2$$

- Rewrite period-1 demand using this:

$$x_{1,i}^* = \frac{\tau_{p_2}^2}{\gamma} \left(\underbrace{\mathbb{E} \left[\int_0^1 \mathbb{E}[v | \eta_j, \nu, \mu_1, \mu_2] dj - \frac{\gamma}{\tau_v^2} u_2 \middle| \eta_i, \nu, \mu_1 \right]}_{\text{2nd-order expectation}} - p_1 \right)$$

- I -trader demand in period 1 is thus a function of a **second-order expectation**: The more i expects J traders to value the asset, the more he buys

26

Results

- In the paper, Kondor considers a more general information structure where there is a common factor about which I and J both learn.
- He then defines *weakly correlated* information structures in which the common factor is not too important – otherwise we're back in Brown and Jennings [1989] world
- In the above, we have disregarded the common factor, so what we analyzed is automatically a weakly correlated information structure

Main result

If the information structure is *weakly correlated*, then trading intensity, volume and informational content of prices **increase** in both periods when there is more public info.

- Public signals create trade, due to their effect on second-order beliefs

27

Model 2

Heterogenous trading horizons

- Timing
 - 1 I-traders and J-traders trade
 - 2 I-traders sell all their holdings to J-traders and quit
 - 3 v is realized and J-traders consume
- Let μ be the proportion of J-traders

Interpretation

- Traders with different trading horizons co-exist in the market
- For instance day-traders and pension savers

28

Model 2: Results

- When μ is high, most traders trade with each other: the market is well-integrated
- When μ is small, the results of model 2 are close to those of model 1
- But when μ is high, public information crowds out private information, and public signals have the usual effect
 - I.e., there will be less disagreement and less trade
- Thus, integration is key to the results (what happens to ST speculation as market becomes more integrated?)

29

Relation to empirics

- In general, the model provides an explanation for trade after public announcements
- Bailey, Karolyi, and Salva [2006] find that price volatility and trading volumes increase after earnings announcements
 - They find that the effect is larger for cross-listed stocks
 - Kondor argues that cross-listing is roughly equivalent to lower market integration: lower μ

30

Kondor: Conclusion

- Public announcements can affect second-order beliefs, thereby generating trade and increasing price volatility!
- This requires some very specific assumptions on the information structure in the market though...
- The model goes primarily towards explaining some empirical puzzles; not clear whether we should base welfare analysis on it.
- But it should allow us to predict better which stocks will react strongly to announcements

31

References I

- W. Bailey, G. A. Karolyi, and C. Salva. The economic consequences of increased disclosure: Evidence from international cross-listings. *Journal of Financial Economics*, 81(1):175–213, 2006. URL <https://doi.org/10.1016/j.jfineco.2005.06.002>. Publisher: Elsevier.
- D. P. Brown and R. H. Jennings. On Technical Analysis. *The Review of Financial Studies*, 2(4):527–551, 1989. ISSN 0893-9454. URL <https://www.jstor.org/stable/2962067>. Publisher: [Oxford University Press, Society for Financial Studies].
- M. F. Hellwig. On the aggregation of information in competitive markets. *Journal of Economic Theory*, 22(3):477–498, June 1980. ISSN 0022-0531. doi: 10.1016/0022-0531(80)90056-3. URL <https://www.sciencedirect.com/science/article/pii/0022053180900563>.
- P. Kondor. The more we know about the fundamental, the less we agree on the price. *The Review of Economic Studies*, 79(3):1175–1207, 2012. URL <https://doi.org/10.1093/restud/rdr051>. Publisher: Oxford University Press.

Financial Markets Microstructure

Lecture 18

Asset Price Bubbles

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Previously on FMM

- Information and trading volumes:
 - Private signals create disagreement → generate trade, which reveals and aggregates private info
 - Public signals should theoretically mitigate disagreement and lead to less trade
 - But IRL trading volumes increase around public announcements
 - **Kondor**: Possible explanation through second-order beliefs

Today

Two stories for why **bubbles** may occur

- Rational herding
 - Herding: following the actions of others, even when this goes against one's own private information
 - We will look at different explanations for why this may be rational
- Lack of common knowledge/coordination (Abreu and Brunnermeier [2003])
 - There is a difference between everybody knowing that an asset is overpriced, and everybody knowing that everybody knows...
 - Again, speculation depends on these *higher-order beliefs*

3

Bubbles

- **Wikipedia:** “Trade in high volumes at prices that are considerably at variance with intrinsic values”
- **Investopedia:** “A surge in equity prices, often more than warranted by the fundamentals and usually in a particular sector, followed by a drastic drop in prices as a massive sell-off occurs”
- **Chicago Fed:** “...a bubble exists when the market price of an asset exceeds its price determined by fundamental factors by a significant amount for a prolonged period”

4

Herding: Introduction

- **Herding**: ignoring private information in favor of “wisdom of the crowd”
 - 1 Herding may be the efficient response to new information or similarity between agents...
 - 2 ...but it may also be the inefficient result of a certain decision making process. Here: focus on the latter
- In **financial markets**: momentum trading, positive-feedback trading
- In 1992, a wave (a herd?) of articles showing that herding could be result of rational [informational cascade](#)
- Today we look at this and other ‘rational’ explanations
- Note: I will follow the presentation in Bikhchandani and Sharma [2000].
See Bikhchandani et al. [2024] for a comprehensive review of the literature.

9

Preview

- Agents arrive at the market sequentially and need to make a decision
- Every agent has a private signal and observes decisions of previous agents (but not their signals!)
- Ideally: pool private information to find best decision
- But here: [sequential decision making](#)
 - First-comers: make decisions based on information
 - But as time goes by, people may start disregarding their own information and just choose the most popular action
 - Herding ensues, but it is *fully rational*: public information swamps private information
- [Informational cascade](#): a few pieces of information may determine everyone’s choice

10

Model

- **State** (fundamental) $v \in \{L, H\}$;
- In each period $t \in \{1, 2, \dots\}$ an individual arrives and needs to make a **decision** $d_t \in \{0, 1\}$ (invest or not);
- **Payoffs** $u(d_t, v)$:
 - $u(0, v) = 0$,
 - $u(1, L) = L - m < 0$,
 - $u(1, H) = H - m > 0$,
 - (say for now that price (midquote) m is fixed and there is no spread: $a = b = m$)

11

Model: Beliefs

- Represent all beliefs p about v as probabilities of $v = H$ conditional on relevant info.
- “Market belief” q_t
 - q_0 is the public prior belief – e.g., $1/2$;
 - q_t incorporates info contained in decisions at $t = 1, 2, \dots, t - 1$
- Period- t agent observes q_t and a private signal η_t and forms private belief r_t .
 - Suppose $\eta_t \in \{h, l\}$ with $\mathbb{P}(\eta_t = h | v = H) = \mathbb{P}(\eta_t = l | v = L) = \rho$.
- All beliefs calculated using Bayes’ rule

12

Decisions

- Agent behaves optimally $\Rightarrow$ chooses $d_t = 1$ iff $r_t \geq \bar{r} \equiv \frac{m-L}{H-L}$.
- Use Bayes' rule to compute $r_t(\eta_t, q_t)$:

$$r_t(h, q_t) = \frac{q_t \rho}{q_t \rho + (1 - q_t)(1 - \rho)}$$
$$r_t(l, q_t) = \frac{q_t(1 - \rho)}{q_t(1 - \rho) + (1 - q_t)\rho}$$

Note $r_t(h, q_t) > q_t > r_t(l, q_t)$.

13

Herds and cascades

- If $r_t(h, q_t) > \bar{r} > r_t(l, q_t)$ then the agent follows their signal
 - $\rightarrow$ their action is informative
 - $\rightarrow$ public belief is updated: $q_{t+1}(q_t, d_t = 1) = r_t(h, q_t)$; $q_{t+1}(q_t, d_t = 0) = r_t(l, q_t)$.
- If $r_t(h, q_t) > r_t(l, q_t) > \bar{r}$ then the agent chooses $d_t = 1$ regardless of private signal
 - $\rightarrow$ action uninformative $\rightarrow q_{t+1} = q_t$
 - $\rightarrow$ next agent will also ignore private signal!
 - Everyone ignores private signals and chooses the same action (we have a **herd**)
 - Market belief q_t is frozen in place; private information is not aggregated!
 - This **herd may be incorrect** (unless $q_t = 0$ or $q_t = 1$)
 - (Same happens if $\bar{r} > r_t(h, q_t) > r_t(l, q_t)$ with $d_t = 0$)

14

What triggers the herd?

- A few incorrect signals can be enough to set off a herd
- A small amount of information 'cascades' through the system
- Each agent is rational, but together they may seem stupid: their information taken together is very precise, but information aggregation fails
- A specific example with numbers of how a herd may arise: [here](#)

15

Herds and cascades: comments

- Incorrect herds only occur if distribution of η_i is bounded – otherwise strong enough signals could overpower the public info
- In a slightly richer model, the opposite outcome is also possible – state of permanent uncertainty in which everyone acts solely on their private signal, ignoring public information
- Terminology:
 - **Herd** = action convergence ($d_{t+1} = d_t$ from some point onwards)
 - **Cascade** = public belief convergence ($q_{t+1} = q_t$ from some point onwards)
 - Distinction is not super important for our purposes

16

What if we introduce a price?

- As before: unknown asset value v , and each trader receives a private signal η_i
- With probability π the trader is a noise trader, who buys/sells/abstains with equal probability, w.p. $1 - \pi$ rational as above
- A risk-neutral market maker quotes competitive bid-ask **prices**
- What will happen?
 - This is standard Glosten-Milgrom, so no herds!
 - Prices adjust in such a way that following private signals is optimal, and prices themselves then incorporate all private signals!

17

More layered model

- However, the GM result is in part due to model simplicity
- Avery and Zemsky [1998] expand on this analysis
 - Suppose $v \in \{0, \frac{1}{2}, 1\}$
 - If $v = \frac{1}{2}$, this is perfectly revealed by the private signal: $\eta_t = \frac{1}{2}$
 - If $v \in \{0, 1\}$, trader t receives informative signal with precision $\rho_t \equiv \mathbb{P}(\eta_t = v)$
 - Furthermore, a proportion $\mu \in \{\mu^H, \mu^L\}$ of traders have perfect information: $\rho_t = 1$
 - The remaining $1 - \mu$ traders have noisy info: $\rho_t \in (\frac{1}{2}, 1)$
- Three levels of uncertainty:
 - Event uncertainty: $v = 1/2$ (**no event**) or $v \in \{0, 1\}$ (**event**)
 - Value uncertainty: if event, $v = 1$ or $v = 0$
 - Composition uncertainty: many informed traders (μ^H : **well-informed economy**) or few (μ^L : **poorly-informed economy**)

18

Herding can occur with pricing and multi-layered uncertainty

- In GM, price mechanism worked as a screening device: made sure that high types bought and low types sold
- But now, it is possible to 'misprice' such that herding occurs, at least temporarily
 - Can be because $\mu = \mu_H$ so all traders know v , but MM does not (non-speculative bubble)
 - Can be because $\mu = \mu_L$ but traders do not know that and perceive past order flow as more informative than it is (speculative bubble)
- This allows bubbles to occur

See numerical example [here](#)

19

Other types of herding: Reputational herding

A brief example.

- **Two managers:** Invest or not in project of unknown value (gets imperfect signal $\eta \in \{1, 0\}$)
- **Type:** Each manager either smart or dumb (doesn't know). If both managers smart, observe same η ; if one or both dumb, observe independent signals η
- **Payoffs:** Managers [maximizes their reputation](#) (want to appear smart)
- **Herding:** Manager 1 moves first, then manager 2
 - If manager 1 invests, manager 2 can deduce that $\eta_1 = 1$. Suppose $\eta_2 = 0$. Then one of the two (or both) must be dumb.
 - If manager 2 does not invest, he reveals this. If player 1's investment then succeeds, people will assume that manager 2 is the dumb manager
 - Investing might be better: even if the investment fails, people might think that both managers are smart, but got an 'unlucky' signal

20

Herding: Conclusion

- **Herding**: may occur when private information cannot be easily aggregated
- **Price mechanism**: alleviates problem by providing private incentives to trade and thus reveal information, which is then incorporated into prices
- **Multi-layered uncertainty**: can make the herds occur even with flexible prices
- **Aside**: *Momentum trading* often assumed to be a 'behavioral feature': but it may be perfectly rational
- **Empirical estimation of herding**: some conclusions can be tested in the data but to a very limited extent; see Bikhchandani and Sharma [2000] and Bikhchandani et al. [2024] for details.

21

Abreu and Brunnermeier [2003]: Introduction

- Efficient market hypothesis: bubbles are impossible
 - In finite model, use backward induction argument: no mispricing in last period, which should be foreseen in second-last period, etc. → unravelling
- This requires that all traders agree on when the bubble will collapse (at least on the distribution of times)
- Here: a model in which coordination is needed to cause a crash
- Coordination, in turn, depends on beliefs about others
 - Back to higher order beliefs

23

Model

- **Single asset traded:** value at t is v_t
- **Progress:** At $t = 0$, technological progress makes v_t grow at rate g
- **Slowdown:** at some random time t_0 , there is a slowdown and v_t growth slows to $r < g$
- **Price:** Price grows at rate g until either
 - At least a fraction κ of rational traders sell the asset (κ is the *absorption capacity* of the economy), or
 - The market is exogenously corrected at time $t_0 + \bar{\tau}$
- **Gradual learning:** Each period, a fraction $1/\eta$ of rational traders become aware of the mispricing
 - But they don't know t_0 , and hence don't know how many others know of the mispricing

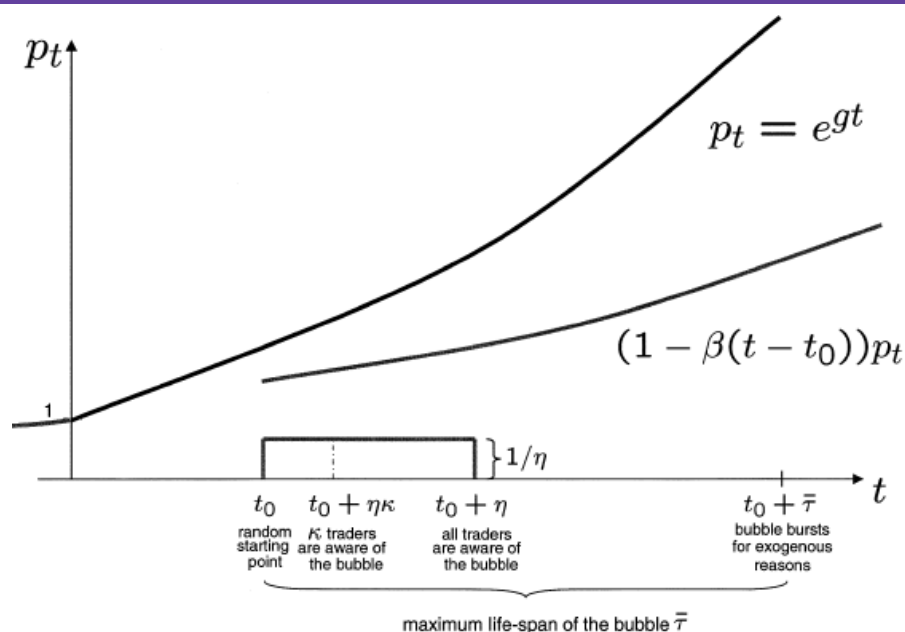
24

Traders

- **Behavioral traders** think progress will last forever
 - When enough rational traders sell, absorption capacity of behavioral traders is reached and price must drop
 - At this point, everybody becomes aware of the mispricing
 - If this doesn't happen, then market exogenously collapses when mispricing is too big
- **Rational traders** know progress is temporary, but not when it stops
 - Implication: when you learned that progress has stopped, you are not sure what other people believe
 - Suppose you learn at t' . At $t' + \eta$ you know that everybody knows
 - But somebody else might learn that progress stopped at $t'' > t'$; he will not know that everybody knows until $t'' + \eta > t' + \eta$

25

Graphically



(Note: graph is from the paper and it has a mistake)

26

Why bubbles then?

- In this model there is never *common knowledge*: you are never sure what the others know. This prevents usual backward induction
- At least κ traders must sell to burst bubble: coordination important
- But it is hard to coordinate without common knowledge
- Therefore, mispricing can go on for a long time

Even if traders realize the market will crash, they don't know exactly when, incentivizing them to ride the bubble

27

Definition of a bubble

- In this setting, let's use a very strict definition of a bubble:
mispricing should be well-known to traders to count as a bubble

Definition

A bubble is persistent mispricing *beyond* $t_0 + \eta\kappa$

- After $t_0 + \eta\kappa$, the mispricing is known by enough traders to correct it, should they want to

28

Some observations

The following is shown in the paper:

- Traders either take the maximum long (buy) or short (sell) position
- When a rational trader 'goes short', all traders who learned of the mispricing before him will already have gone short
- Once a rational trader goes short, he never re-enters the market: he waits for the bubble to burst

29

Bubbles and crashes

Generally, in the model, there are two possible types of equilibria

- Exogenous crash
 - When the growth rate g under the bubble is high, dispersion η and absorption capacity κ are high, informed traders sell out very slowly
 - In effect, they take a chance on 'riding the bubble'
 - As a result, selling will never be sufficient to burst the bubble, which will burst at the exogenous date $t_0 + \bar{\tau}$
- Endogenous crash
 - When traders have incentives to sell quick, this leads to unraveling: enough traders will sell to make the bubble burst
 - However, 'bubble incentives' remain: the sooner the price crashes, the smaller the cost of riding the bubble
 - Therefore, the bubble will be smaller but still exist

30

The role of 'sunspots'

- **Sunspot equilibria:** refers to equilibria where *economically irrelevant information* has an influence
- In our case: suppose an uninformative event is observed with a certain probability
- If informed traders decide to coordinate their actions around this event (eg. use the strategy 'sell when event occurs'), it can become pivotal
- Example from article: in 1980s trade data had big market impact in the US; in 1990s Alan Greenspan's statements were more influential
 - If sufficiently many people react to an event, then you must too - even if the event carries little/no information by itself

31

Abreu and Brunnermeier: Conclusion

- Standard arguments against bubbles rely a lot on common knowledge between agents
- When we dispense with common knowledge, belief dispersion among agents can cause mispricings to persist even after everyone has observed the mispricing
- Thus, bubbles may not be fixed by the market
- As a side-effect, seemingly insignificant events can serve to coordinate actions and cause crashes

32

Herding example: Setup

- This example follows that in Bikhchandani and Sharma [2000]
- Suppose $q_0 = 1/2$
- **Payoffs:** Let $L - m = -1$; $H - m = 1$
 - Then $\bar{r} = 1/2$
 - Suppose that agents flip a 50/50 coin whenever indifferent
- Suppose the following signal sequence realized: $\{h, l, l, l, \dots\}$
- Denote by $\mathbb{I}_i$ agent i 's information set

Herding example: Analysis

First agent

- Information set $\mathbb{I}_1 = \{\eta_1 = h\}$

- Then

$$r_1(h, 1/2) = \mathbb{P}(v = H | \mathbb{I}_1) = \frac{\frac{1}{2}\rho}{\frac{1}{2}\rho + \frac{1}{2}(1-\rho)} = \rho$$

- Since $\rho > 1/2$: invest.
- **Thus**, given $\eta_1 = h$, first agent invests: $d_1 = 1$

Herding example: Analysis (2)

Second agent

- Second agent can perfectly deduce first agent's signal: $\eta_1 = h$ if $d_1 = 1$, $\eta_1 = l$ otherwise.
 - In other words, $q_2 = r_1 = \rho$, no information is lost.
 - Second agent receives $\eta_2 = l$, his information set is $\mathbb{I}_2 = \{\eta_1 = 1, \eta_2 = -1\}$

- Signals are *symmetric*, so

$$r_2(l, \rho) = \mathbb{P}(v = H | \mathbb{I}_2) = \frac{\frac{1}{2}\rho(1-\rho)}{\frac{1}{2}\rho(1-\rho) + \frac{1}{2}(1-\rho)\rho} = 1/2$$

- Thus, second agent is indifferent, flips a coin to decide. If $d_2 = 0$ then we are back to $q_3 = 1/2$.
- But suppose the coin-toss decides invest: $d_2 = 1$

Herding example: Analysis (3)

Third agent

- Third agent can also perfectly deduce the first agent's signal
- Information set is $\mathbb{I}_3 = \{\eta_1 = h, d_2 = 1, \eta_3 = l\}$. Furthermore,

$$\mathbb{P}(d_2 = 1 | v = H) = \rho + (1 - \rho)(1/2) = (1 + \rho)/2$$

$$\mathbb{P}(d_2 = 1 | v = L) = \rho(1/2) + (1 - \rho) = 1 - \rho/2$$

$$\Rightarrow q_3(q_2 = \rho, d_2 = 1) = \frac{\rho^{\frac{1+\rho}{2}}}{\rho^{\frac{1+\rho}{2}} + (1 - \rho) \left(1 - \frac{\rho}{2}\right)}$$

$$\Rightarrow r_3(l, q_3) = \frac{\rho^{\frac{1+\rho}{2}}(1 - \rho)}{\rho^{\frac{1+\rho}{2}}(1 - \rho) + (1 - \rho) \left(1 - \frac{\rho}{2}\right) \rho} = \frac{1 + \rho}{3} > \frac{1}{2}$$

- Hence, $d_3 = 1$

Herding example: Analysis (4)

Fourth agent

- Information set $\mathbb{I}_4 = \{\eta_1 = h, d_2 = d_3 = 1, \eta_4 = l\}$
- Agent four knows that agent three would have invested regardless of his signal
- So he doesn't learn anything from d_3 . So $q_4 = q_3$ and $r_4(l, q_4) = r_3(l, q_3)$
- Hence $d_4 = 1$
- Same for all subsequent agents: $d_i = 1$ for all i , regardless of η_i !
- We have a herd! – and a very inefficient one at that! (Look at signals)

[back](#)

Herding with prices example

Take an extreme case of Avery and Zemsky's model: suppose the following parameter values

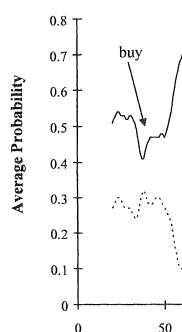
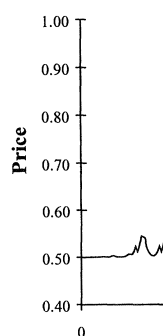
- $\mathbb{P}(v = 1/2) = 0.9999$: very small prior probability of an 'event';
 $\mathbb{P}(v = 1) = \mathbb{P}(v = 0) = 0.00005$
- $\frac{\mathbb{P}(\mu = \mu^H)}{\mathbb{P}(\mu = \mu^L)} = 99$: high prior probability of a well-informed economy
- If economy is poorly informed: all traders have $\rho_i = 0.51$, i.e. very poor signal about value

Suppose we're in an unlikely state of the world

- 1 Value is low ($v = 0$) implying that there is an event
- 2 The economy is poorly informed ($\mu = \mu^L$)

How will market learn state of the world? Let's look at a simulation

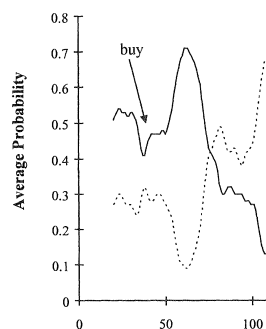
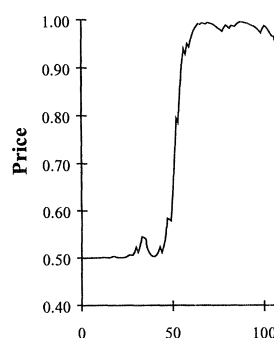
Herding with prices example (2)



Three of first five traders buy

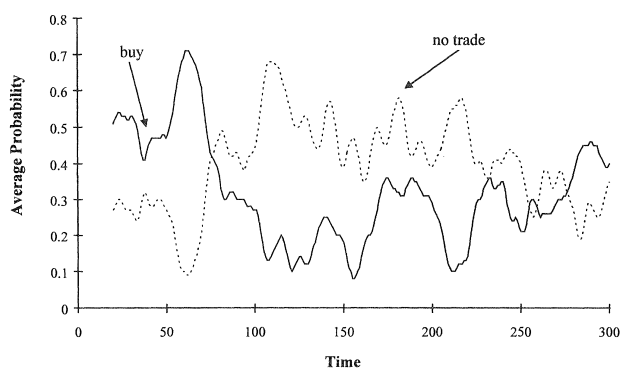
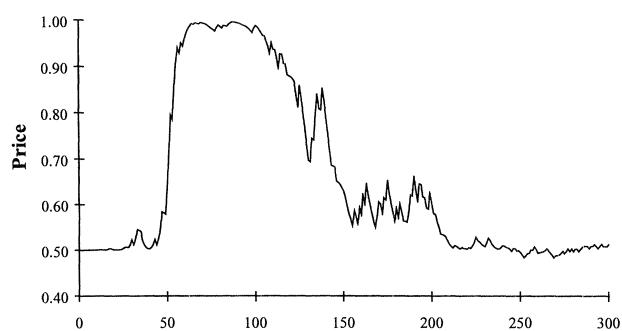
- Since economy poorly informed: herd buying starts
- MM thinks it's likely that economy is well informed: price goes up

Herding with prices example (3)



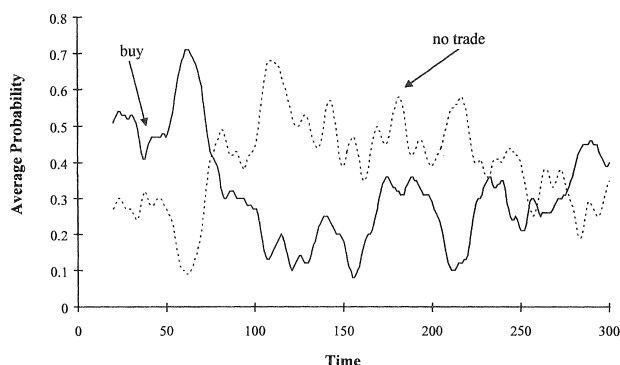
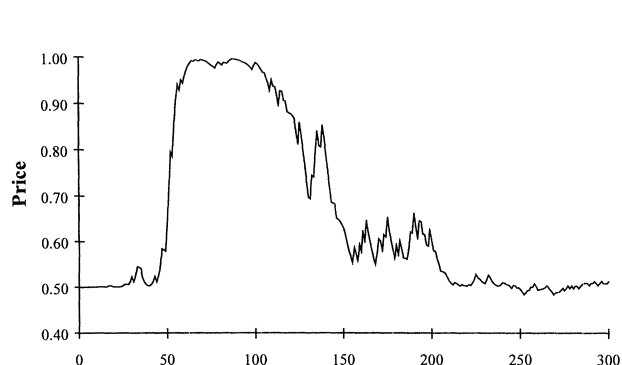
As the price goes up, the herd is broken: trading volume diminishes as rational traders drop out

Herding with prices example (4)



As herd becomes apparent, MM realizes that only a few informative trades have been made → price toward $1/2$

Herding with prices example (5)



Rational traders only re-enter in period 220: trade based on information (buy if high/sell if low)

In the example, the price is persistently above the level that would ensue if traders pooled their information

[Back](#)

Written Exam Economics Summer 2020

Financial Markets Microstructure

June 12, 2020

This exam question consists of 5 pages in total

Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '127.pdf') and uploaded to Digital Exam.

Be careful not to cheat at exams!

Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules laid down in part 4.12 of the Faculty of Social Science's common part of the curriculum on cooperation/sparring

You can read more about the rules on exam cheating on your Study Site and in part 4.12 of the Faculty of Social Science's common part of the curriculum.

Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

Problem 1

In the Glosten-Milgrom model, as well as many other models we had in the class (e.g., Glosten, Parlour, Duffie-Garleanu-Pedersen, etc), strategic traders are restricted to trading at most one unit of the asset. Without this restriction, they would be willing to buy or sell infinite amounts.

1. What factors preclude or disincentivize such behavior in the real world?
2. Is it reasonable to model these factors as an exogenous constraint on trade size? If not, how would you incorporate them in the Glosten-Milgrom model?

Problem 2

Consider an asymmetric Glosten-Milgrom model in which the fundamental value v is $v = 10$ with probability $\gamma < 1/2$ and $v = 5$ w.p. $1 - \gamma$. The arriving trader can submit a buy or a sell order for one unit of the asset. The trader is informed w.p. π and is a noise trader w.p. $1 - \pi$. In the latter case the trader submits a buy order w.p. $\rho > 1/2$ and a sell order w.p. $1 - \rho$ independently of v . The informed trader knows v and trades so as to maximize profit. The dealer is risk-neutral and competitive.

1. Derive the ask and bid quotes set by the dealer.
2. Calculate the bid-ask spread. How would the spread react to an increase in ρ ? How does your answer depend on π ?
3. What is the intuition behind this dependence on π ?

Problem 3

This question is based on the Kondor model. Suppose that the fundamental value of the asset is given by the sum of L individual components:

$$\theta = \theta_1 + \theta_2 + \dots + \theta_L,$$

where each component $\theta_l \sim \text{i.i.d.} \mathcal{N}(0, \sigma^2)$. There are two strategic traders, label them i and j . Trader i observes the first I components of θ (denote their sum as $x_i = \theta_1 + \dots + \theta_I$), while trader j observes the last J components (denote their sum as $x_j = \theta_{L-J+1} + \dots + \theta_L$). Assume $I + J < L$, so there are no components observed by both traders, but there are some which are not observed by either one (denote their sum as $x_k = \theta_{I+1} + \dots + \theta_{L-J}$). In addition, both traders observe the same public signal $y = \theta + \epsilon$, where $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$.

In answering the questions below, you can use the following fact:

If $q = v + u$, where $v \sim \mathcal{N}(\mu_v, \sigma_v^2)$ and $u \sim \mathcal{N}(\mu_u, \sigma_u^2)$, and the two are independent, then: (1) $q \sim \mathcal{N}(\mu_v + \mu_u, \sigma_v^2 + \sigma_u^2)$, and (2) $\mathbb{E}[v|q] = \mathbb{E}[v] + (q - \mathbb{E}[q]) \frac{\mathbb{C}(v,q)}{\mathbb{V}(q)}$.

1. Calculate $\mathbb{E}[\theta|x_j, y]$, i.e., trader j 's asset valuation conditional on the information available to him.¹
2. Calculate the second-order expectation $\mathbb{E}[\mathbb{E}[\theta|x_j, y] | x_i, y]$, i.e., trader i 's expectation of trader j 's valuation, conditional on i 's information.

HINT: you should get an expression of the form $\mathbb{E}[\mathbb{E}[\theta|x_j, y] | x_i, y] = \alpha y + \beta(y - x_i)$. You need to find α and β .

3. Take the coefficient β you have obtained in part 2 (if you did everything correctly, it should be positive). It captures the degree of divergence of second-order beliefs: the higher is trader I 's private signal of asset value, the lower he expects J 's valuation to be, and the higher is β , the stronger this effect is. How does β depend on I , the number of components that trader i observes? Explain the intuition behind this.
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NOTE: if you could not solve part 2, you can still try to provide an educated guess for the directions and the reasons of the effects in parts 3 and 4.

Problem 4

Below you can find an Economist article from July 17, 2019 on the liquidity of the corporate bond market.² The article mentions that the actual liquidity in that market is well below what the investors seem to expect it to be. What are the possible consequences of such misalignment? Should we attempt to alleviate it by making the market more transparent? How does it relate to the discussion of market transparency we had in class?

Buttonwood – Why everybody is concerned about corporate-bond liquidity

In September 2007 Britain suffered its first bank run in a century. Television pictures showed a long queue of depositors outside a branch of Northern Rock. Alistair Darling watched in dismay from Portugal, where he and his fellow European Union finance ministers were gathered. “They’re behaving perfectly rationally, you know,” Mervyn King, the governor of the Bank of England, said

¹In general, conditioning on x_j is not the same as conditioning on $(\theta_{L-J+1}, \dots, \theta_L)$, since the latter contains more information. But the two are equivalent in this problem.

²Also available at: <https://www.economist.com/finance-and-economics/2019/07/11/why-everybody-is-concerned-about-corporate-bond-liquidity>

in the smarty-pants manner that economists are cherished for. Mr Darling was uncharmed. “It was not what I wanted to hear,” he recalled.

What Lord King probably had in mind was a well-thumbed textbook model. Banks have a liquidity mismatch. One side of the balance-sheet is hard-to-sell loans; the other side is deposits that can be withdrawn in a trice. If depositors believe that a bank is sound, there will be no runs on it. But if enough start to demand their deposits back, it makes sense for everybody to join the rush.

This model can also be applied in other areas. Take the corporate-bond market. Every policy body of stature, from the IMF to the European Central Bank (ECB), has worried about a growing mismatch between investors’ expectations that they can sell out at any moment and an underlying shortage of liquidity in the market. More investors are using corporate-bond funds as an alternative to cash. But fewer dealers are willing to trade bonds in size. A big scare could feasibly start a run.

The dynamics of capital-market runs are similar to those of bank runs. You see them in currency crises. Foreign-exchange reserves, say, are slim relative to the scale of local-currency assets held by flighty investors. Should enough of those investors sell out, others will soon follow. The result is a rout. There is a similar pattern with investment funds that promise speedy withdrawals but hold assets that cannot be sold quickly. Bad news prompts withdrawals. The speedy get paid. Other investors then try to get out, too. But the fund cannot sell assets fast enough. It is forced to suspend redemptions.

Such trouble is especially likely with corporate bonds, which are inherently illiquid. In contrast with trading in shares, where buy and sell orders are matched on electronic order books, corporate bonds are traded over-the-counter. Bonds are not as standardised as shares. A company may have bonds of several different maturities. If you want to buy or sell, you call a dealer.

The ease with which investors can trade bonds—the market’s liquidity—depends a lot, then, on the readiness of dealer banks to stockpile securities. Where there is heavy selling, dealers would ideally warehouse cheaper bonds for when people want to buy again. But since the financial crisis new rules have made it less cost-effective for banks to use capital for trading of any kind. The inventory of corporate bonds held by dealers has fallen sharply in the past decade.

As the role of dealers has shrunk, the thirst for instant liquidity has increased. Derisory yields on the safest government debt have drawn investors towards riskier securities, including corporate bonds. A cheap and convenient way to invest in them is to buy an exchange-traded fund, or ETF. These are low-cost investment funds that hold a basket of bonds, usually mirroring a benchmark index. They trade on stock exchanges just as listed shares do. The ease of buying and selling bond ETFs is a big part of their appeal. They are also often used as depositories for spare cash. Studies are divided on whether ETFs make the underlying bonds more or less liquid. But there are concerns that in a stressed market, outflows from ETFs might make a bad situation worse. And it is not hard to make a case that the corporate-bond market has become more fragile. Many firms in America have issued lots of bonds to buy back their own shares. With extra leverage comes more risk. Half of all investment-grade bonds have a credit rating of BBB. In a recession a chunk of those

bonds will be downgraded to junk. Many mutual funds and ETF s can hold only investment-grade bonds. If a lot of bonds have to change hands quickly, that could easily overwhelm the market's limited liquidity. Prices might fall a long way.

Just how messy the next big shake-out in the corporate-bond market is depends on many things: on how weak the economy gets; on how many BBB borrowers can avert a downgrade; on how quickly funds can be raised to buy at fire-sale prices. For now, it seems rational to hold bonds that afford a little extra yield. Smart-alecks say this will surely end badly. But who wants to hear that?

Financial Markets Microstructure

Final Exam with Solutions

Københavns Universitet

June 2020

Problem 1

In the Glosten-Milgrom model, as well as many other models we had in the class (e.g., Glosten, Parlour, Duffie-Garleanu-Pedersen, etc), strategic traders are restricted to trading at most one unit of the asset. Without this restriction, they would be willing to buy or sell infinite amounts.

1. What factors preclude or disincentivize such behavior in the real world?
2. Is it reasonable to model these factors as an exogenous constraint on trade size? If not, how would you incorporate them in the Glosten-Milgrom model?

Solution:

The possible factors include the following:

- **Limited liquidity/positive price impact.** The models under consideration effectively assume traders to be competitive, i.e., (1) infinitesimally small, and for that reason (2) not internalizing their impact on the market price. In the real world any trade of nontrivial quantity of the asset will have a non-zero price impact. This is not captured in the GM model, but it is the main driving force behind Kyle model, which can be seen as “GM model with price impact”.
- **Risk aversion.** If traders are risk-averse, they would not be willing to assume infinite positions, since those entail infinite risks. If we assume that informed traders in the GM model are risk-averse then we can dispose with the assumption of fixed trade size, although this gives rise to the next issue.
- **Need to blend in with noise traders.** The informed traders obtain a better price on their trades by mimicking the behavior of uninformed traders. If uninformed traders have particular trade patterns (e.g., always trade a single unit at a time) then informed

traders have incentives to mimic these patterns (restrict their order sizes). This aspect can be introduced into the model without affecting its conclusions: it can be shown that if informed traders can submit larger orders, they will never do so as long as noise traders operate in unit orders only.

- **Funding liquidity/leverage/short-selling constraints.** In order to take a large long position in an asset that is currently underpriced, the trader must have the liquid funds to do so. Similarly, short sales typically require a margin (collateral). All investors have some limits on the funds at their disposal, which yields a natural upper bound on the size of positions they can take. Relatedly, borrowing money in order to trade is typically subject to leverage constraints. Capturing all of these constraints as an exogenous limit on trade size is reasonable, but one should keep in mind that these limits may differ across traders.

Problem 2

Consider an asymmetric Glosten-Milgrom model in which the fundamental value v is $v = 10$ with probability $\gamma < 1/2$ and $v = 5$ w.p. $1 - \gamma$. The arriving trader can submit a buy or a sell order for one unit of the asset. The trader is informed w.p. π and is a noise trader w.p. $1 - \pi$. In the latter case the trader submits a buy order w.p. $\rho > 1/2$ and a sell order w.p. $1 - \rho$ independently of v . The informed trader knows v and trades so as to maximize profit. The dealer is risk-neutral and competitive.

1. Derive the ask and bid quotes set by the dealer.
2. Calculate the bid-ask spread. How would the spread react to an increase in ρ ? How does your answer depend on π ?
3. What is the intuition behind this dependence on π ?

Solution:

1. Dealer is competitive, so the quotes will capture the expected value of the asset conditional

on buy and sell orders respectively.

$$\begin{aligned}
a &= \mathbb{E}[v|\text{buy}] \\
&= \mathbb{E}[v|\text{buy,noise}] \mathbb{P}(\text{noise}|\text{buy}) + \mathbb{E}[v|\text{buy,informed}] \mathbb{P}(\text{informed}|\text{buy}) \\
&= (10\gamma + 5(1 - \gamma)) \frac{(1 - \pi)\rho}{\pi\gamma + (1 - \pi)\rho} + 10 \frac{\pi\gamma}{\pi\gamma + (1 - \pi)\rho} \\
&= \mu + 5(1 - \gamma) \frac{\pi\gamma}{\pi\gamma + (1 - \pi)\rho},
\end{aligned}$$

where $\mu = \mathbb{E}[v] = 5 + 5\gamma$. Similarly,

$$b = \mathbb{E}[v|\text{sell}] = \mu - 5\gamma \frac{\pi(1 - \gamma)}{\pi(1 - \gamma) + (1 - \pi)(1 - \rho)}.$$

2. The spread is given by

$$S = a - b = \frac{5\pi\gamma(1 - \gamma)}{[\pi\gamma + (1 - \pi)\rho][\pi(1 - \gamma) + (1 - \pi)(1 - \rho)]}.$$

Its derivative w.r.t. ρ is

$$\frac{\partial S}{\partial \rho} = -S \frac{(1 - \pi)[\pi(1 - 2\gamma) + (1 - \pi)(1 - 2\rho)]}{[\pi\gamma + (1 - \pi)\rho][\pi(1 - \gamma) + (1 - \pi)(1 - \rho)]}.$$

The denominator is positive, hence the spread reacts positively to an increase in ρ if and only if

$$\pi(1 - 2\gamma) + (1 - \pi)(1 - 2\rho) < 0 \Leftrightarrow \pi < \frac{\rho - 1/2}{\rho - \gamma}.$$

(The assumption $\gamma < 1/2 < \rho$ is important for the sign of the final inequality.)

3. In this problem the spread is asymmetric across buy and sell orders. When ρ increases, there is relatively less informed trading on the buy side (so the buy-side half-spread $a - \mu$ shrinks), but more on the sell side. The final effect on S is thus determined by which of these two components dominates.

Each half-spread is most sensitive to parameters when the probability of informed trading in the respective direction is close to 50% – this is the state of maximal uncertainty, – while otherwise the order flow is very likely to be either informative, or not, and comparable parameter value changes do not affect this probability as strongly. Given our parameter assumptions, if $\pi < \frac{\rho - 1/2}{\rho - \gamma}$ then $\mathbb{P}(\text{informed}|\text{sell})$ is closer to 1/2 than $\mathbb{P}(\text{informed}|\text{buy})$, hence the sell-side effect dominates, and vice versa.

Problem 3

This question is based on the Kondor model. Suppose that the fundamental value of the asset is given by the sum of L individual components:

$$\theta = \theta_1 + \theta_2 + \dots + \theta_L,$$

where each component $\theta_l \sim \text{i.i.d.} \mathcal{N}(0, \sigma^2)$. There are two strategic traders, label them i and j . Trader i observes the first I components of θ (denote their sum as $x_i = \theta_1 + \dots + \theta_I$), while trader j observes the last J components (denote their sum as $x_j = \theta_{L-J+1} + \dots + \theta_L$). Assume $I + J < L$, so there are no components observed by both traders, but there are some which are not observed by either one (denote their sum as $x_k = \theta_{I+1} + \dots + \theta_{L-J}$). In addition, both traders observe the same public signal $y = \theta + \epsilon$, where $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$.

In answering the questions below, you can use the following fact:

If $q = v + u$, where $v \sim \mathcal{N}(\mu_v, \sigma_v^2)$ and $u \sim \mathcal{N}(\mu_u, \sigma_u^2)$, and the two are independent, then: (1) $q \sim \mathcal{N}(\mu_v + \mu_u, \sigma_v^2 + \sigma_u^2)$, and (2) $\mathbb{E}[v|q] = \mathbb{E}[v] + (q - \mathbb{E}[q]) \frac{\mathbb{C}(v, q)}{\mathbb{V}(q)}$.

1. Calculate $\mathbb{E}[\theta|x_j, y]$, i.e., trader j 's asset valuation conditional on the information available to him.¹
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HINT: you should get an expression of the form $\mathbb{E}[\mathbb{E}[\theta|x_j, y] | x_i, y] = \alpha y + \beta(y - x_i)$. You need to find α and β .

3. Take the coefficient β you have obtained in part 2 (if you did everything correctly, it should be positive). It captures the degree of divergence of second-order beliefs: the higher is trader I 's private signal of asset value, the lower he expects J 's valuation to be, and the higher is β , the stronger this effect is. How does β depend on I , the number of components that trader i observes? Explain the intuition behind this.
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NOTE: if you could not solve part 2, you can still try to provide an educated guess for the directions and the reasons of the effects in parts 3 and 4.

¹In general, conditioning on x_j is not the same as conditioning on $(\theta_{L-J+1}, \dots, \theta_L)$, since the latter contains more information. But the two are equivalent in this problem.

Solution:

1. We have

$$\begin{aligned}
\mathbb{E}[\theta | x_j, y] &= \mathbb{E} \left[\sum_{l=1}^L \theta_l \mid x_j, y \right] \\
&= \mathbb{E} [x_j + x_i + x_k \mid x_j, y] \\
&= x_j + \mathbb{E} [x_i + x_k \mid x_j, y]
\end{aligned} \tag{1}$$

To calculate the latter expectation we notice that $y = x_j + x_i + x_k + \epsilon$, where x_j is known (to j). Applying the fact given in the problem, we infer that

$$\mathbb{E} [x_i + x_k \mid x_j, y] = \alpha(y - x_j),$$

where $\alpha = \frac{(L-J)\sigma^2}{(L-J)\sigma^2 + \sigma_\epsilon^2}$. Plugging it back into (1), we get

$$\mathbb{E} [\theta | x_j, y] = \alpha y + (1 - \alpha)x_j. \tag{2}$$

2. Plugging (2) into the target expectation, we get

$$\begin{aligned}
\mathbb{E} [\mathbb{E} [\theta | x_j, y] \mid x_i, y] &= \mathbb{E} [\alpha y + (1 - \alpha)x_j \mid x_i, y] \\
&= \alpha y + (1 - \alpha)\mathbb{E} [x_j \mid x_i, y]
\end{aligned} \tag{3}$$

Recalling again that $y = x_i + x_j + x_k + \epsilon$ and applying the fact we obtain

$$\mathbb{E} [x_j \mid x_i, y] = (y - x_i) \frac{J\sigma^2}{(L - I)\sigma^2 + \sigma_\epsilon^2}$$

Plugging this back into (3), we finally get the desired expression

$$\begin{aligned}
\mathbb{E} [\mathbb{E} [\theta | x_j, y] \mid x_i, y] &= \alpha y + \beta (y - x_i), \\
\text{where } \beta &= (1 - \alpha) \frac{J\sigma^2}{(L - I)\sigma^2 + \sigma_\epsilon^2} \\
&= \frac{J\sigma^2\sigma_\epsilon^2}{[(L - J)\sigma^2 + \sigma_\epsilon^2][(L - I)\sigma^2 + \sigma_\epsilon^2]} > 0
\end{aligned}$$

3. Weight β is increasing in I , i.e., as trader i gets to observe more components, the divergence amplifies. For example, suppose $\theta_1 > 0$ and $\theta_2 = 0$. Then increasing I from one to two will lead trader i to amplify the inference he makes from θ_1 and decrease his opinion about j 's valuation – even though θ_2 by itself is uninformative. However, even though θ_2 does

not contain any information about the actual asset value in this example, it leads trader i to believe that information in $y - x_i$ is more likely to be observed by trader j . This is because i knows that j observes share $J/(L - I + 1)$ of components that constitute $y - x_i$.

4. Weight β is increasing in J , i.e., as trader j gets to observe more components, the divergence amplifies:

$$\frac{\partial \beta_2}{\partial J} = \beta_2 \left(\frac{1}{J} + \frac{\sigma^2}{(L - J)\sigma^2 + \sigma_\epsilon^2} \right) < 0$$

The intuition is the same as in part 3: as J increases, trader i perceives it to be more probable that j observes whatever components are responsible for the information in $y - x_i$.

Problem 4

Below you can find an Economist article from July 17, 2019 on the liquidity of the corporate bond market.² The article mentions that the actual liquidity in that market is well below what the investors seem to expect it to be. What are the possible consequences of such misalignment? Should we attempt to alleviate it by making the market more transparent? How does it relate to the discussion of market transparency we had in class?

Solution: A surface-level consequence is that the investors are exposed to more risk than they account for, since they may be unable to unwind their positions when they decide to do so. This is, in itself, an inefficiency that we would typically want to correct.

The counterargument is also described in the article: this false belief in liquidity leads investors to hold bonds, which they would or could not invest in otherwise due to liquidity risk. This by itself creates some liquidity, supplied by such traders. An attempt to correct the investors' belief would lead to the market's suffering liquidity dropping even further, completely paralyzing the bond market and leading to a crash, which is not the most desirable outcome.

In the end, the blanket conclusion we made about opaque markets – that opaqueness hurts the uninformed traders while benefitting other market participants, – gets an extra layer in this setting, in that the aggregate effects of transparency may be so catastrophic that the uninformed traders will lose more than they gain if the market is made more transparent.

²Also available at: <https://www.economist.com/finance-and-economics/2019/07/11/why-everybody-is-concerned-about-corporate-bond-liquidity>

Buttonwood – Why everybody is concerned about corporate-bond liquidity

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What Lord King probably had in mind was a well-thumbed textbook model. Banks have a liquidity mismatch. One side of the balance-sheet is hard-to-sell loans; the other side is deposits that can be withdrawn in a trice. If depositors believe that a bank is sound, there will be no runs on it. But if enough start to demand their deposits back, it makes sense for everybody to join the rush.

This model can also be applied in other areas. Take the corporate-bond market. Every policy body of stature, from the IMF to the European Central Bank (ECB), has worried about a growing mismatch between investors’ expectations that they can sell out at any moment and an underlying shortage of liquidity in the market. More investors are using corporate-bond funds as an alternative to cash. But fewer dealers are willing to trade bonds in size. A big scare could feasibly start a run.

The dynamics of capital-market runs are similar to those of bank runs. You see them in currency crises. Foreign-exchange reserves, say, are slim relative to the scale of local-currency assets held by flighty investors. Should enough of those investors sell out, others will soon follow. The result is a rout. There is a similar pattern with investment funds that promise speedy withdrawals but hold assets that cannot be sold quickly. Bad news prompts withdrawals. The speedy get paid. Other investors then try to get out, too. But the fund cannot sell assets fast enough. It is forced to suspend redemptions.

Such trouble is especially likely with corporate bonds, which are inherently illiquid. In contrast with trading in shares, where buy and sell orders are matched on electronic order books, corporate bonds are traded over-the-counter. Bonds are not as standardised as shares. A company may have bonds of several different maturities. If you want to buy or sell, you call a dealer.

The ease with which investors can trade bonds—the market’s liquidity—depends a lot, then, on the readiness of dealer banks to stockpile securities. Where there is heavy selling, dealers would ideally warehouse cheaper bonds for when people want to buy again. But since the financial crisis new rules have made it less cost-effective for banks to use capital for trading of any kind. The inventory of corporate bonds held by dealers has fallen sharply in the past decade.

As the role of dealers has shrunk, the thirst for instant liquidity has increased. Derisory yields on the safest government debt have drawn investors towards riskier securities, including corporate bonds. A cheap and convenient way to invest in them is to buy an exchange-traded fund, or ETF. These are low-cost investment funds that hold a basket of bonds, usually mirroring a benchmark index. They trade on stock exchanges just as listed shares do. The ease of buying and selling

bond ETFs is a big part of their appeal. They are also often used as depositories for spare cash. Studies are divided on whether ETFs make the underlying bonds more or less liquid. But there are concerns that in a stressed market, outflows from ETFs might make a bad situation worse. And it is not hard to make a case that the corporate-bond market has become more fragile. Many firms in America have issued lots of bonds to buy back their own shares. With extra leverage comes more risk. Half of all investment-grade bonds have a credit rating of BBB. In a recession a chunk of those bonds will be downgraded to junk. Many mutual funds and ETF s can hold only investment-grade bonds. If a lot of bonds have to change hands quickly, that could easily overwhelm the market's limited liquidity. Prices might fall a long way.

Just how messy the next big shake-out in the corporate-bond market is depends on many things: on how weak the economy gets; on how many BBB borrowers can avert a downgrade; on how quickly funds can be raised to buy at fire-sale prices. For now, it seems rational to hold bonds that afford a little extra yield. Smart-alecks say this will surely end badly. But who wants to hear that?

Written Exam Economics summer 2021

Financial Markets Microstructure

June 16, 2021

This exam question consists of 3 pages in total

Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.

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- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
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Problem 1

We have discussed throughout the class that larger presence of uninformed traders in the market improves market liquidity. However, some recent evidence shows that more active trading through Robinhood (trading app used by predominantly retail investors) leads to *lower* liquidity in the exposed markets.

Give a possible explanation, intuitive or via a model, to this empirical phenomenon. Use the knowledge you have obtained in the course and/or any external sources you can find (remember to give appropriate credit to your sources).

Problem 2

Consider the Biais-Foucault-Moinas model of high-frequency trading we discussed in class, but assume now instead that adverse selection is mild: $\epsilon < \delta$.

In particular, suppose there is a unit continuum of profit-maximizing institutions $i \in [0, 1]$, of which share $\alpha \in (0, 1)$ has access to HFT technology (fast institutions). There is a single asset; institutions value the asset at $u_i = v + y_i$, where $v \in \{\mu - \epsilon, \mu + \epsilon\}$ with equal probabilities is the fundamental value, and $y_i \in \{-\delta, \delta\}$ with equal probabilities, independent across i , is the institution i 's idiosyncratic value for the asset. Given a trading opportunity, the institution can submit a market order to buy or sell one unit of the asset. At the time of trade, a fast institution knows both v and y_i , while a slow institution only knows y_i . Further, trading opportunities arrive to fast institutions with probability 1, while slow institutions only receive a trading opportunity with probability ρ . Risk-neutral liquidity providers are competitive and provide bid and ask quotes for one unit of the asset so as to get zero expected profit on any trade.

Answer the following questions given $\epsilon < \delta$ and $\alpha > 0$.

1. Equilibrium multiplicity is still an issue in this case. Focusing on the ask side of the market, what are the equilibria that can arise in such a market? Calculate the equilibrium ask price in each of these equilibria.
2. When do each of these equilibria exist? Derive the exact existence conditions.
3. Calculate the expected profits of slow and fast institutions in the Pareto-dominant equilibrium. How do they depend on α ?
HINT: Pareto-dominant is the equilibrium with the largest amount of trade.
4. (Conditional on the Pareto-dominant equilibrium:) How do incentives to invest in speed depend on α ? Do you think these incentives will result in efficient investment or will there be over-/underinvestment? Explain why (intuitively). Explain how this relates to the case of severe adverse selection ($\epsilon > \delta > \epsilon/2$) that we explored in class.

Problem 3

Blockchain markets attempt to experiment with novel ways to organize markets and provide liquidity. For example, Uniswap trading protocol on the Ethereum blockchain implements the “liquidity pools” scheme described below. You are asked to analyze the implications of this market design for liquidity provision.¹

If a trader is willing to provide liquidity for a given asset, they must buy into a liquidity pool for that asset. If the liquidity pool consists of A units of the stock and C units of cash (or cryptocurrency), then to enter the trader needs to contribute a units of asset and c units of cash s.t. $a/c = A/C$. The trader would then own fraction $\frac{a}{A+a}$ of the liquidity pool indefinitely. If the trader wants to quit the pool, they can claim their share of the asset and the cash and withdraw from the pool.

Any market order must necessarily trade against the whole liquidity pool. I.e., you can think that every liquidity provider executes a part of every incoming market order equal to their share of the liquidity pool. The price per unit for an infinitesimally small trade is given by $p = C/A$, but trading any non-trivial amount would have a price impact. For this problem, assume that the average price at which a trade of size q executes is given by $p = C/A + \lambda q$, where q is trade size, and the price impact coefficient λ is determined by the size of the pool (larger pool leads to lower λ).²

Answer the following questions.

1. Assume that all elements of the model except for liquidity providers are the same as in the (single-period) Kyle model: the fundamental value is $v \sim \mathcal{N}(\mu, \sigma^2)$, the midquote is $C/A = \mu$, the order flow is given by $q = x + u$, where x comes from a profit-optimizing informed trader who knows v , and $u \sim \mathcal{N}(0, \sigma_u^2)$ comes from uninformed traders. Further, assume free entry into the liquidity pool – i.e., the liquidity providers enter until the expected profit from entry is zero.

Derive the equilibrium pool depth λ . How does it compare to depth that would arise in the presence of a single dealer? How does it compare to depth that would be generated by a limit order book?

HINT: when considering entry decisions, ignore the cost of buying into the pool and focus on profit from incoming trades only, since the latter will dominate if the trader stays in the pool long enough.

2. What are the benefits and drawbacks of the liquidity pool as opposed to dealer markets? As opposed to the limit order book? (In dimensions other than market depth.)

¹Note: this problem explores a modification of the Uniswap V2 protocol, not the just-released V3 protocol.

²The actual Uniswap V2 protocol has a more sophisticated pricing algorithm, in which the price the trader gets is such that in the end, $\frac{A+\Delta A}{C+\Delta C} = \kappa$ for some exogenously fixed constant κ .

Financial Markets Microstructure

Final Exam with Solutions

Københavns Universitet

June 2021

Problem 1

We have discussed throughout the class that larger presence of uninformed traders in the market improves market liquidity. However, some recent evidence shows that more active trading through Robinhood (trading app used by predominantly retail investors) leads to *lower* liquidity in the exposed markets.

Give a possible explanation, intuitive or via a model, to this empirical phenomenon. Use the knowledge you have obtained in the course and/or any external sources you can find (remember to give appropriate credit to your sources).

Solution: The mentioned evidence is presented by Eaton et al (2021).¹ The explanation they provide is tied to the nature of Robinhood as not simply a trading platform, but also a social platform, on which individual traders can share trading tips and mention possible trading opportunities. This enables mob mentality and herding: many traders focus their trading on relatively few stocks. (Note that such herding can even be perfectly rational, as we saw when discussing the Smith & Sørensen model.) Importantly, what this herding implies for the liquidity providers is greater inventory risk because the order flow becomes more autocorrelated. Greater inventory risk is what leads to worse quotes supplied by the liquidity providers and lower overall market liquidity.

Problem 2

Consider the Biais-Foucault-Moinas model of high-frequency trading we discussed in class, but assume now instead that adverse selection is mild: $\epsilon < \delta$.

¹Eaton, Gregory W., T. Clifton Green, Brian Roseman, and Yanbin Wu. “Zero-Commission Individual Investors, High Frequency Traders, and Stock Market Quality.” SSRN Scholarly Paper. Rochester, NY: Social Science Research Network, January 1, 2021. <https://doi.org/10.2139/ssrn.3776874>.

In particular, suppose there is a unit continuum of profit-maximizing institutions $i \in [0, 1]$, of which share $\alpha \in (0, 1)$ has access to HFT technology (fast institutions). There is a single asset; institutions value the asset at $u_i = v + y_i$, where $v \in \{\mu - \epsilon, \mu + \epsilon\}$ with equal probabilities is the fundamental value, and $y_i \in \{-\delta, \delta\}$ with equal probabilities, independent across i , is the institution i 's idiosyncratic value for the asset. Given a trading opportunity, the institution can submit a market order to buy or sell one unit of the asset. At the time of trade, a fast institution knows both v and y_i , while a slow institution only knows y_i . Further, trading opportunities arrive to fast institutions with probability 1, while slow institutions only receive a trading opportunity with probability ρ . Risk-neutral liquidity providers are competitive and provide bid and ask quotes for one unit of the asset so as to get zero expected profit on any trade.

Answer the following questions given $\epsilon < \delta$ and $\alpha > 0$.

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HINT: Pareto-dominant is the equilibrium with the largest amount of trade.

4. (Conditional on the Pareto-dominant equilibrium:) How do incentives to invest in speed depend on α ? Do you think these incentives will result in efficient investment or will there be over-/underinvestment? Explain why (intuitively). Explain how this relates to the case of severe adverse selection ($\epsilon > \delta > \epsilon/2$) that we explored in class.

Solution:

Parts 1 and 2. We now have that $\mu + \epsilon + \delta > \mu + \delta > \mu - \epsilon + \delta > \mu$, i.e., that $u_{GH}^F > u_H^S > u_{BH}^F > \mu$, with all other institution types' valuations being below μ . Different possible equilibria would have the ask prices in different intervals relative to these values. Let us see whether we can construct an equilibrium for each such relation between a and $\{u_{GH}^F, u_H^S, u_{BH}^F\}$.

1. $a \in [\mu, \mu - \epsilon + \delta]$: in this case, the institutions buy with probabilities $\beta_{GH}^F = \beta_H^S = \beta_{BH}^F = 1$, hence the zero-profit condition for liquidity providers implies

$$\begin{aligned} a &= \mathbb{E}[v|\text{Buy}] \\ &= \frac{\frac{\alpha}{4}}{\frac{\alpha}{2} + \rho \frac{1-\alpha}{2}} (\mu + \epsilon) + \frac{\rho \frac{1-\alpha}{2}}{\frac{\alpha}{2} + \rho \frac{1-\alpha}{2}} \mu + \frac{\frac{\alpha}{4}}{\frac{\alpha}{2} + \rho \frac{1-\alpha}{2}} (\mu - \epsilon) \\ &= \mu. \end{aligned}$$

It does indeed hold that $\mu < \mu - \epsilon + \delta$, hence this equilibrium exists under no additional conditions.

2. $a \in [\mu - \epsilon + \delta, \mu + \delta]$: then $\beta_{GH}^F = \beta_H^S = 1$ and $\beta_{BH}^F = 0$. From the zero-profit condition:

$$\begin{aligned} a &= \mathbb{E}[v|\text{Buy}] \\ &= \frac{\frac{\alpha}{4}}{\frac{\alpha}{4} + \rho^{\frac{1-\alpha}{2}}}(\mu + \epsilon) + \frac{\rho^{\frac{1-\alpha}{2}}}{\frac{\alpha}{4} + \rho^{\frac{1-\alpha}{2}}}\mu \\ &= \mu + \frac{\alpha}{\alpha + 2\rho(1-\alpha)}\epsilon. \end{aligned}$$

Check whether it belongs to the suggested interval: on the one hand, we need

$$\begin{aligned} \mu + \frac{\alpha}{\alpha + 2\rho(1-\alpha)}\epsilon &\leq \mu + \delta \\ \iff \alpha\epsilon &\leq (\alpha + 2\rho(1-\alpha))\delta \\ \iff \alpha &\leq \frac{2\rho\delta}{2\rho\delta - (\delta - \epsilon)}, \end{aligned}$$

which always holds because $\alpha \in (0, 1]$, and the right-hand side is greater than one. The second condition is

$$\begin{aligned} \mu + \frac{\alpha}{\alpha + 2\rho(1-\alpha)}\epsilon &\geq \mu - \epsilon + \delta \\ \iff (2\alpha + 2\rho(1-\alpha))\epsilon &\geq (\alpha + 2\rho(1-\alpha))\delta \\ \iff \alpha &\geq 1 - \frac{2\epsilon - \delta}{(2\rho - 1)(\delta - \epsilon) + \epsilon} \end{aligned}$$

(which is only possible if $\epsilon \geq \delta/2$).

3. $\alpha \in [\mu + \delta, \mu + \epsilon + \delta]$: then $\beta_{GH}^F = 1$ and $\beta_H^S = \beta_{BH}^F = 0$, hence $a = \mathbb{E}[v|\text{Buy}] = \mu + \epsilon$. This, however, violates the assumption $\alpha \geq \mu + \delta$ because $\delta > \epsilon$, hence an equilibrium like this can not exist.

Note: it can be showed $a < \mu$ is impossible in equilibrium as well. This argument is omitted from this solution.

In the end, we have type-1 equilibrium which always exists, and type-2 equilibrium that only exists under particular conditions. It is immediate that type-1 is the Pareto-dominant equilibrium: all institutions get to trade and they do so at best possible prices (liquidity providers get zero profit in all equilibria, so they are indifferent).

Part 3. The expected profits of the Fast Institutions are

$$\begin{aligned}\phi &= \frac{1}{4}(\mu + \epsilon + \delta - \mu) + \frac{1}{4}(\mu - \epsilon + \delta - \mu) + \frac{1}{4}(\mu - (\mu - \epsilon - \delta)) + \frac{1}{4}(\mu - (\mu + \epsilon - \delta)) \\ &= \delta.\end{aligned}$$

The expected profits of the Slow Institutions are

$$\begin{aligned}\psi &= \frac{1}{2}\rho(\mu + \delta - \mu) + \frac{1}{2}\rho(\mu - (\mu - \delta)) \\ &= \rho\delta.\end{aligned}$$

Both are positive and independent of α .

Part 4. The incentives to invest in speed are given by the difference between the profits of fast and slow institutions: $\phi - \psi = (1 - \rho)\delta$. In words, the only benefit of HFT technology is in finding trading opportunities, while having early access to information about v does not entail any benefit, since trading decisions are driven by the idiosyncratic valuation y_i in either case. Due to this, investment in HFT does not produce any externalities on other institutions (while in the basic model it resulted in worse prices offered to everyone), hence the equilibrium level of investment will be efficient.

Problem 3

Blockchain markets attempt to experiment with novel ways to organize markets and provide liquidity. For example, Uniswap trading protocol on the Ethereum blockchain implements the “liquidity pools” scheme described below. You are asked to analyze the implications of this market design for liquidity provision.²

If a trader is willing to provide liquidity for a given asset, they must buy into a liquidity pool for that asset. If the liquidity pool consists of A units of the stock and C units of cash (or cryptocurrency), then to enter the trader needs to contribute a units of asset and c units of cash s.t. $a/c = A/C$. The trader would then own fraction $\frac{a}{A+a}$ of the liquidity pool indefinitely. If the trader wants to quit the pool, they can claim their share of the asset and the cash and withdraw from the pool.

Any market order must necessarily trade against the whole liquidity pool. I.e., you can think that every liquidity provider executes a part of every incoming market order equal to their share of the liquidity pool. The price per unit for an infinitesimally small trade is given by $p = C/A$,

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but trading any non-trivial amount would have a price impact. For this problem, assume that the average price at which a trade of size q executes is given by $p = C/A + \lambda q$, where q is trade size, and the price impact coefficient λ is determined by the size of the pool (larger pool leads to lower λ).³

Answer the following questions.

1. Assume that all elements of the model except for liquidity providers are the same as in the (single-period) Kyle model: the fundamental value is $v \sim \mathcal{N}(\mu, \sigma^2)$, the midquote is $C/A = \mu$, the order flow is given by $q = x + u$, where x comes from a profit-optimizing informed trader who knows v , and $u \sim \mathcal{N}(0, \sigma_u^2)$ comes from uninformed traders. Further, assume free entry into the liquidity pool – i.e., the liquidity providers enter until the expected profit from entry is zero.

Derive the equilibrium pool depth λ . How does it compare to depth that would arise in the presence of a single dealer? How does it compare to depth that would be generated by a limit order book?

HINT: when considering entry decisions, ignore the cost of buying into the pool and focus on profit from incoming trades only, since the latter will dominate if the trader stays in the pool long enough.

2. What are the benefits and drawbacks of the liquidity pool as opposed to dealer markets? As opposed to the limit order book? (In dimensions other than market depth.)

Solution:

Part 1. Like in the basic Kyle model, given a linear pricing schedule $p = \mu + \lambda q$, the informed trader's optimal trading strategy is given by $x = \beta(v - \mu)$ with $\beta = \frac{1}{2\lambda}$.

The condition that pins down λ is mentioned in the text: the marginal entrant into the pool must get zero expected profit. However, the expected profits of all pool participants are proportional to their pool shares, meaning that in the presence of free entry the expected profit of the whole pool will be zero:

$$\mathbb{E}_{v,q} [q(v - p)] = 0,$$

where the subscripts at the expectation denote the variables, with respect to which the expectation is taken.

Note that this condition would be satisfied if $p = \mathbb{E}_v[v|q]$: in that case $\mathbb{E}_v[v - p | q] = 0$, so $\mathbb{E}_v[q(v - p) | q] = 0$, and taking the expectation over q we conclude that $\mathbb{E}_{v,q} [q(v - p)] = 0$.

³The actual Uniswap V2 protocol has a more sophisticated pricing algorithm, in which the price the trader gets is such that in the end, $\frac{A+\Delta A}{C+\Delta C} = \kappa$ for some exogenously fixed constant κ .

However, $p = \mathbb{E}_v[v|q]$ is exactly the schedule that competitive dealers offer in the Kyle model! Repeating the derivations we did for the Kyle model, we obtain that in equilibrium, $\lambda = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2}$, and solving that jointly with $\beta = \frac{1}{2\lambda}$, we get that $\beta = \frac{\sigma_u}{\sigma_v}$ and $\lambda = \frac{\sigma_v}{2\sigma_u}$.

The depth of liquidity pool is trivially the same as the depth of a dealer market would have been given this liquidity demand. Invoking the comparisons between the Kyle and the Glosten models we made in class, we can then say that market traders can get a better price on small orders from a dealer, and a better price on large orders in a LOB market (where the marginal price of y th unit of the asset is given by $\mathbb{E}[v | q \geq y]$).

Part 2. One benefit of liquidity pools is that the “limit orders” are priced and supplied automatically by the market algorithm, and liquidity providers do not need to do anything manually (or use any algorithm on their side). This reduces the direct costs of liquidity provision – albeit at the cost of liquidity pools tying up a lot more capital in order to provide the same amount of liquidity as regular markets. On the other hand, liquidity demanders also have more guarantees on the prices they will receive, since liquidity providers cannot frontrun market orders and cancel their limit orders before what they think is an unfavorable market order. (They can still withdraw from the pool, but this will only affect the price impact, not the best quotes, so small market orders always execute at the advertised price.)

Comparing the liquidity pool (under linear pricing mechanism) and a dealer market, the quotes supplied by the two, as we saw above, are the same. The potential differences thus lie outside the model. Firstly, we assume in Kyle model that the dealers are competitive, which is not completely true in reality, with dealers having some market power and ability to extract profit from the market – while the free entry into the liquidity pool is much more likely to lead to competitive outcomes and competitive quotes. Secondly, the liquidity pool is unable to offer price improvements to individual liquidity demanders like a dealer could. Together with the frontrunning argument above, this implies that whereas a dealer could discriminate based on trader identity, the liquidity pool treats all liquidity demanders equally, leading to worse outcomes for uninformed traders, but improvements for informed traders. (See the in-class discussion on in-trade transparency.)

Comparing the liquidity pool to the limit order book, the main argument in favor of the pool is risk sharing. Namely, all liquidity providers in the pool participate in executing all market orders, as opposed to some providers in the limit order book having profitable opportunities at the cost of other limit traders being picked off. This implicit risk-sharing together with automatic repricing of limit orders based on pool composition should make liquidity provision more attractive to traders. However, the very same risk sharing eliminates any profits that liquidity providers could get in a LOB with time priority, thereby reducing incentives to participate in the market (similarly to how it happened in our discussion of time priority vs pro rata allocation

rule in LOB markets). Finally, the pool eliminates the traders' choice between supplying and demanding liquidity: a patient trader who wants to buy the asset can no longer get a price improvement by using a limit order as opposed to a market order. Liquidity provision is now an activity in itself and can not be used as a part of active trading strategy. This can potentially reduce the number of traders who would be interested in providing liquidity compared to LOB markets.

Written Exam Economics Summer 2020

Financial Markets Microstructure

August 23, 2020

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Answers only in English.

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Problem 1

The Parlour model explores traders' choices between market and limit orders. Its bottom line, as we argued in class, is that limit orders result in more favorable prices but at the cost of the execution risk. In particular, we ignored the potential effects of asymmetric information. Suppose now instead that with some probability public information (news) can arrive between periods t and $t + 1$ (but not at any other time, and all of these facts are commonly known). This will produce an asymmetry in the sense that trader at $t + 1$ will be able to act upon this information, while limit orders submitted by trader t are independent of this news.

Answer the following questions using convincing intuitive arguments. Proceed via backwards induction: i.e., holding previous traders' strategies fixed, state how the strategy of a trader in a given period changes. You do not need to analyze a formal model, but you are welcome to do so if you want to.

1. How will the behavior of traders who arrive at $t + 2$ or later change due to the possible arrival of news?
2. What about the trader who arrives at $t + 1$?
3. What about the trader who arrives at t ?
4. Now suppose that if the period- t trader submitted a limit order, he can revise/cancel his limit order after the news arrives. How do your answers to parts 1-3 change?

Problem 2

This question investigates the inventory risk in uncertain environments within the Stoll model framework. Consider a three-period model, $t \in \{0, 1, 2\}$. There is one asset, whose fundamental value evolves as $\mu_{t+1} = \mu_t + \epsilon_t$, where $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$. The respective μ_t is observed at the beginning of period t .

In periods $t \in \{0, 1\}$ the representative dealer must provide quote schedule $p(q)$ for any incoming order size q (where $q > 0$ means a buy order and $q < 0$ means a sell order). In period $t = 0$ one trader arrives for sure and submits an order denoted by q_0 . In period 1 one trader arrives with probability λ and, if he does, submits an order $q_1 = -q_0$. In period $t = 2$ the asset is paid out: every owner of the asset receives a payment μ_2 per unit and the asset has no future value.

Dealer has mean-variance preferences over his final wealth w_2 . I.e., in every period he maximizes

$$U(w_2) = \mathbb{E}[w_2] - \frac{\rho}{2} \mathbb{V}(w_2).$$

His initial position in the asset is neutral: $z_0 = 0$. The initial cash holdings c_0 are also normalized to zero. (The dealer can borrow cash and short the asset at no cost). The dealer behaves competitively (is a price-taker).

1. Consider period $t = 1$. Denote the dealer's position at the beginning of the period as z_1 . Derive the dealer's quote schedule $p_1(q)$ given z_1 .
2. What is the price at which trade will happen at $t = 1$?
3. Consider period $t = 0$. Derive the dealer's quote schedule $p_0(q)$.
4. Explain how $p_0(q)$ depends on λ and why.

NOTE: if you could not solve parts 1-3, you can still try to make an educated guess here.

5. The problem assumes $q_1 = -q_0$, i.e., that the order flow is perfectly negatively autocorrelated. How justified is this assumption? How does it relate to the dealer's pricing decisions?

NOTE: if you could not solve parts 1-4, you can still answer this question.

Problem 3

Many real-world trading platforms have “circuit breakers” – automatic safeguards which halt all trading in an asset if its price changes too drastically over a short time. Using the material of the course, discuss what consequences can the existence of such circuit breakers have in terms of market outcomes (liquidity, traders' risk exposure and willingness to trade, price discovery).

Financial Markets Microstructure

Final Re-Exam with Solutions

Københavns Universitet

August 2020

Problem 1

The Parlour model explores traders' choices between market and limit orders. Its bottom line, as we argued in class, is that limit orders result in more favorable prices but at the cost of the execution risk. In particular, we ignored the potential effects of asymmetric information. Suppose now instead that with some probability public information (news) can arrive between periods t and $t + 1$ (but not at any other time, and all of these facts are commonly known). This will produce an asymmetry in the sense that trader at $t + 1$ will be able to act upon this information, while limit orders submitted by trader t are independent of this news.

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3. What about the trader who arrives at t ?
4. Now suppose that if the period- t trader submitted a limit order, he can revise/cancel his limit order after the news arrives. How do your answers to parts 1-3 change?

Solution:

1. All quotes will reflect the new information, but the trade-off between market and limit orders will remain the same.

2. The trader at $t + 1$ will have a wider range of possible asset valuations, since in addition to idiosyncratic valuation he will know the innovation to the fundamental value. Hence holding fixed the limit order strategy of t -trader, the trader at $t + 1$ will use market orders with higher probability. This effect is state-dependent: good news increases the probability of a market buy order and decreases the probability of any sell order (if the distribution of idiosyncratic valuations y_t is not too strange). The effect is flipped for bad news.
3. Public information affects the strategic environment of time- t trader in two dimensions. On the one hand, his limit order, if submitted, has a higher execution probability. This makes limit orders more appealing and would lead to time- t trader using more limit orders and less market orders. On the other hand, limit orders can now be picked off: a sell limit order becomes more likely to be executed in case of “good news” about asset valuation – i.e., the t -trader sells too cheaply on average. This will make limit orders less appealing, since price improvement they yield may be completely offset by adverse selection.

The formal analysis can help answer which of the two effects above dominates and whether time- t trader will end up using limit orders more or less actively relative to market orders, although the exact conclusion will depend on the assumptions about the distributions of y_t (the idiosyncratic component) and ϵ_t (the public news component).

4. If time- t trader can cancel and revise his limit order before it can be picked off by the $t + 1$ -trader, then the revision will incorporate the public information, and the expected payoff of time- t trader from submitting a limit order will remain exactly the same as in the absence of news. Therefore, his choice between market and limit orders will not be affected.

If there is some chance that $t + 1$ -trader can manage to trade before t -trader can revise his quotes, then the effect from part 3 applies to the extent proportional to that chance. (Note parallels to the Budish-Cramton-Shim model.)

Problem 2

This question investigates the inventory risk in uncertain environments within the Stoll model framework. Consider a three-period model, $t \in \{0, 1, 2\}$. There is one asset, whose fundamental value evolves as $\mu_{t+1} = \mu_t + \epsilon_t$, where $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$. The respective μ_t is observed at the beginning of period t .

In periods $t \in \{0, 1\}$ the representative dealer must provide quote schedule $p(q)$ for any incoming order size q (where $q > 0$ means a buy order and $q < 0$ means a sell order). In period $t = 0$ one

trader arrives for sure and submits an order denoted by q_0 . In period 1 one trader arrives with probability λ and, if he does, submits an order $q_1 = -q_0$. In period $t = 2$ the asset is paid out: every owner of the asset receives a payment μ_2 per unit and the asset has no future value.

Dealer has mean-variance preferences over his final wealth w_2 . I.e., in every period he maximizes

$$U(w_2) = \mathbb{E}[w_2] - \frac{\rho}{2}\mathbb{V}(w_2).$$

His initial position in the asset is neutral: $z_0 = 0$. The initial cash holdings c_0 are also normalized to zero. (The dealer can borrow cash and short the asset at no cost). The dealer behaves competitively (is a price-taker).

1. Consider period $t = 1$. Denote the dealer's position at the beginning of the period as z_1 . Derive the dealer's quote schedule $p_1(q)$ given z_1 .
2. What is the price at which trade will happen at $t = 1$?
3. Consider period $t = 0$. Derive the dealer's quote schedule $p_0(q)$.
4. Explain how $p_0(q)$ depends on λ and why.

NOTE: if you could not solve parts 1-3, you can still try to make an educated guess here.

5. The problem assumes $q_1 = -q_0$, i.e., that the order flow is perfectly negatively autocorrelated. How justified is this assumption? How does it relate to the dealer's pricing decisions?

NOTE: if you could not solve parts 1-4, you can still answer this question.

Solution:

1. This part is identical to the Stoll model we had in class. The dealer is a price-taker, so he chooses the optimal asset supply y_1 given any price p . We derive the resulting asset supply schedule $y_1(p)$, invert it and invoke market clearing condition $y_1 = q_1$ to derive the price schedule $p_1(q)$. Asset supply y_1 is chosen to maximize

$$\begin{aligned} U_1(y_1) &= \mathbb{E}[w_2] - \frac{\rho}{2}\mathbb{V}(w_2) \\ &= py_1 + \mu_1(z_1 - y_1) - \frac{\rho}{2}(z_1 - y_1)^2\sigma_\epsilon^2 \end{aligned}$$

Taking the first order condition and substituting $q = y_1$, we end up with

$$p_1(q) = \mu_1 + \rho\sigma_\epsilon^2(q - z_1).$$

2. The problem states that $q_1 = -q_0$, meaning that $q - z_1 = 0$, so $p_1 = \mu_1$.

3. Whenever the dealer trades y_0 at $t = 0$, two continuations are possible: with probability λ an order $y_1 = -y_0$ will arrive at $t = 1$, in which case

$$w_2 = p_0 y_0 + p_1 y_1 = (p_0 - \mu_1) y_0.$$

With probability $1 - \lambda$ no trader will arrive at $t = 1$, so

$$w_2 = p_0 y_0 + \mu_2 z_2 = (p_0 - \mu_2) y_0.$$

Therefore, the dealer's objective function at time $t = 0$ is

$$U_0(y_0) = (p_0 - \mu_0) y_0 - \frac{\rho}{2} [\lambda \sigma_\epsilon^2 + (1 - \lambda) 2 \sigma_\epsilon^2] y_0^2 \quad (1)$$

Maximizing this expression w.r.t. y_0 for given p_0 and invoking the market clearing condition $q_0 = y_0$, we obtain

$$p_0(q) = \mu_0 + \rho(2 - \lambda) \sigma_\epsilon^2 q \quad (2)$$

4. Larger λ decreases the price impact coefficient. Higher λ means that the dealer faces a higher chance of unwinding his inventory at $t = 1$ and not staying stuck with it until $t = 2$. This reduces the dealer's exposure to volatility in the asset's fundamental value μ_t (which is priced because the dealer is risk-averse), and so reduces the dealer's required risk premium.
5. In the real world, order flow from traders is typically positively autocorrelated (although this depends on the time scale). This may be due to traders splitting their large orders and feeding them to the market as a series of small orders, or due to different traders acting upon the same piece of news. In this respect the assumption is disconnected from the real world.

However, in this problem the dealer dislikes holding on to any inventory due to the risk this exposes him to, and this feeds into his pricing decisions so as to create negative autocorrelation in the order flow. E.g., if the dealer has purchased a unit of the asset at $t = 0$, then he would set lower prices $p_1(q)$ in the following period so as to incentivize buyers and discourage sellers, which creates respective pressure on the order flow.

That said, we are looking at the representative dealer in this model – a fictional agent meant to represent the aggregate of all individual dealers in the market. While any single dealer can be non-trivially risk-averse, the market as a whole can absorb quite significant positions before distorting the prices due to inventory concerns. Therefore, it may be reasonable to believe that the representative dealer's degree of risk aversion is quite small, so this driver of negative autocorrelation should be weak.

Problem 3

Many real-world trading platforms have “circuit breakers” – automatic safeguards which halt all trading in an asset if its price changes too drastically over a short time. Using the material of the course, discuss what consequences can the existence of such circuit breakers have in terms of market outcomes (liquidity, traders’ risk exposure and willingness to trade, price discovery).

Solution: The most basic arguments that can be made are as follows.

- Circuit breakers halt trade on a platform, which by definition kills all liquidity for the duration of the halt, thus existence of CBs can be seen as a liquidity risk.
- CBs are triggered by the excess volatility of the asset price, hence they effectively put an upper bound on this volatility (on the particular time frame of minutes, but not on the micro-scale of milliseconds, and not on the long-run volatility). This provides a form of insurance to traders who hold positions in this asset, and so decreases their risk exposure. This is likely to increase the investors’ willingness to trade, more so for those who are not capable or willing to monitor the asset price continuously – i.e., the less sophisticated and the less informed investors. As we know, presence of such investors makes the market more liquid, and this boost to liquidity can reasonably outweigh the liquidity risk mentioned above, improving the overall market liquidity.
- By design, CBs may hurt price discovery, since the asset price can no longer adjust rapidly in response to news. Higher share of uninformed investors in the market mentioned above also slows down price discovery in the presence of CBs. Higher-order arguments can be made (e.g., “uninformed investors slow the price discovery down by just enough to avoid triggering CBs, which in the end improves the average speed of price discovery”).

Many of the more subtle arguments, as well as empirical estimates of the actual effects, can be found in ESMA working paper No. 1, 2020 (“Market impacts of circuit breakers – Evidence from EU trading venues”, available at https://www.esma.europa.eu/sites/default/files/library/esmawp-2020-1_market_impacts_of_circuit_breakers.pdf).

Written Exam Economics summer 2021

Financial Markets Microstructure

August 21, 2021

This exam question consists of 3 pages in total

Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.

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- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules laid down in part 4.12 of the Faculty of Social Science's common part of the curriculum on cooperation/sparring

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Problem 1

This problem explores the Glosten-Milgrom model with feedback, in which the firm can use the stock market to gauge the attractiveness of an investment project.

In particular, suppose that the firm is facing a binary investment decision. If it invests in the project (e.g., decides to develop a new product), this project will yield net return v_ω , which depends on the state of the world $\omega \in \{l, h\}$ with $\mathbb{P}(h) = 1/2$. If the firm does not invest, it gets zero. The returns are such that $v_h > 0 > v_l$, i.e., the firm wants to invest in the project if and only if the state is $\omega = h$. The baseline value of the firm is μ ; it changes to $\mu + v_\omega$ if the firm invests and remains at μ otherwise.

The timeline is as follows: the firm announces the investment project to the public, one period of trading in the financial market follows, the firm observes trading outcomes and decides whether to proceed with investing in the project or not.

The financial market is modelled as a standard Glosten-Milgrom setting: one trader can submit a buy or a sell order for one unit of the asset. The trader is a profit-maximizing insider with probability $\pi \in (0, 1)$, in which case he knows the true state ω . (Think of the insider as an expert in this industry.) With probability $1 - \pi$, the trader is a noise trader, who submits a buy or a sell order with equal probabilities regardless of ω . Orders are executed by a representative competitive dealer, who provides bid and ask quotes.

1. Suppose that the insider buys the asset when $\omega = h$ and sells when $\omega = l$.
 - (a) What is the expected net value of investment for the firm when it observes a buy order in the market? When it observes a sell order?
 - (b) For which values of π is it optimal for the firm to “follow the market”, i.e., to proceed with the investment when its announcement generates demand for its stocks and to revert its decision when the announcement triggers a “sell-off”? What does this condition mean intuitively?
 - (c) Assuming that the condition you derived in (b) holds and that the firm thus follows the market, derive the bid and ask prices quoted by the dealer.
 - (d) Assuming the condition from (b) holds and given everything you derived, is it optimal for the insider to follow the strategy we assumed? Conclude whether the situation described above constitutes an equilibrium.
2. Assume now the condition from (1b) does not hold and that $\bar{v} = \frac{v_h + v_l}{2} < 0$. Derive formally the pure-strategy equilibrium that occurs in this case. Explain intuitively what happens in this equilibrium and why.
3. Assume now the condition from (1b) does not hold and that $\bar{v} = \frac{v_h + v_l}{2} > 0$. Derive formally the pure-strategy equilibrium that occurs in this case. Explain intuitively what happens in this equilibrium and why.

Problem 2

We have discussed in class that corporate bond markets operate on the basis of RFQs (requests for quotes). The reality is slightly more intricate. The traders typically have a choice between calling a dealer on the phone (voice trading) and using an electronic platform to submit RFQs to a set of dealers (electronic trading). These two methods of trading have coexisted for some time, with electronic trading gradually gaining market share.

Answer the following questions to the best of your ability, relying on the knowledge you have obtained throughout the course. Provide at least two reasons/arguments/suggestions when answering each question.

1. Given the option to trade electronically, why could the traders and dealers prefer to use voice trading?
2. There is some evidence that the spread of electronic trading has led to better quotes being offered in voice trading, and that dealers with more electronic trading in a given bond tend to provide better prices in their voice trading. Why, in your opinion, could this happen?
3. Suppose you are contracted as a consultant by a small electronic exchange, with the goal of increasing the market share of this exchange in corporate bond trading. What suggestions can you give to the exchange that would allow it to attract trading flow?

Financial Markets Microstructure

Final Re-Exam with Solutions

Københavns Universitet

August 2021

Problem 1

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In particular, suppose that the firm is facing a binary investment decision. If it invests in the project (e.g., decides to develop a new product), this project will yield net return v_ω , which depends on the state of the world $\omega \in \{l, h\}$ with $\mathbb{P}(h) = 1/2$. If the firm does not invest, it gets zero. The returns are such that $v_h > 0 > v_l$, i.e., the firm wants to invest in the project if and only if the state is $\omega = h$. The baseline value of the firm is μ ; it changes to $\mu + v_\omega$ if the firm invests and remains at μ otherwise.

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Solution:

Part 1.

- (a) The probability that all initially uninformed parties (the firm and the dealer) assign to the state being $\omega = h$ after observing a Buy order is

$$\mathbb{P}(h|\text{Buy}) = \frac{\frac{\pi}{2} + \frac{1-\pi}{4}}{\frac{\pi}{2} + \frac{1-\pi}{2}} = \frac{1+\pi}{2}.$$

The expected return on investment after observing a Buy order is then

$$\mathbb{E}(v_\omega|\text{Buy}) = \frac{1+\pi}{2}v_h + \left(1 - \frac{1+\pi}{2}\right)v_l = \frac{1+\pi}{2}v_h + \frac{1-\pi}{2}v_l.$$

For the sale order the two are equal to, respectively:

$$\begin{aligned}\mathbb{P}(h|\text{Sell}) &= \frac{\frac{1-\pi}{4}}{\frac{\pi}{2} + \frac{1-\pi}{2}} = \frac{1-\pi}{2}, \\ \mathbb{E}(v_\omega|\text{Sell}) &= \frac{1-\pi}{2}v_h + \left(1 - \frac{1-\pi}{2}\right)v_l = \frac{1-\pi}{2}v_h + \frac{1+\pi}{2}v_l.\end{aligned}$$

- (b) For it to be optimal for the firm to follow the market's advice, it must be that $\mathbb{E}(v_\omega|\text{Buy}) \geq 0 \geq \mathbb{E}(v_\omega|\text{Sell})$. The former inequality is equivalent to

$$\begin{aligned}\frac{1+\pi}{2}v_h + \frac{1-\pi}{2}v_l &\geq 0 \\ \iff \pi &\geq \frac{-2\bar{v}}{v_h - v_l},\end{aligned}$$

where $\bar{v} = \frac{v_h + v_l}{2}$. If $\bar{v} \geq 0$ then this condition always holds. Similarly, $\mathbb{E}(v_\omega | \text{Sell}) \leq 0$ is equivalent to $\pi \geq \frac{2\bar{v}}{v_h - v_l}$. This always holds if $\bar{v} \leq 0$. Both conditions are satisfied if

$$\pi \geq \frac{2|\bar{v}|}{v_h - v_l}. \quad (\star)$$

Intuitively, this condition requires that there are enough insiders in the market for the price to be informative. If $\bar{v} = 0$ then the firm is ex ante indifferent between investing and not, so any positive or negative signal could convince it to do the respective decision. But if, for example, $\bar{v} > 0$, then the investment project looks appealing ex ante, so the negative price signal should be informative enough of ω for the firm to decide to go back on its investment decision.

- (c) Let $I \in \{0, 1\}$ denote the firm's final investment decision. Then $I = 1$ after a buy order and $I = 0$ after a sell order. The dealer is competitive, so we can obtain the quoted prices from the zero-profit condition:

$$a = \mathbb{E}(\mu + Iv_\omega | \text{Buy}) = \mu + \frac{1+\pi}{2}v_h + \frac{1-\pi}{2}v_l; \quad (1)$$

$$b = \mathbb{E}(\mu + Iv_\omega | \text{Sell}) = \mu. \quad (2)$$

- (d) If $\omega = h$, the insider's profit is

$$\Pi_s(h) = \begin{cases} (\mu + v_h) - (\mu + \frac{1+\pi}{2}v_h + \frac{1-\pi}{2}v_l) = \frac{1-\pi}{2}(v_h - v_l) > 0 & \text{if Buy;} \\ 0 & \text{if Pass;} \\ (\mu) - (\mu) = 0 & \text{if Sell} \end{cases}$$

Buying is thus the optimal decision in state $\omega = h$. On the other hand, if $\omega = l$:

$$\Pi_s(l) = \begin{cases} (\mu + v_l) - (\mu + \frac{1+\pi}{2}v_h + \frac{1-\pi}{2}v_l) = -\frac{1+\pi}{2}(v_h - v_l) < 0 & \text{if Buy;} \\ 0 & \text{if Pass;} \\ (\mu) - (\mu) = 0 & \text{if Sell} \end{cases}$$

so selling is weakly optimal.

In the end, if $(\star)$ holds, we have an equilibrium, in which the dealer quotes the ask and bid prices as given by (1) and (2) respectively, the informed trader buys if $\omega = h$ and sells if $\omega = l$, and the firm invests if and only if it observes a buy order (equivalently, if the price of its stock goes up after the investment project is announced).

Part 2. Now $(\star)$ is violated and $\bar{v} < 0$, meaning that the firm never invests in the project, regardless of how the market reacts. This means that $\mathbb{E}(\mu + Iv_\omega \mid \text{Buy}) = \mathbb{E}(\mu + Iv_\omega \mid \text{Sell}) = \mu$ because $I = 0$. The equilibrium then is such that the dealer sets $a = b = \mu$, the firm never invests, and the insider's strategy is arbitrary (since all actions yield zero profit).

Since the firm never invests, the insider's knowledge is irrelevant to the firm value, and there is no adverse selection in the market.

Part 3. If $(\star)$ is violated and $\bar{v} > 0$, the firm always invests. In that case, it is weakly optimal for the insider to buy if $\omega = h$, since then $\Pi_s(h) = \mu + v_h - a$ and $a = \mathbb{E}(\mu + v_\omega \mid \text{Buy}) \leq \mu + v_h$, and by selling the insider would get $\Pi_s(h) = b - (\mu + v_h) \leq 0$ because $b \leq \mu + v_h$. Similarly, it is weakly optimal for the insider to sell if $\omega = l$. Assuming the insider behaves this way, we can then derive the dealer's quotes as

$$a = \mathbb{E}(\mu + Iv_\omega \mid \text{Buy}) = \mu + \frac{1 + \pi}{2}v_h + \frac{1 - \pi}{2}v_l;$$

$$b = \mathbb{E}(\mu + Iv_\omega \mid \text{Sell}) = \mu + \frac{1 - \pi}{2}v_h + \frac{1 + \pi}{2}v_l.$$

Therefore, there exists an equilibrium, in which the dealer sets the prices as above, the insider buys if $\omega = h$ and sells if $\omega = l$, and the firm always invests in the project.

In this case the project is so appealing ex ante that the market can not dissuade the firm from pursuing it. The insider's trading thus does not affect the firm's investment decisions, and we are back to the standard Glosten-Milgrom model.

As in part 1 above, there may, in principle, exist mixed-strategy equilibria in this case, in which the speculator is indifferent in one of the states and mixes between buying and passing or between selling and passing.

Problem 2

We have discussed in class that corporate bond markets operate on the basis of RFQs (requests for quotes). The reality is slightly more intricate. The traders typically have a choice between calling a dealer on the phone (voice trading) and using an electronic platform to submit RFQs to a set of dealers (electronic trading). These two methods of trading have coexisted for some time, with electronic trading gradually gaining market share.

Answer the following questions to the best of your ability, relying on the knowledge you have obtained throughout the course. Provide at least two reasons/arguments/suggestions when answering each question.

1. Given the option to trade electronically, why could the traders and dealers prefer to use voice

trading?

2. There is some evidence that the spread of electronic trading has led to better quotes being offered in voice trading, and that dealers with more electronic trading in a given bond tend to provide better prices in their voice trading. Why, in your opinion, could this happen?
3. Suppose you are contracted as a consultant by a small electronic exchange, with the goal of increasing the market share of this exchange in corporate bond trading. What suggestions can you give to the exchange that would allow it to attract trading flow?

Solution: This problem follows many points and observations mentioned by O'Hara and Zhou (2021).¹

Part 1.

- (a) This may be the result of a miscoordination (as we discussed when talking about market fragmentation): traders do not engage in electronic markets because dealers are not present there, and dealers do not enter the electronic markets because there are no traders there.
- (b) Advertising a trade among many dealers on the electronic market may not be desirable to a trader if this trade is based on private information. Advertising the trade would thus reveal the trader's private information and limit the potential profits the trader could extract from this information. Voice trading, on the other hand, limits the disclosure of information to the particular dealer the trader is engaging with, and is thus more beneficial for such trades. As we discussed in relation to market transparency, the dealers also prefer having informational advantage relative to the rest of the market, and so they would be willing to offer low commissions on voice trading in order to acquire this private information about order flow.

Part 2.

- (a) Increasing competition from electronic trading venues can force dealers to provide more competitive prices in their voice trading (related to our discussion of trading costs in fragmented markets).
- (b) Electronic trading reduces the costs for searching for the right counterparties. So dealers who are more active in the electronic market are able to unravel their inventory more easily, and due to this they are able to provide better quotes to their clients in voice trading.

- (c) Dealers' pricing in their traditional voice trading could be improved by information they learn from both trade interests and actual trades on electronic trading platforms. Having more information about asset value means that dealers are less exposed to adverse selection and are able to quote tighter spreads.

Part 3. This question builds on top of our discussion of market fragmentation and market transparency; any of the many arguments used in those discussions could be invoked in this answer. Below are some examples.

- (a) The first suggestion is to be an appealing platform: offer low trading costs, convenient interface, and be convenient to use in all other respects.
- (b) As we discussed, liquidity begets liquidity, meaning that offering good service is not by itself sufficient to attract business – the platform needs to attract a critical mass of trades before it can attract more. One option is to negotiate with dealers from other platforms and to attempt to lure them into the platform (with monetary incentives like rebates). Another option is to employ new market makers who would provide liquidity in the market.
- (c) In addition to attracting traders with better liquidity and lower trading costs, the platform can make transparency guarantees, making it easy for traders to access information about quotes, past trades, and counterparty identity. As we argued, this would attract uninformed traders to the market (which, in turn, would allow dealers to supply better quotes and further improve liquidity).

¹O'Hara, Maureen, and Xing Alex Zhou. "The Electronic Evolution of Corporate Bond Dealers." *Journal of Financial Economics* 140, no. 2 (May 1, 2021): 368–90. <https://doi.org/10.1016/j.jfineco.2021.01.001>.

Written Exam Economics summer 2022

Financial Markets Microstructure

June 9, 2022

This exam question consists of 04 pages in total

Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.

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Final exam

Write up your responses to questions below and submit them to Digital Exam. The deadline to submit the responses is Jun 9, 22:00. No cooperation with other students is permitted.

Be concise, but show your work and explain your answers. Some questions may require you to make additional assumptions beyond those provided in the question; be clear about the assumptions you make. Some questions are open ended in that they may not have a unique correct answer. You are allowed to refer to textbooks, lecture notes, slides, problem sets, etc.

Problem 1: Twitter buyout

In April 2022, when Elon Musk has announced his intent to buy Twitter at \$54.20 per share, but no official acceptance from Twitter's Board of Directors has been issued, the following tweet has been made by the Saudi Prince Alwaleed bin Talal, head of Kingdom Holding Company, which owns 5.2 percent of Twitter:

"I don't believe that the proposed offer by @elonmusk (\$54.20) comes close to the intrinsic value of @Twitter given its growth prospects ... I reject this offer."

(https://twitter.com/Alwaleed_Talal/status/1514615956986757127)

Twitter stock closing price on NYSE on April 13th (day before the tweet) was \$45.85, and on April 14th (day of the tweet) was \$45.08.

1. Explain how the price movement on April 14th could be rationalized by the quoted tweet.
2. The Prince claimed the intrinsic value of Twitter was much higher than \$54.20 per stock, but the market price was significantly lower than that. Based on the course material, provide at least three possible explanations for this discrepancy.

Problem 2: Liquidity premium with liquidity shocks

When we looked at the Amihud-Mendelson model of illiquidity premium in class, we assumed that all traders have the same holding period h . This problem asks you to reconsider this model when instead of a fixed holding period h , all traders have a fixed probability λ with which a need to sell the asset arises in any given period.

In particular, consider an asset, whose (mid-)price grows at a constant rate R : $\mu_t = \mu_{t-1}(1 + R)$, and which is traded with a constant relative half-spread $\frac{s}{2}$. Consider a market consisting of identical investors who consider buying the asset in period t at the current ask price $a_t = \mu_t(1 + \frac{s}{2})$. In every period $\tau > t$, if the investor is still holding the asset, a liquidity shock arrives with probability λ , which forces the investor to sell the asset at then-current bid price b_τ . The asset yields no dividends. The investor's outside option is putting money in a bank which yields return r and is a perfectly liquid investment.

1. Derive the nominal return R on the asset that should establish in equilibrium. (It should make the investors exactly indifferent between investing in the asset and pursuing their outside options.)
2. How does R depend on λ ? On s ? Explain. *NOTE: a formal argument coupled with an intuitive explanation is preferred. You can provide a purely intuitive argument for partial credit if you were unable to derive the exact expression for R .*

Problem 3: Dynamic limit order book with adverse selection: Effects of algorithmic trading

This problem explores the effects of informed trading in a version of the Parlour model that we have seen in class. Suppose that there is one asset, whose fundamental value v is unknown, and whose market valuation evolves according to $\mu_t = \mathbb{E}[v \mid \Omega_t] = \mu_{t-1} + \epsilon_t$, where $\epsilon_t \in \{-\sigma, \sigma\}$ with equal probabilities is period- t news, publicly announced at the end of period t (after any period- t orders are submitted).¹ In every period t , one risk-neutral trader arrives at the market. With probability π the trader is *informed* and already knows this period's news ϵ_t . With probability $1 - \pi$ the trader is *uninformed* but has an idiosyncratic valuation $y_t \in \{-\sigma, \sigma\}$ with equal probabilities, which is independent of all $\{\epsilon_t\}$. The period- t uninformed trader thus values the asset at $v + y_t$.

Suppose that in every period, there is one ask price $a_t = \mu_{t-1} + S$ and one bid price $b_t = \mu_{t-1} - S$, where S denotes the half-spread, constant across periods. Each arriving trader can choose between submitting a limit order for one unit at the respective price or a market order against an existing order in the limit order book. A limit order is valid for one period and is automatically cancelled if it is not traded against by the next trader. Suppose further that a limit order submitted in period t can be automatically cancelled or repriced when ϵ_t is revealed, so a limit sell order submitted in period t is effectively priced at $a_{t+1} = \mu_t + S$, and a limit buy order at $b_{t+1} = \mu_t - S$. Let $d_t \in \{\emptyset, MS, LS, LB, MB\}$ denote the order submitted by period- t trader, where $d_t = \emptyset$ means the trader abstains from trading, and the other four denote, respectively, the market sell, limit sell, limit buy, and market buy orders.

1. What is the expected utility of a period- t *informed* trader from using a limit buy order, as a function of its execution probability p_{MS} ?
2. What is the expected utility of a period- t *uninformed* trader from using a limit buy order, as a function of its execution probability p_{MS} ?
3. What are the expected utilities that the informed and uninformed traders get from using a market buy order (assuming a limit sell order is in the book)?
4. Conjecture that when $y_t = +\sigma$, the *uninformed* trader uses a market buy order with probability α , assuming one is available, and a limit buy order w.p. $1 - \alpha$; when $\epsilon_t = +\sigma$, the *informed* trader uses MB w.p. β and LB w.p. $1 - \beta$; and symmetric strategies are used when $y_t/\epsilon_t = -\sigma$. Calculate the spread level S^U (as a function of α, β) that renders the uninformed traders indifferent between market and limit orders.
5. Calculate the spread level S^I (as a function of α, β) that renders the uninformed traders indifferent between market and limit orders. How does it compare to S^U ? Explain this relation intuitively: which group of traders is more willing to provide liquidity and why?
6. Taking S as exogenous, what kind of pure-strategy equilibria can arise for different levels of S ? (Note that relation of S and S^I, S^U determines α and β , which, in turn, determine S^I and S^U .)
7. Where do you expect the equilibrium spread S to be, relative to the interval that you identified. (Who has the power to determine S ? Would these traders prefer higher or lower S ? Calculate the equilibrium S if you can.)

NOTE: you can attempt to answer this question intuitively for partial credit if you have not answered some or all of the parts 1-6 above.

8. Turns out, this problem was about algorithmic trading all along! In particular, suppose that it is exactly algorithmic trading that gives the limit traders their ability to reprice their limit orders before

¹Object Ω_t denotes all public information available to the market at (the end of) period t .

they are picked off. Explain intuitively the implications of algorithmic trading in the context of this model.

NOTE: you are expected to make an educated guess about the results in the absence of algo trading; you are not expected to analyze the whole model without repricing. You can attempt to answer this question even if you have not answered some or all of the parts 1-7 above.

Final exam solutions

Problem 1: Twitter buyout

In April 2022, when Elon Musk has announced his intent to buy Twitter at \$54.20 per share, but no official acceptance from Twitter's Board of Directors has been issued, the following tweet has been made by the Saudi Prince Alwaleed bin Talal, head of Kingdom Holding Company, which owns 5.2 percent of Twitter:

"I don't believe that the proposed offer by @elonmusk (\$54.20) comes close to the intrinsic value of @Twitter given its growth prospects ... I reject this offer."
(https://twitter.com/Alwaleed_Talal/status/1514615956986757127)

Twitter stock closing price on NYSE on April 13th (day before the tweet) was \$45.85, and on April 14th (day of the tweet) was \$45.08.

1. Explain how the price movement on April 14th could be rationalized by the quoted tweet.
2. The Prince claimed the intrinsic value of Twitter was much higher than \$54.20 per stock, but the market price was significantly lower than that. Based on the course material, provide at least three possible explanations for this discrepancy.

Solution

1. A major shareholder sent a signal that they might not agree to the buyout, reducing the probability that other investors will receive the buyout value of \$54.20 (which is above the market consensus valuation), which reduced the expected payoff of twitter stock as perceived by the investors.
(The Prince also claimed the fundamental value is above \$54.20, which should have been a good signal about the fundamental value but wasn't, possibly because other traders disagree or did not find this claim credible.)
2. More explanations are possible, but the simplest ones are: (1) The Prince values Twitter stock more than other investors due to some idiosyncratic preference (like it giving him access to internal Twitter data that could be used for political purposes); (2) the Prince has some insider/private information about Twitter's true value; (3) investors vary in their risk-aversion, with most investors being more risk-averse than the Prince and thus having lower valuation of the Twitter stock, which leads to the market valuation being lower as well.

Problem 2: Liquidity premium with liquidity shocks

When we looked at the Amihud-Mendelson model of illiquidity premium in class, we assumed that all traders have the same holding period h . This problem asks you to reconsider this model when instead of a fixed holding period h , all traders have a fixed probability λ with which a need to sell the asset arises in any given period.

In particular, consider an asset, whose (mid-)price grows at a constant rate R : $\mu_t = \mu_{t-1}(1 + R)$, and which is traded with a constant relative half-spread $\frac{s}{2}$. Consider a market consisting of identical investors who consider buying the asset in period t at the current ask price $a_t = \mu_t(1 + \frac{s}{2})$. In every period $\tau > t$, if the investor is still holding the asset, a liquidity shock arrives with probability λ , which forces the investor to sell the asset at then-current bid price b_τ . The asset yields no dividends. The investor's outside option is putting money in a bank which yields return r and is a perfectly liquid investment.

1. Derive the nominal return R on the asset that should establish in equilibrium. (It should make the

investors exactly indifferent between investing in the asset and pursuing their outside options.)

2. How does R depend on λ ? On s ? Explain. *NOTE: a formal argument coupled with an intuitive explanation is preferred. You can provide a purely intuitive argument for partial credit if you were unable to derive the exact expression for R .*

Solution

1. The invested amount should be exactly equal to the expected discounted cash flow from a sale, where the discount rate is r :

$$\mu_t \left(1 + \frac{s}{2}\right) = \lambda \frac{\mu_{t+1} \left(1 - \frac{s}{2}\right)}{1+r} + (1-\lambda)\lambda \frac{\mu_{t+2} \left(1 - \frac{s}{2}\right)}{(1+r)^2} + (1-\lambda)^2 \lambda \frac{\mu_{t+3} \left(1 - \frac{s}{2}\right)}{(1+r)^3} + \dots \quad (1)$$

The idea behind the expression above is as follows: with probability λ the investor receives a liquidity shock and has to sell in the first period after investment, in which case the return is the asset sale price $b_{t+1} = \mu_{t+1} \left(1 - \frac{s}{2}\right)$, and the investor discounts that return by factor $1+r$. The right-hand side of the expression above consists of an infinite series of terms like this, for each period $\tau \in \{t+1, t+2, t+3, \dots\}$. The probability of receiving a liquidity shock in period τ is equal to the joint probability of not receiving a shock in any of the periods from $t+1$ to $\tau-1$ and getting a shock at τ , which amounts to $(1-\lambda)^{\tau-(t+1)}\lambda$. The sale price in period τ is $b_\tau = \mu_\tau \left(1 - \frac{s}{2}\right) = \mu_t(1+R)^{\tau-t} \left(1 - \frac{s}{2}\right)$. Finally, the discount factor for period τ is $(1+r)^{-(\tau-t)}$.

Equation (1) can then be rewritten as

$$\mu_t \left(1 + \frac{s}{2}\right) = \lambda \mu_t \left(1 - \frac{s}{2}\right) \left[\frac{1+R}{1+r} + \frac{(1-\lambda)(1+R)^2}{(1+r)^2} + \frac{(1-\lambda)^2(1+R)^3}{(1+r)^3} + \dots \right]$$

Note that the big bracket on the right-hand side is an infinite geometric series $\Sigma = \beta_0 + \beta_1 + \beta_2 + \dots$ with first term $\beta_0 = \frac{1+R}{1+r}$ and factor $\delta = \frac{(1-\lambda)(1+R)}{1+r}$ (so $\beta_k = \beta_0 \delta^k$). The equality in (1) must hold, meaning that the sum must be convergent, so $\delta < 1$. Using the formula for the sum of an infinite geometric progression, $\Sigma = \frac{\beta_0}{1-\delta}$, we get:

$$\begin{aligned} \mu_t \left(1 + \frac{s}{2}\right) &= \lambda \mu_t \left(1 - \frac{s}{2}\right) \frac{\frac{1+R}{1+r}}{1 - \frac{(1-\lambda)(1+R)}{1+r}} \\ \iff \left(1 + \frac{s}{2}\right) &= \lambda \left(1 - \frac{s}{2}\right) \frac{1+R}{1+r - (1-\lambda)(1+R)} \\ \iff (1+r) \left(1 + \frac{s}{2}\right) &= (1+R) \left[\lambda \left(1 - \frac{s}{2}\right) + (1-\lambda) \left(1 + \frac{s}{2}\right) \right] \\ \iff 1+R &= (1+r) \frac{1 + \frac{s}{2}}{1 + \frac{s}{2}(1-2\lambda)} \end{aligned} \quad (2)$$

2. From (2) we see that given r , R is increasing in both s and λ , which is quite intuitive: s increases the asset illiquidity that the investor is exposed to, which increases the illiquidity premium required by the investor, and λ increases the probability with which (decreases the average time before) the investor is exposed to this illiquidity, which has the same effect.

Problem 3: Dynamic limit order book with adverse selection: Effects of algorithmic trading

This problem explores the effects of informed trading in a version of the Parlour model that we have seen in class. Suppose that there is one asset, whose fundamental value v is unknown, and whose market valuation evolves according to $\mu_t = \mathbb{E}[v \mid \Omega_t] = \mu_{t-1} + \epsilon_t$, where $\epsilon_t \in \{-\sigma, \sigma\}$ with equal probabilities is period- t news, publicly announced at the end of period t (after any period- t orders are submitted).¹ In every period t , one risk-neutral trader arrives at the market. With probability π the trader is *informed* and already knows this period's news ϵ_t . With probability $1 - \pi$ the trader is *uninformed* but has an idiosyncratic valuation $y_t \in \{-\sigma, \sigma\}$ with equal probabilities, which is independent of all $\{\epsilon_t\}$. The period- t uninformed trader thus values the asset at $v + y_t$.

Suppose that in every period, there is one ask price $a_t = \mu_{t-1} + S$ and one bid price $b_t = \mu_{t-1} - S$, where S denotes the half-spread, constant across periods. Each arriving trader can choose between submitting a limit order for one unit at the respective price or a market order against an existing order in the limit order book. A limit order is valid for one period and is automatically cancelled if it is not traded against by the next trader. Suppose further that a limit order submitted in period t can be automatically cancelled or repriced when ϵ_t is revealed, so a limit sell order submitted in period t is effectively priced at $a_{t+1} = \mu_t + S$, and a limit buy order at $b_{t+1} = \mu_t - S$. Let $d_t \in \{\emptyset, MS, LS, LB, MB\}$ denote the order submitted by period- t trader, where $d_t = \emptyset$ means the trader abstains from trading, and the other four denote, respectively, the market sell, limit sell, limit buy, and market buy orders.

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6. Taking S as exogenous, what kind of pure-strategy equilibria can arise for different levels of S ? (Note that relation of S and S^I, S^U determines α and β , which, in turn, determine S^I and S^U .)
7. Where do you expect the equilibrium spread S to be, relative to the interval that you identified. (Who has the power to determine S ? Would these traders prefer higher or lower S ? Calculate the equilibrium S if you can.)

NOTE: you can attempt to answer this question intuitively for partial credit if you have not answered some or all of the parts 1-6 above.

8. Turns out, this problem was about algorithmic trading all along! In particular, suppose that it is exactly algorithmic trading that gives the limit traders their ability to reprice their limit orders before

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they are picked off. Explain intuitively the implications of algorithmic trading in the context of this model.

NOTE: you are expected to make an educated guess about the results in the absence of algotrading; you are not expected to analyze the whole model without repricing. You can attempt to answer this question even if you have not answered some or all of the parts 1-7 above.

Solution

This solution assumes that all limit orders are always automatically repriced, in line with the suggestion to consider LS order in period t as priced at a_{t+1} and LB as priced at b_{t+1} . One could alternatively consider a problem, in which a limit trader strategically decides whether to reprice or not (which would only be relevant for an uninformed trader). Such a setup may yield different results, but would still be considered correct.

1. The informed trader's expected utility from a limit buy order is

$$\begin{aligned} U_{LB}^I &= \mathbb{E}[(v - b_{t+1})p_{MS} \mid \mu_{t-1}, \epsilon_t, d_{t+1} = MS] \\ &= \left((\mu_t + \mathbb{E}[\epsilon_{t+1} \mid \epsilon_t, d_{t+1} = MS]) - (\mu_t - S) \right) p_{MS} \\ &= \left(S + \mathbb{E}[\epsilon_{t+1} \mid \epsilon_t, d_{t+1} = MS] \right) p_{MS}. \end{aligned} \quad (3)$$

Note that while the informed trader knows ϵ_t , they do not know ϵ_{t+1} , and the event that their limit order executes may be informative about ϵ_{t+1} .

2. The uninformed trader's expected utility from a limit buy order is

$$\begin{aligned} U_{LB}^U &= \mathbb{E}[(v + y_t - b_{t+1})p_{MS} \mid \mu_{t-1}, d_{t+1} = MS] \\ &= \left((\mu_{t-1} + \mathbb{E}[\epsilon_t + \epsilon_{t+1} \mid d_{t+1} = MS] + y_t) - (\mu_{t-1} + \mathbb{E}[\epsilon_t \mid d_{t+1} = MS] - S) \right) p_{MS} \\ &= \left(\mathbb{E}[\epsilon_{t+1} \mid d_{t+1} = MS] + y_t + S \right) p_{MS}. \end{aligned} \quad (4)$$

The uninformed trader has no prior knowledge of either ϵ_t or ϵ_{t+1} , but the former is irrelevant (since the trader can reprice after ϵ_t is revealed), and the trader can make inferences about the latter from the event $d_{t+1} = MS$.

3. The informed trader's expected utility from using a market buy order is given by

$$\begin{aligned} U_{MB}^I &= \mathbb{E}[v \mid \mu_{t-1}, \epsilon_t] - a_t \\ &= (\mu_{t-1} + \epsilon_t) - (\mu_{t-1} + S) \\ &= \epsilon_t - S. \end{aligned} \quad (5)$$

For the uninformed trader we have

$$\begin{aligned} U_{MB}^U &= (\mathbb{E}[v \mid \mu_{t-1}] + y_t) - a_t \\ &= (\mu_{t-1} + y_t) - (\mu_{t-1} + S) \\ &= y_t - S. \end{aligned} \quad (6)$$

4. Given α and β , we can calculate p_{MS} and the expectations that enter the utilities above:

$$p_{MS} = \pi \frac{\beta}{2} + (1 - \pi) \frac{\alpha}{2},$$

$$\mathbb{E}[\epsilon_{t+1} \mid \epsilon_t, d_{t+1} = MS] = \mathbb{E}[\epsilon_{t+1} \mid d_{t+1} = MS] = \frac{\pi \frac{\beta}{2}}{\pi \frac{\beta}{2} + (1 - \pi) \frac{\alpha}{2}} \cdot (-\sigma) + \frac{(1 - \pi) \frac{\alpha}{2}}{\pi \frac{\beta}{2} + (1 - \pi) \frac{\alpha}{2}} \cdot 0$$

$$= \frac{-\pi \beta \sigma}{\pi \beta + (1 - \pi) \alpha}.$$

The cutoff S^U must be such that $U_{MB}^U = U_{LB}^U$ (it is trivial to verify that the indifference on the sell side yields the same condition):

$$\sigma - S^U = \left(\frac{-\pi \beta \sigma}{\pi \beta + (1 - \pi) \alpha} + \sigma + S^U \right) \cdot \left(\pi \frac{\beta}{2} + (1 - \pi) \frac{\alpha}{2} \right)$$

$$\iff S^U = \frac{2 - (1 - \pi) \alpha}{2 + (1 - \pi) \alpha + \pi \beta} \sigma.$$

5. The cutoff S^I must be such that $U_{MB}^I = U_{LB}^I$:

$$\sigma - S^U = \left(\frac{-\pi \beta \sigma}{\pi \beta + (1 - \pi) \alpha} + S^U \right) \cdot \left(\pi \frac{\beta}{2} + (1 - \pi) \frac{\alpha}{2} \right)$$

$$\iff S^I = \frac{2 + \pi \beta}{2 + (1 - \pi) \alpha + \pi \beta} \sigma.$$

It is easy to see that $S^I > S^U$. Since trader of type $\theta \in \{I, U\}$ prefers market orders when $S < S^\theta$ and limit orders when $S > S^\theta$, it follows then that informed traders use market orders at least as often as the uninformed traders ($\beta \geq \alpha$ in equilibrium). This is because the informed traders trade based on their private information and have no other trading concerns – but their informational advantage can only be capitalized on via market orders, since ϵ_t is publicly revealed before any trade can happen with a limit order.

6. Three cases are possible with pure strategies:

- (a) $S > S^I$: then $\alpha = \beta = 0$, so no traders use market orders (an indirect way to notice this is to observe that market orders yield negative profit). The cutoff then evaluates to $S_1^I = \sigma$. However, if no traders use market orders, then there is no point to using limit orders, leading to a market breakdown. Therefore, if $S > \sigma$, then there is no trade in equilibrium.
- (b) $S \in (S^U, S^I]$: then $\alpha = 0, \beta = 1$. The cutoffs in this case evaluate to $S_2^I = \sigma$, $S_2^U = \frac{2}{2+\pi} \sigma$. Therefore, when $S \in \left(\frac{2}{2+\pi} \sigma, \sigma \right]$, there exists an equilibrium, in which the uninformed traders use limit orders, while the informed traders use market traders whenever possible (a simple verification confirms that both types get weakly positive profits from their respective strategies, so abstaining is not strictly optimal, and the remaining order types – LS/MS when a buy is prescribed and LB/MB when a sell is prescribed, – yield negative expected profit).
- (c) $S \leq S^U$: then $\alpha = \beta = 1$. The cutoff then evaluates to $S_3^U = \frac{1+\pi}{3} \sigma$. So when $S \leq \frac{1+\pi}{3} \sigma$, there exists an equilibrium in which all traders use market orders *whenever possible* and limit orders otherwise (a similar verification to the one mentioned in the previous point is required). Note that this is, indeed, an equilibrium: despite all traders' preference for market orders, not all of them have the opportunity to trade via a MO (since an appropriate limit order may not be in the LOB), in which case they will have to submit a limit order.

Note that if $S \in \left(\frac{1+\pi}{3}\sigma, \frac{2}{2+\pi}\sigma\right)$ then no pure-strategy equilibrium exists (but a mixed-strategy equilibrium exists, in which $\beta = 1$, and α solves $S = S^U$).

7. The prices are set by the (endogenously selected) limit order traders, whereas traders submitting market orders can only choose whether to trade or not at a given price. It is in the limit order traders' interest to maximize S , since this increases their profits – as long as future traders are willing to trade against their limit orders. This is a reason to believe that S would be up to either $\frac{1+\pi}{3}\sigma$, or σ . Comparing the expected profits from submitting limit orders at the two price levels yields that both the informed and the uninformed traders get higher expected profit when setting $S = \frac{1+\pi}{3}\sigma$, since then the gain from a lower execution risk outweighs the loss from a worse price, compared to $S = \sigma$.

A possible countervailing force is competition. If limit order traders are competitive, the competition may drive the spread down to $S = 0$. When taken literally, however, this model has no competition, since there can be at most one limit order per period and limit orders are automatically cancelled after one period.

8. Algorithmic trading that allows automatic repricing of limit orders mitigates the risks of limit orders, mainly the risk of being picked off after unfavorable news is revealed. This reduces the liquidity providers' exposure to adverse selection, thereby making liquidity provision more attractive and improving market liquidity as a result.²

²The actual logic at play is slightly more convoluted. The equilibrium (half-)spread S derived in part 6 is given by “the largest spread that uninformed traders are willing to tolerate while still trading with market orders”. Reducing adverse selection and making limit orders more appealing reduces this level. So the equilibrium liquidity is improved due to reducing the limit order traders' market power, which, in turn, is due to the limit orders becoming more appealing.

Written Exam Economics summer 2022

Financial Markets Microstructure

August 27, 2022

This exam question consists of 03 pages in total

Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.

Be careful not to cheat at exams!

Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules laid down in part 4.12 of the Faculty of Social Science's common part of the curriculum on cooperation/sparring

You can read more about the rules on exam cheating on your Study Site and in part 4.12 of the Faculty of Social Science's common part of the curriculum.

Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

Final re-exam

Write up your responses to questions below and submit them to Digital Exam. The deadline to submit the responses is Aug 27, 22:00. No cooperation with other students is permitted.

Be concise, but show your work and explain your answers. Be clear about the assumptions you make. Some questions are open ended in that they may not have a unique correct answer. You are allowed to refer to textbooks, lecture notes, slides, problem sets, etc.

Problem 1: Isolated markets

The war in Ukraine led to the isolation of the Russian financial market: both Russian companies' and Russian investors' access to foreign capital markets got heavily restricted, and vice versa, as a result of other countries' sanctions and Russia's own regulations.

Discuss the implications of such an isolation for:

1. Allocative efficiency: will Russian and foreign assets be allocated efficiently among investors? Why/why not?
2. Informational efficiency: will Russian and foreign assets be priced efficiently? Why/why not?

Problem 2: Islandsbanki

COPENHAGEN, April 19 2022 (Reuters) – Iceland said on Tuesday a sale of the state's remaining share in Islandsbanki would not go ahead as planned due to lack of transparency in a sale of the bank's shares last month. ...

"The implementation of the sale did not fully meet the government's expectations, including transparency and clear disclosure of information," it said in a statement. ...

The state sold off 35% of Islandsbanki last year in Iceland's largest ever initial public offering. ... In January, state holding company Icelandic State Financial Investments (ISFI) got permission to sell its remaining 65% stake, of which 22.5% were sold last month ... in an oversubscribed auction.¹

In the context of the case presented above, answer the following question: how can the lack of transparency and clear disclosure of information affect the results during such a stock offering?

Problem 3: Dynamic limit order book with adverse selection

NOTE: this problem is long and difficult. Make as much progress as you can and show your work.

This problem explores the effects of informed trading in a version of the Parlour model that we have seen in class. Suppose that there is one asset, whose fundamental value v is unknown, and whose market valuation evolves according to $\mu_t = \mathbb{E}[v \mid \Omega_t] = \mu_{t-1} + \epsilon_t$, where $\epsilon_t \in \{-\sigma, \sigma\}$ with equal probabilities is period- t news, publicly announced at the end of period t (after any period- t orders are submitted).² In every period t , one risk-neutral trader arrives at the market. With probability π the trader is *informed* and already knows this period's news ϵ_t . With probability $1 - \pi$ the trader is *uninformed* but has an idiosyncratic valuation $y_t \in \{-\sigma, \sigma\}$ with equal probabilities, which is independent of all $\{\epsilon_t\}$. The period- t uninformed trader thus values the asset at $v + y_t$.

¹<https://www.reuters.com/article/iceland-banks-islandsbanki-idUSL2N2WH0NS>

²Object Ω_t denotes all public information available to the market at (the end of) period t .

Suppose that in every period, there is one ask price $a_t = \mu_{t-1} + S$ and one bid price $b_t = \mu_{t-1} - S$, where S denotes the half-spread, constant across periods. Each arriving trader can choose between submitting a limit order for one unit at the respective price or a market order against an existing order in the limit order book. A limit order is valid for one period and is automatically cancelled if it is not traded against by the next trader.³ Let $d_t \in \{\emptyset, MS, LS, LB, MB\}$ denote the order submitted by period- t trader, where $d_t = \emptyset$ means the trader abstains from trading, and the other four denote, respectively, the market sell, limit sell, limit buy, and market buy orders.

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2. What is the expected utility of a period- t *uninformed* trader from using a limit buy order, as a function of its execution probability p_{MS} ?
3. What are the expected utilities that the informed and uninformed traders get from using a market buy order (assuming a limit sell order is in the book)? How do they depend on ϵ_{t-1} ?

We shall look for equilibrium in which traders use a market order if they get a good price, while otherwise an uninformed trader uses a limit order, and an informed trader abstains. I.e., in the presence of a fitting limit order in the book, trading strategies $d_t^I(\epsilon_t, \epsilon_{t-1})$ and $d_t^U(y_t, \epsilon_{t-1})$ for the informed and the uninformed traders are given by:

$$\begin{array}{ll} d_t^I(+\sigma, +\sigma) = MB & d_t^U(+\sigma, +\sigma) = MB \\ d_t^I(+\sigma, -\sigma) = LS & d_t^U(+\sigma, -\sigma) = LB \\ d_t^I(-\sigma, +\sigma) = LB & d_t^U(-\sigma, +\sigma) = LS \\ d_t^I(-\sigma, -\sigma) = MS & d_t^U(-\sigma, -\sigma) = MS \end{array}$$

4. Derive the execution probabilities p_{MS} . Are they different for the informed and the uninformed traders? Why/why not?
5. For which values of S is it optimal for a period- t trader to use a market buy order when $\epsilon_{t-1} = \sigma$ and $\epsilon_t/y_t = +\sigma$ (for the informed/uninformed trader, respectively)?
6. For which values of S are $d_t^I(+\sigma, -\sigma) = LS$ and $d_t^U(+\sigma, -\sigma) = LB$ the optimal strategies in their respective cases?
7. Argue that your answers to the two previous questions apply also to the four remaining cases (a very short verbal argument is expected). Derive the final conditions that S must satisfy for the strategies assumed above to constitute an equilibrium.
8. Do the conditions you derived above impose any restrictions on π ? If yes, explain intuitively why the desired equilibrium may not exist when π is out of bounds.
9. The equilibrium under consideration involves informed traders sometimes trading *against* their private information (when $\epsilon_t = +\sigma$ they may submit a limit *sell* order and vice versa). Discuss intuitively why such a situation arises.

NOTE: you can attempt to answer this question even if you have not answered some or all of the parts 4-9 above.

10. Finally, most of the problem above looked at one specific equilibrium. Are there any other equilibria of this game (for a given S)?

NOTE: this is a bonus question, i.e., it is extremely long and difficult. You are welcome to attempt it and show your work after you have solved the rest of the exam, but you are not expected to provide a

³To be clear: a limit order submitted in period t can **not** be cancelled or repriced when ϵ_t is revealed.

complete answer.

Final re-exam solutions

Problem 1: Isolated markets

The war in Ukraine led to the isolation of the Russian financial market: both Russian companies' and Russian investors' access to foreign capital markets got heavily restricted, and vice versa, as a result of other countries' sanctions and Russia's own regulations.

Discuss the implications of such an isolation for:

1. Allocative efficiency: will Russian and foreign assets be allocated efficiently among investors? Why/why not?
2. Informational efficiency: will Russian and foreign assets be priced efficiently? Why/why not?

Solution

1. No, it is likely efficient for Russians to hold some foreign assets in their portfolios and vice versa, for diversification reasons (country risk in financial assets is both significant and easily diversifiable). Removing such an opportunity hurts the allocative efficiency in financial markets.
2. In the short run, the invasion and the isolation had created a significant uncertainty about the Russian firms' future prospects and their fundamental value. Unless asset prices are strong-form efficient, which they never are, this implies that asset prices would, on average, be further away from the fundamental value, i.e., less efficient.

In the medium to long run, the uncertainty above would be resolved. However, there would still be fewer participants in both Russian and foreign financial markets – due to Russian investors being cut off from the foreign markets and vice versa. This leads to worse price discovery and less efficient prices in both markets. One could, however, argue that such “cross-pollination” was not significant enough in the first place for its shutdown to have a meaningful effect.

Problem 2: Islandsbanki

COPENHAGEN, April 19 2022 (Reuters) – Iceland said on Tuesday a sale of the state's remaining share in Islandsbanki would not go ahead as planned due to lack of transparency in a sale of the bank's shares last month. ...

“The implementation of the sale did not fully meet the government's expectations, including transparency and clear disclosure of information,” it said in a statement. ...

The state sold off 35% of Islandsbanki last year in Iceland's largest ever initial public offering. ... In January, state holding company Icelandic State Financial Investments (ISFI) got permission to sell its remaining 65% stake, of which 22.5% were sold last month ... in an oversubscribed auction.¹

In the context of the case presented above, answer the following question: how can the lack of transparency and clear disclosure of information affect the results during such a stock offering?

Solution

We have seen in class that transparency typically benefits the uninformed and harms the informed investors, since it partly negates the informational advantage that the latter have. If the effect on the uninformed

¹<https://www.reuters.com/article/iceland-banks-islandsbanki-idUSL2N2WH0NS>

investors dominates, their reduced willingness to participate in non-transparent public offerings leads to lower prices and thus lower revenue for the seller.

If the market is mainly driven by the informed investors, they would be more willing to participate in an opaque auction. This is not necessarily better for the seller, since the informed traders' profit comes at a cost to both the uninformed traders and the seller (assuming the seller is uninformed; this seller's loss can be missed from only looking at competitive dealer markets, which we did throughout the course, and which does not apply to the situation at hand). So in the end, opaqueness likely harms the seller regardless of the composition of the buyer population.

Problem 3: Dynamic limit order book with adverse selection

NOTE: this problem is long and difficult. Make as much progress as you can and show your work.

This problem explores the effects of informed trading in a version of the Parlour model that we have seen in class. Suppose that there is one asset, whose fundamental value v is unknown, and whose market valuation evolves according to $\mu_t = \mathbb{E}[v | \Omega_t] = \mu_{t-1} + \epsilon_t$, where $\epsilon_t \in \{-\sigma, \sigma\}$ with equal probabilities is period- t news, publicly announced at the end of period t (after any period- t orders are submitted).² In every period t , one risk-neutral trader arrives at the market. With probability π the trader is *informed* and already knows this period's news ϵ_t . With probability $1 - \pi$ the trader is *uninformed* but has an idiosyncratic valuation $y_t \in \{-\sigma, \sigma\}$ with equal probabilities, which is independent of all $\{\epsilon_t\}$. The period- t uninformed trader thus values the asset at $v + y_t$.

Suppose that in every period, there is one ask price $a_t = \mu_{t-1} + S$ and one bid price $b_t = \mu_{t-1} - S$, where S denotes the half-spread, constant across periods. Each arriving trader can choose between submitting a limit order for one unit at the respective price or a market order against an existing order in the limit order book. A limit order is valid for one period and is automatically cancelled if it is not traded against by the next trader.³ Let $d_t \in \{\emptyset, MS, LS, LB, MB\}$ denote the order submitted by period- t trader, where $d_t = \emptyset$ means the trader abstains from trading, and the other four denote, respectively, the market sell, limit sell, limit buy, and market buy orders.

1. What is the expected utility of a period- t *informed* trader from using a limit buy order, as a function of its execution probability p_{MS} ?
2. What is the expected utility of a period- t *uninformed* trader from using a limit buy order, as a function of its execution probability p_{MS} ?
3. What are the expected utilities that the informed and uninformed traders get from using a market buy order (assuming a limit sell order is in the book)? How do they depend on ϵ_{t-1} ?

We shall look for equilibrium in which traders use a market order if they get a good price, while otherwise an uninformed trader uses a limit order, and an informed trader abstains. I.e., in the presence of a fitting limit order in the book, trading strategies $d_t^I(\epsilon_t, \epsilon_{t-1})$ and $d_t^U(y_t, \epsilon_{t-1})$ for the informed and the uninformed traders are given by:

$$\begin{array}{ll} d_t^I(+\sigma, +\sigma) = MB & d_t^U(+\sigma, +\sigma) = MB \\ d_t^I(+\sigma, -\sigma) = LS & d_t^U(+\sigma, -\sigma) = LB \\ d_t^I(-\sigma, +\sigma) = LB & d_t^U(-\sigma, +\sigma) = LS \\ d_t^I(-\sigma, -\sigma) = MS & d_t^U(-\sigma, -\sigma) = MS \end{array}$$

4. Derive the execution probabilities p_{MS} . Are they different for the informed and the uninformed traders?

²Object Ω_t denotes all public information available to the market at (the end of) period t .

³To be clear: a limit order submitted in period t can **not** be cancelled or repriced when ϵ_t is revealed.

Why/why not?

5. For which values of S is it optimal for a period- t trader to use a market buy order when $\epsilon_{t-1} = \sigma$ and $\epsilon_t/y_t = +\sigma$ (for the informed/uninformed trader, respectively)?
6. For which values of S are $d_t^I(+\sigma, -\sigma) = LS$ and $d_t^U(+\sigma, -\sigma) = LB$ the optimal strategies in their respective cases?
7. Argue that your answers to the two previous questions apply also to the four remaining cases (a very short verbal argument is expected). Derive the final conditions that S must satisfy for the strategies assumed above to constitute an equilibrium.
8. Do the conditions you derived above impose any restrictions on π ? If yes, explain intuitively why the desired equilibrium may not exist when π is out of bounds.
9. The equilibrium under consideration involves informed traders sometimes trading *against* their private information (when $\epsilon_t = +\sigma$ they may submit a limit *sell* order and vice versa). Discuss intuitively why such a situation arises.

NOTE: you can attempt to answer this question even if you have not answered some or all of the parts 4-9 above.

10. Finally, most of the problem above looked at one specific equilibrium. Are there any other equilibria of this game (for a given S)?

NOTE: this is a bonus question, i.e., it is extremely long and difficult. You are welcome to attempt it and show your work after you have solved the rest of the exam, but you are not expected to provide a complete answer.

Solution

1. The informed trader's expected utility from a limit buy order is

$$\begin{aligned}
 U_{LB}^I &= \left(\mathbb{E}[v \mid \mu_{t-1}, \epsilon_t, d_{t+1} = MS] - b_t \right) p_{MS} \\
 &= \left((\mu_t + \mathbb{E}[\epsilon_{t+1} \mid \epsilon_t, d_{t+1} = MS]) - (\mu_{t-1} - S) \right) p_{MS} \\
 &= \left(\epsilon_t + \mathbb{E}[\epsilon_{t+1} \mid \epsilon_t, d_{t+1} = MS] + S \right) p_{MS}.
 \end{aligned} \tag{1}$$

Note that while the informed trader knows ϵ_t , they do not know ϵ_{t+1} , and the event that their limit order executes may be informative about ϵ_{t+1} .

2. The uninformed trader's expected utility from a limit buy order is

$$\begin{aligned}
 U_{LB}^U &= \left(\mathbb{E}[v \mid \mu_{t-1}, d_{t+1} = MS] + y_t - b_t \right) p_{MS} \\
 &= \left((\mu_{t-1} + \mathbb{E}[\epsilon_t + \epsilon_{t+1} \mid d_{t+1} = MS] + y_t) - (\mu_{t-1} - S) \right) p_{MS} \\
 &= \left(\mathbb{E}[\epsilon_t + \epsilon_{t+1} \mid d_{t+1} = MS] + y_t + S \right) p_{MS}.
 \end{aligned} \tag{2}$$

The uninformed trader has no prior knowledge of either ϵ_t or ϵ_{t+1} , but can make inferences about them from the event $d_{t+1} = MS$.

3. The informed trader's expected utility from using a market buy order is given by

$$\begin{aligned}
 U_{MB}^I &= \mathbb{E}[v \mid \mu_{t-1}, \epsilon_t] - a_{t-1} \\
 &= (\mu_{t-1} + \epsilon_t) - (\mu_{t-2} + S) \\
 &= (\mu_{t-1} - \mu_{t-2}) + \epsilon_t - S \\
 &= \epsilon_{t-1} + \epsilon_t - S.
 \end{aligned} \tag{3}$$

For the uninformed trader we have

$$\begin{aligned}
 U_{MB}^U &= (\mathbb{E}[v \mid \mu_{t-1}] + y_t) - a_{t-1} \\
 &= (\mu_{t-1} + y_t) - (\mu_{t-2} + S) \\
 &= (\mu_{t-1} - \mu_{t-2}) + y_t - S \\
 &= \epsilon_{t-1} + y_t - S.
 \end{aligned} \tag{4}$$

The limit sell order placed at $t-1$ is priced at $a_{t-1} = \mu_{t-2} + S$, hence it does not incorporate the last period's news shock ϵ_{t-1} . So if $\epsilon_{t-1} = \sigma$, asset value has increased since the limit order was placed, which is not reflected in a_{t-1} , and so the period- t trader gets to buy the asset cheaply and gets higher utility because of that. Conversely, if $\epsilon_{t-1} = -\sigma$, the limit sell order from $t-1$ overprices the asset relative to its market valuation in t , so using a market buy order yields lower utility.

4. For the uninformed trader, p_{MS} and all the relevant expectations given the assumed equilibrium strategies are given by:

$$p_{MS}^U = \frac{\pi}{4} + \frac{1-\pi}{4} = \frac{1}{4}; \tag{5}$$

$$\begin{aligned}
 \mathbb{E}[\epsilon_{t+1} \mid d_{t+1} = MS] &= -\sigma \cdot \mathbb{P}(I \mid d_{t+1} = MS) + \frac{\sigma - \sigma}{2} \cdot \mathbb{P}(U \mid d_{t+1} = MS) \\
 &= -\sigma\pi;
 \end{aligned} \tag{6}$$

$$\mathbb{E}[\epsilon_t \mid d_{t+1} = MS] = -\sigma. \tag{7}$$

For the informed trader:

$$p_{MS}^I(\epsilon_t) = \mathbb{P}(d_{t+1} = MS \mid \epsilon_t) = \begin{cases} 0 & \text{if } \epsilon_t = +\sigma, \\ \frac{\pi}{2} + \frac{1-\pi}{2} = \frac{1}{2} & \text{if } \epsilon_t = -\sigma; \end{cases} \tag{8}$$

$$\begin{aligned}
 \mathbb{E}[\epsilon_{t+1} \mid \epsilon_t = -\sigma, d_{t+1} = MS] &= \frac{\pi}{\pi + (1-\pi)}(-\sigma) + \frac{1-\pi}{\pi + (1-\pi)} \cdot \frac{\sigma - \sigma}{2} \\
 &= -\pi\sigma;
 \end{aligned} \tag{9}$$

and we do not compute $\mathbb{E}[\epsilon_{t+1} \mid \epsilon_t = +\sigma, d_{t+1} = MS]$ since it is irrelevant (because $p_{MS} = 0$ in that case).

As argued above, ϵ_t affects the appeal of a market order relative to a limit order for trader at $t+1$ – and the assumed strategy profile does indeed assume that the informed trader's behavior at $t+1$ depends on ϵ_t . Therefore, the execution probability p_{MS} of a limit buy order depends on ϵ_t , and the knowledge of ϵ_t allows the informed trader to better estimate p_{MS} .

5. Look first at the uninformed trader when $\epsilon_{t-1} = y_t = +\sigma$. Plugging these values and (5)–(7) into (2)

and (4), we get

$$\begin{aligned} U_{MB}^U(+\sigma, +\sigma) &= 2\sigma - S \\ U_{LB}^U(+\sigma, +\sigma) &= \left((- \sigma - \pi\sigma) + \sigma + S\right) \frac{1}{4} = \frac{S - \pi\sigma}{4} \\ U_{LS}^U(+\sigma, +\sigma) &= U_{LB}^U(-\sigma, -\sigma) = \left((- \sigma - \pi\sigma) - \sigma + S\right) \frac{1}{4} = \frac{S - (2 + \pi)\sigma}{4} \\ U_{MS}^U(+\sigma, +\sigma) &= U_{MB}^U(-\sigma, -\sigma) = -2\sigma - S, \end{aligned}$$

where the first equations in the two latter lines follow from the symmetry of the model. Note that $U_{LB}^U(+\sigma, +\sigma) > U_{LS}^U(+\sigma, +\sigma)$ and $U_{MS}^U(+\sigma, +\sigma) < 0$. Then MB is optimal for U if $U_{MB}^U(+\sigma, +\sigma) \geq \max\{0, U_{LB}^U(+\sigma, +\sigma)\}$, which is equivalent to $S \leq \min\{2\sigma, \frac{8+\pi}{5}\sigma\} \iff S \leq \frac{8+\pi}{5}\sigma$.

Now consider the informed trader when $\epsilon_{t-1} = \epsilon_t = +\sigma$. Plugging these values and (8)–(9) into (1) and (3), we get

$$\begin{aligned} U_{MB}^I(+\sigma, +\sigma) &= 2\sigma - S \\ U_{LB}^I(+\sigma, +\sigma) &= 0 \\ U_{LS}^I(+\sigma, +\sigma) &= U_{LB}^I(-\sigma, -\sigma) = \left(-\sigma - \pi\sigma + S\right) \frac{1}{2} \\ U_{MS}^I(+\sigma, +\sigma) &= U_{MB}^I(-\sigma, -\sigma) = -2\sigma - S, \end{aligned}$$

where the second line follows from $p_{MS}^I(\epsilon_t = +\sigma) = 0$. Note that $U_{MS}^I(+\sigma, +\sigma) < 0$. Then MB is optimal for I if $U_{MB}^I(+\sigma, +\sigma) \geq \max\{0, U_{LS}^I(+\sigma, +\sigma)\}$, which is equivalent to $S \leq \min\{2\sigma, \frac{5+\pi}{3}\sigma\} \iff S \leq \frac{5+\pi}{3}\sigma$.

Market buy order is then optimal in both cases when $S \leq \min\{\frac{8+\pi}{5}\sigma, \frac{5+\pi}{3}\sigma\} \iff S \leq \frac{8+\pi}{5}\sigma$.

6. Start again with the uninformed trader when $\epsilon_{t-1} = -\sigma$ and $y_t = +\sigma$. From (2) and (4) we get

$$\begin{aligned} U_{MB}^U(+\sigma, -\sigma) &= -S \\ U_{LB}^U(+\sigma, -\sigma) &= \left((- \sigma - \pi\sigma) + \sigma + S\right) \frac{1}{4} = \frac{S - \pi\sigma}{4} \\ U_{LS}^U(+\sigma, -\sigma) &= U_{LB}^U(-\sigma, +\sigma) = \left((- \sigma - \pi\sigma) - \sigma + S\right) \frac{1}{4} = \frac{S - (2 + \pi)\sigma}{4} \\ U_{MS}^U(+\sigma, -\sigma) &= U_{MB}^U(-\sigma, +\sigma) = -S. \end{aligned}$$

Note that either market order yields negative profit, and $U_{LB}^U(+\sigma, -\sigma) > U_{LS}^U(+\sigma, -\sigma)$. Then LB is optimal if $U_{LB}^U(+\sigma, -\sigma) \geq 0 \iff S \geq \pi\sigma$.

For the informed trader, when $\epsilon_{t-1} = -\sigma$ and $\epsilon_t = +\sigma$, (1) and (3) yield

$$\begin{aligned} U_{MB}^I(+\sigma, -\sigma) &= -S \\ U_{LB}^I(+\sigma, -\sigma) &= 0 \\ U_{LS}^I(+\sigma, -\sigma) &= U_{LB}^I(-\sigma, +\sigma) = \left(-\sigma - \pi\sigma + S\right) \frac{1}{2} \\ U_{MS}^I(+\sigma, -\sigma) &= U_{MB}^I(-\sigma, +\sigma) = -S, \end{aligned}$$

where either market order yields negative profit, so LS is optimal if $U_{LS}^I(+\sigma, -\sigma) \geq 0 \iff S \geq (1 + \pi)\sigma$.

7. The model is fully symmetric w.r.t. buy/sell sides, hence the conditions for the optimality of market buy orders are exactly the same as the conditions we would get for the optimality of market sell orders

in their respective cases. The same applies to the two remaining cases, $d_t^I(-\sigma, +\sigma)$ and $d_t^U(-\sigma, +\sigma)$, which are completely symmetric to the respective cases $d_t^I(+\sigma, -\sigma)$ and $d_t^U(+\sigma, -\sigma)$, respectively.

Combining the conditions from the previous two questions, we get that the desired equilibrium can be sustained only if

$$S \in \left[\max \{ \pi\sigma, (1 + \pi)\sigma \}, \min \left\{ \frac{8 + \pi}{5}\sigma, \frac{5 + \pi}{3}\sigma \right\} \right] \\ \iff S \in \left[(1 + \pi)\sigma, \frac{8 + \pi}{5}\sigma \right]. \quad (10)$$

8. For the interval in (10) to be nonempty, we need $1 + \pi \leq \frac{8 + \pi}{5} \iff \pi \leq 3/4$. The bounds on S come from two sources (actually more than that, but these two are the tightest). Firstly, $U_{LS}^I(+\sigma, -\sigma) \geq 0$: the spread must be large enough to reward limit orders from informed traders. Putting in a limit order, however, is associated with adverse selection, since a limit order is more likely to execute when $\epsilon_{t+1} = -\sigma$. The higher is π , the more severe this adverse selection is. Secondly, $U_{MB}^U(+\sigma, +\sigma) \geq U_{LB}^U(+\sigma, +\sigma)$: the spread must be small enough for the market orders to not be too costly relative to limit orders for the uninformed traders. While higher π makes limit orders less appealing for the same reason as above, thus relaxing this constraint, this channel is not as strong for uninformed investors because their primary concern is spread S . This is in the sense that U are choosing between MB and LB in equilibrium, while I are choosing between LS and abstaining – so the uninformed investors are twice as sensitive to the spread.
9. Such a countertrading occurs in cases when both past and future price movements turn out to be unfavorable. E.g., if $\epsilon_t = +\sigma$, the informed trader in period t has private information that the asset is worth more than the rest of the market currently believes. However, if $\epsilon_{t-1} = -\sigma$, the current limit orders in the LOB do not reflect this current valuation, since they do not reflect the previous period's bad news. This makes market buy orders unappealing, since the ask price is high relative to the current market valuation μ_{t-1} . On the other hand, a limit buy order is also unlikely to succeed, since ϵ_t is revealed before any market orders can be submitted, so any trader who could trade against a limit buy order would know of the asset's high valuation, meaning they are unlikely to sell at the low price b_t . Therefore, the informed trader is unable to capitalize on their knowledge that the asset is currently underpriced (partly because it is not, in fact, underpriced by the LOB). However, if the spread S is large enough, they can capitalize on the knowledge of the future price movements. In the scenario above, $(\epsilon_t, \epsilon_{t-1}) = (+\sigma, -\sigma)$, putting in a limit sell order is profitable because the spread is large enough for such an order to yield a profit, and the fact that this order is underpriced relative to μ_t makes it more likely to execute.
10. From the analysis above, at least one other equilibrium is relatively salient, which is identical to the one under consideration, except $d_t^I(+\sigma, -\sigma) = d_t^I(-\sigma, +\sigma) = \emptyset$: instead of countertrading, the informed traders abstain in case of unfavorable price movements. These trading strategies can be sustained in equilibrium when $S \in [\pi\sigma, \min \{ (1 + \pi)\sigma, \frac{8 + \pi}{5}\sigma \}]$.

My guess from bruteforcing all other possible equilibria is that no other equilibria exist (any conjectured strategy profile leads to a collection of mutually exclusive conditions on S), but it was a very preliminary analysis, so the conclusion may be wrong.

Written Exam Economics summer 2023

Financial Markets Microstructure

June 8th 2023

This exam question consists of 12 pages in total

Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.

Be careful not to cheat at exams!

Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules in the Faculty of Social Science's common part of the curriculum

You can read more about the rules on exam cheating on your Study Site and in the Faculty of Social Science's common part of the curriculum.

Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

Final exam

Write up your responses to questions below and submit them to Digital Exam. The deadline to submit the responses is Jun 8, 21:00. No cooperation with other students is permitted.

Be concise, but show your work and explain your answers. Some questions may require you to make additional assumptions beyond those provided in the question; be clear about the assumptions you make. Some questions are open ended in that they may not have a unique correct answer. You are allowed to refer to textbooks, lecture notes, slides, problem sets, etc.

Problem 1: Estimating spread with informative order flow

Suppose you want to estimate the average spread for a given stock, but only have access to trade data (prices, directions, volumes). You hence decide to use the Roll's model that we have seen in class. At the same time, you find its assumption about the uninformative order flow unrealistic and want to relax it. You believe instead that some traders are informed, and the prices reflect that (as in, e.g., the Glosten-Milgrom world), which leads to $\mathbb{E}(d_t \epsilon_t) = \rho > 0$ in the Roll's model.

1. Can you estimate ρ from trade data? If yes, explain in a few words how you would do it (you do not need to spell out the formal estimation procedure). If no, explain why and suggest another way to estimate spread.
2. Assuming you know (or have a good estimate for) the value of ρ , provide an estimator for spread S .

Problem 2: Order splitting

Splitting a large order into multiple smaller orders is an important instrument that traders have for moderating their price impact, but it also has general equilibrium implications for order pricing. This problem explores these implications in the context of the Glosten-Milgrom model.

There is one asset traded in a dealer market with a representative (competitive) risk-neutral dealer, who is tasked with quoting an ask price a_1 and a bid price b_1 for one unit of the asset at all times. The dealer believes that the asset's fundamental value is distributed as $v \sim U[\mu - \sigma, \mu + \sigma]$.

The dealer's belief is that any arriving trader is risk-neutral and either informed with probability π , or uninformed with probability $1 - \pi$. An informed trader knows v and selects the profit-maximizing trade given v . An uninformed (noise) trader wants to buy or sell with equal probabilities, and would like to trade either one (with probability $1 - \gamma$) or two (w.p. γ) units of the asset. To trade two units, the trader remains in the market after the first trade and submits another order. The dealer observes the trader's identity and can offer different quotes a_2 and b_2 to a repeat trader.

1. Explain why in such a market, an informed trader would never profit by trading more than two units.
2. Calculate the ask price a_1 that a dealer would quote for the first unit.
3. Calculate the ask price for "repeat" purchases, a_2 .
4. Compare these quotes with what would have been offered in a pure limit order book market (without a dealer), according to the Glosten model (with no order display cost). Would the market-order-traders prefer to trade in a dealer market or a LOB market?
5. Now assume instead that the dealer cannot see the trader's identity, so quote a_2 is also "public". This means that it is taken either by the original trader who wanted to trade two units, or by a second trader willing to buy (the dealer's beliefs about the second trader are the same as about the first). Calculate the ask quote a_2^O . How does it compare to the quote a_2 derived above? Would the traders

prefer their identity to be observable or not?

Problem 3: The fall of FTX

FTX, a large crypto exchange, has collapsed in the Fall of 2022. You can find the timeline of the collapse in the article from Investopedia attached at the end of this exam.¹ You are to answer the following question:

“How does the fall of FTX affect the other participants of the crypto market?”

1. Consult a chatbot/AI/LLM (hereinafter referred to as LLM) on this issue.² Show both your prompt and the response (and mention which LLM you used).

NOTE: for best results, you may want to phrase your own prompt, rather than just copy the question above.

2. Discuss the response you got. Does it make sense? Is it applicable to the situation at hand?
3. Is there anything you would like to add to the LLM response, using the knowledge you obtained in the course?

¹Original article available at <https://www.investopedia.com/what-went-wrong-with-ftx-6828447>.

²For example, you can use openAI's ChatGPT: <https://chat.openai.com>.

The Collapse of FTX: What Went Wrong with the Crypto Exchange?

Full Bio

11–14 minutes

Cryptocurrency exchange FTX and its founder and former CEO, Sam Bankman-Fried, are intricately entwined. The swift and damaging [collapse of FTX in late 2022](#) will have repercussions on the international crypto community for years to come.

Learn more about what went wrong with FTX.

Key Takeaways

- FTX collapsed in early November 2022 following a report by CoinDesk highlighting potential leverage and solvency concerns involving FTX-affiliated trading firm Alameda Research.
- FTX's collapse shook the volatile crypto market, which lost billions at the time, falling below a \$1 trillion valuation.
- FTX in November 2022 faced a liquidity crisis and searched for bailout funds; rival exchange Binance considered buying portions of the company but quickly backed out.
- By Nov. 11, 2022, FTX's CEO stepped down and the company filed for bankruptcy.
- In the hours following, FTX experienced a possible hack in which hundreds of millions worth of tokens were stolen.

- FTX founder and ex-CEO Sam Bankman-Fried was arrested in The Bahamas and extradited to the United States in late December. He pleaded innocent to all criminal charges on Jan. 3, 2023.

What Happened to FTX?

FTX's collapse took place over a 10-day period in November 2022. The catalyst was a Nov. 2 scoop by crypto news site CoinDesk that revealed that Alameda Research, the [quantitative trading](#) firm also run by [Bankman-Fried](#), held a position valued at \$5 billion in FTT, the [native token](#) of FTX.

The report disclosed that Alameda's investment foundation was also in FTT, the token that its sister company had invented, not a [fiat currency](#) or other [cryptocurrency](#).¹ That prompted concern across the cryptocurrency industry regarding Bankman-Fried's companies' undisclosed [leverage](#) and [solvency](#).²

FTX Collapse's Sequence of Events

The following is a recap of the events that led to FTX's failure, bankruptcy filing, and its former CEO's jailing and extradition to the United States to face a series of criminal and civil charges.

FTX Collapse Timeline—2022

- **Nov. 6:** Rival exchange Binance sells all FTT tokens.
- **Nov. 7:** FTX announces liquidity crisis, seeks bailout from venture capitalists, then Binance.
- **Nov. 8:** Binance says it will buy FTX's non-U.S. business.
- **Nov. 9:** Binance walks away from FTX acquisition after conducting due diligence.

- **Nov. 10:** The Bahamas freezes assets of FTX's subsidiary there; Bankman-Fried admits non-U.S. businesses' liquidity crisis, says affiliate Alameda Research to wind down.
- **Nov. 11:** Bankman-Fried steps down as FTX CEO, is replaced by a court-appointed CEO with restructuring experience. FTX files for Chapter 11 bankruptcy protection.
- **Nov. 12:** FTX reports an alleged hack, suspected to be up to \$477 million, and moves its digital assets to cold storage for security reasons.
- **Nov. 18:** The Bahamas takes control of FTX assets held there.
- **Dec. 12:** Bankman-Fried is arrested by Bahamian authorities. He's later extradited to the U.S.
- **Dec. 22:** Bankman-Fried is released on a \$250 million bond, the largest in history, by a federal judge.

Binance Says It Will Sell All Its FTT Tokens

[Binance](#), the world's biggest crypto exchange, announced on Nov. 6 that it would [sell its entire position in FTT tokens](#)—roughly 23 million FTT tokens valued at about \$529 million. Binance CEO [Changpeng “CZ” Zhao](#) said the decision to [liquidate](#) the exchange's FTT position was based on [risk management](#), following the [collapse of the Terra \(LUNA\)](#) crypto token earlier in 2022.³⁴

FTX Liquidity Crisis and Binance Deal

By the next day, FTX was experiencing a [liquidity crisis](#). Bankman-Fried attempted to reassure FTX investors that its assets were stable, but customers demanded withdrawals worth \$6 billion in the days immediately following the CoinDesk report.

Bankman-Fried searched for additional money from venture capitalists before turning to rival Binance. The value of FTT fell by more than 80% in two days.⁵⁶

On Nov. 8, Binance announced that it had reached a nonbinding agreement to buy the non-U.S. business of FTX for an undisclosed sum—effectively the world’s largest cryptocurrency exchange [bailing out](#) its close rival.⁷

Binance Cancels Deal to Bail Out FTX

The promise of a rescue was short-lived, as Binance backed out of the deal a day later. On Nov. 9, the exchange said that it would cancel the FTX deal after corporate [due diligence](#) raised concerns about the mishandling of customer funds, among other issues.⁸

FTX Assets Frozen and Other Implications

On Nov. 10, The Bahamas’ securities regulator froze the assets of FTX Digital Markets, FTX’s Bahamian subsidiary, following news that Bankman-Fried was seeking up to \$8 billion in [capital](#) to bail out the exchange.⁹ On the same day, the California Department of Financial Protection and Innovation announced that it had initiated an investigation into FTX.¹⁰

Bankman-Fried apologized the same day for the liquidity crisis and admitted on Twitter that FTX’s non-U.S. exchange had insufficient funds to meet customer demands. He said that “poor internal labeling” caused FTX to miscalculate leverage and [liquidity](#). In the same Twitter thread, he said Alameda would wind down trading.¹¹

Bankman-Fried Steps Down as CEO; FTX Files for Bankruptcy

Bankman-Fried stepped down on Nov. 11 as CEO of FTX, replaced by court-appointed FTX CEO John Ray, who led energy trading firm [Enron](#) through bankruptcy proceedings years before.¹²

FTX filed for [Chapter 11 bankruptcy](#) protection the same day, revealing that roughly 130 other affiliated companies were also part of the proceedings. The bankruptcy filings indicated that FTX had [assets](#) and [liabilities](#) each in the range of \$10 billion to \$50 billion.¹³

Investors and customers have lost billions, and not all of it will be recovered, the new FTX CEO told a U.S. House of Representatives committee hearing on the FTX debacle held on Dec. 13.

‘Unauthorized Transactions’ on FTX

Within hours of filing for bankruptcy, FTX said it was the victim of “unauthorized transactions” and that it would move its digital assets to [cold storage](#) for security purposes.¹⁴ Outside analysts said they suspect that about \$477 million was stolen from FTX in the alleged hack.¹⁵

Lawsuit Against FTX, Celebrity Promoters Filed

On Nov. 16, [a class-action lawsuit was filed](#) in a Florida federal court, alleging that Bankman-Fried created a fraudulent cryptocurrency scheme designed to take advantage of unsophisticated investors from across the U.S. Others named in the lawsuit include celebrities and professional athletes Steph Curry, Shaquille O’Neal, Shohei Ohtani, Naomi Osaka, Larry David, and Kevin O’Leary, who allegedly helped Bankman-Fried facilitate the plan.¹⁶

The Bahamas Takes Control of FTX Digital Assets

The Securities Commission of The Bahamas (SCB) [took control](#) of cryptocurrency assets held by bankrupt exchange FTX on Nov. 18. The SCB said it instructed Bankman-Fried to move crypto assets to the regulator's wallet to protect creditors.

Arrest, Charges Against Bankman-Fried

Bahamian authorities arrested Bankman-Fried on Dec. 12, 2022, and jailed him in connection with multiple fraud charges involving FTX, including those stemming from an indictment from the U.S. Attorney of the Southern District of New York. U.S. Attorney Damian Williams said when announcing the charges against the former CEO that it was one of the largest financial crimes in American history. New FTX CEO Ray told the U.S. House committee on Dec. 13 that FTX practiced "no bookkeeping." He added, "It was old-fashioned embezzlement."¹⁷

Bankman-Fried was extradited to the U.S., then indicted by U.S. District Court in Manhattan on eight counts, including [securities fraud and money laundering](#). Following a court hearing on Dec. 22, a federal judge decided to release Bankman-Fried from custody after his attorneys and federal prosecutors agreed to a \$250 million bond, the largest in history. The 30-year-old former crypto executive will live with his Stanford law professor parents in Palo Alto, Calif., be confined to the Northern California area, and wear an electronic monitoring bracelet.¹⁸

On Jan. 3, 2023, Bankman-Fried pleaded not guilty to all criminal charges in a federal court in New York. He is set to face trial on eight charges on Oct. 2.¹⁹

Future of FTX and Consequences of Collapse

The future of FTX as a cryptocurrency exchange is in serious jeopardy. As of mid-November 2022, withdrawals were disabled and a notice on the FTX website says the company “strongly advise[s] against depositing.”²⁰

The broader consequences of the FTX fiasco for the cryptocurrency industry will take time to unfold. As the largest collapse in the short history of cryptocurrencies, FTX may further deter [investors](#), who already are cautious because of concerns about stability and security. Customers on the FTX platform may not recover their assets, potentially triggering legal action. The U.S. [Securities and Exchange Commission \(SEC\)](#) and other regulators may see the collapse of FTX as justification for tightening regulatory scrutiny of cryptocurrencies, and Congress may be more inclined to step in and create new laws governing digital tokens and exchanges.²¹

The stunning collapse of the third-largest crypto exchange by volume will send shock waves through the crypto universe for some time. To date, crypto lender [BlockFi](#) paused client withdrawals on Nov. 11, 2022, and rumors indicate it may have a dicey future. [Crypto.com](#) saw withdrawals increase Nov. 12–13, 2022. [Genesis Global Trading](#) halted customer withdrawals from its crypto lending unit.

How did FTX fail?

FTX filed for bankruptcy on Nov. 11, 2022, after a surge of customer withdrawals earlier in the month. Then-CEO Sam Bankman-Fried admitted that the company didn’t have sufficient assets [in reserve](#) to meet customer demand.

Did FTX get hacked?

Within hours of filing for bankruptcy, FTX alleged that it was hacked. The exchange noted “unauthorized transactions” that may have stolen close to \$500 million in assets, and that were spotted by Elliptic, a crypto compliance service. The hacker continued to drain wallets for several days, using what analysts called “on-chain spoofing.” The hacker reportedly then invested those funds in Ether (ETH).²²

What happens next to ex-FTX CEO Bankman-Fried?

Bankman-Fried is set to face trial on eight criminal charges on Oct. 2, 2023. After a court hearing on Dec. 22, 2022, a federal judge decided to release Bankman-Fried from custody after his attorneys and federal prosecutors agreed to a \$250 million bond, the largest ever. He will live with his law professor parents in Palo Alto, Calif., be confined to the Northern California area, and wear an electronic monitoring bracelet.¹⁸ On Jan. 3, 2023, Bankman-Fried pleaded not guilty to all criminal charges in a federal court in New York.

The Bottom Line

The fortunes of FTX and its founder and former CEO, Bankman-Fried, are linked. In November 2022, cryptocurrency exchange FTX collapsed over a period of 10 days. Following a report suggesting potential leverage and solvency concerns, the exchange faced a liquidity crisis and tried to negotiate a bailout by rival Binance that quickly fell through. Its CEO was later arrested, extradited to the U.S., released on a \$250 million bond, and now faces trial in October.

The implications for the future of FTX, and the collapse's impact on the broader cryptocurrency industry, are ongoing and difficult to assess.

Final exam solutions

Problem 1: Estimating spread with informative order flow

Suppose you want to estimate the average spread for a given stock, but only have access to trade data (prices, directions, volumes). You hence decide to use the Roll's model that we have seen in class. At the same time, you find its assumption about the uninformative order flow unrealistic and want to relax it. You believe instead that some traders are informed, and the prices reflect that (as in, e.g., the Glosten-Milgrom world), which leads to $\mathbb{E}(d_t \epsilon_t) = \rho > 0$ in the Roll's model.

1. Can you estimate ρ from trade data? If yes, explain in a few words how you would do it (you do not need to spell out the formal estimation procedure). If no, explain why and suggest another way to estimate spread.
2. Assuming you know (or have a good estimate for) the value of ρ , provide an estimator for spread S .

Solution

Question 1. By definition, $\rho \equiv \mathbb{E}(d_t \epsilon_t)$. It is said in the problem that we can observe trade directions d_t . However, $\epsilon_t = m_t - m_{t-1}$ is not directly observed, since midquotes m_t are unknown. It is, however, true, that $m_t = p_t - \frac{d_t S}{2}$, so $\epsilon_t = \Delta p_t - \frac{\Delta d_t S}{2}$ – it is given by the innovation in price, adjusted for the spread. We could then estimate ρ simultaneously with S by using, e.g., the Generalized Method of Moments, given the estimator for S presented below and

$$\hat{\rho} = \mathbb{E} \left[d_t \left(\Delta p_t - \frac{\Delta d_t S}{2} \right) \right].$$

In principle, since we are interested in ϵ_t as an informational innovation, we could also look at a long-run price impact $p_{t+\tau} - p_t$ for some large enough τ instead of Δp_t , adjusted for the spread accordingly. This would filter out any short-run fluctuations caused by inventory risk or order processing costs.

Question 2. Suppose the following:

1. All trades have the same size. $d = 1$: buy, $d = -1$: sell
2. Arriving orders are i.i.d. with $\mathbb{P}(d_t = 1) = \frac{1}{2}$
3. Midquote is random walk: $m_t = m_{t-1} + \epsilon_t$, where ϵ_t are i.i.d. shocks
4. Market orders are informative about price movements: $\mathbb{E}(d_t \epsilon_t) = \rho > 0$ (but maintain $\mathbb{E}(d_t \epsilon_{t+1}) = 0$).
5. Spread $S = a_t - b_t$ is constant.

Then

$$p_t = m_t + \frac{d_t S}{2}.$$

$$\begin{aligned}
Cov(\Delta p_t, \Delta p_{t-1}) &= Cov(p_t - p_{t-1}, p_{t-1} - p_{t-2}) \\
&= Cov\left(\frac{S}{2}d_t - \frac{S}{2}d_{t-1} + \epsilon_t, \frac{S}{2}d_{t-1} - \frac{S}{2}d_{t-2} + \epsilon_{t-1}\right) \\
&= \frac{S^2}{4}Cov\left(d_t - d_{t-1}, d_{t-1} - d_{t-2} + \frac{2}{S}\epsilon_{t-1}\right) \\
&= \frac{S^2}{4}\mathbb{E}\left[(d_t - d_{t-1})\left(d_{t-1} - d_{t-2} + \frac{2}{S}\epsilon_{t-1}\right)\right] \\
&= \frac{S^2}{4}\mathbb{E}\left[-d_{t-1}^2 - \frac{2}{S}\rho\right] = \frac{-S^2 - 2\rho S}{4}
\end{aligned}$$

Denoting $C \equiv Cov(\Delta p_t, \Delta p_{t-1})$, we then have

$$\begin{aligned}
S^2 + 2\rho S + 4C &= 0 \\
\Rightarrow S &= -\rho \pm \sqrt{\rho^2 - 4C}.
\end{aligned}$$

Recall that for $\rho = 0$, our estimator was $\hat{S} = 2\sqrt{-Cov(\Delta p_t, \Delta p_{t-1})}$. This corresponds to the positive root above, hence our new spread estimator is

$$\hat{S} = -\rho + \sqrt{\rho^2 - 4Cov(\Delta p_t, \Delta p_{t-1})}$$

Note: the book talks briefly about the Roll model with adverse selection in Box 5.1. It argues that in the presence of adverse selection, the Roll estimator underestimates the spread by a factor of $\sqrt{(\lambda + \gamma)/\gamma}$. In order for this answer to be accepted, the student must then explain what λ and γ are, and how they can be estimated.

Problem 2: Order splitting

Splitting a large order into multiple smaller orders is an important instrument that traders have for moderating their price impact, but it also has general equilibrium implications for order pricing. This problem explores these implications in the context of the Glosten-Milgrom model.

There is one asset traded in a dealer market with a representative (competitive) risk-neutral dealer, who is tasked with quoting an ask price a_1 and a bid price b_1 for one unit of the asset at all times. The dealer believes that the asset's fundamental value is distributed as $v \sim U[\mu - \sigma, \mu + \sigma]$.

The dealer's belief is that any arriving trader is risk-neutral and either informed with probability π , or uninformed with probability $1 - \pi$. An informed trader knows v and selects the profit-maximizing trade given v . An uninformed (noise) trader wants to buy or sell with equal probabilities, and would like to trade either one (with probability $1 - \gamma$) or two (w.p. γ) units of the asset. To trade two units, the trader remains in the market after the first trade and submits another order. The dealer observes the trader's identity and can offer different quotes a_2 and b_2 to a repeat trader.

1. Explain why in such a market, an informed trader would never profit by trading more than two units.
2. Calculate the ask price a_1 that a dealer would quote for the first unit.
3. Calculate the ask price for "repeat" purchases, a_2 .
4. Compare these quotes with what would have been offered in a pure limit order book market (without a dealer), according to the Glosten model (with no order display cost). Would the market-order-traders prefer to trade in a dealer market or a LOB market?
5. Now assume instead that the dealer cannot see the trader's identity, so quote a_2 is also "public". This

means that it is taken either by the original trader who wanted to trade two units, or by a second trader willing to buy (the dealer's beliefs about the second trader are the same as about the first). Calculate the ask quote a_2^O . How does it compare to the quote a_2 derived above? Would the traders prefer their identity to be observable or not?

Solution

1. An uninformed trader never trades more than two units. Hence if the dealer observes that a trader wants to trade a third unit, the immediate inference is that the trader must be informed. The respective quotes a_3, b_3 are then determined from the zero-profit equations $a_3 = \mathbb{E}[v \mid v \geq a_3]$, $b_3 = \mathbb{E}[v \mid v \leq b_3]$ (which already include the informed trader's optimal strategy "buy only if $v \geq a_3$; sell only if $v \leq b_3$ "). These have unique solutions $a_3 = \mu + \sigma$, $b_3 = \mu - \sigma$. Trading at these prices yields zero profit to an informed trader.

Note that the same logic applies to switching trade directions (submitting a "buy" after a "sell" or vice versa).

2. Let $x \in \{-2, -1, 1, 2\}$ denote the total signed quantity that the trader plans to buy. The informed trader buys the first unit ($x \geq 1$) if the marginal profit from doing so is positive, i.e., if $v - a_1 \geq 0 \iff v \geq a_1$. It is convenient to introduce $\sigma_1 = a_1 - \mu$ and say that the informed trader buys the first unit at price a_1 if $v > \mu + \sigma_1$.

The zero-profit condition for the dealer, $a_1 = \mathbb{E}[v \mid x \geq 1]$, can be expanded using the law of iterated expectations as follows, where I and U denote the events of trader being informed or uninformed, respectively.

$$\begin{aligned} a_1 &= \mathbb{E}[v \mid x \geq 1] \\ &= \mathbb{E}[v \mid U, x \geq 1] \cdot \mathbb{P}[U \mid x \geq 1] + \mathbb{E}[v \mid I, x \geq 1] \cdot \mathbb{P}[I \mid x \geq 1] \\ &= \mu \cdot \frac{(1-\pi)/2}{\pi \frac{\sigma-\sigma_1}{2\sigma} + \frac{1-\pi}{2}} + \left(\frac{(\mu+\sigma) + (\mu+\sigma_1)}{2} \right) \cdot \frac{\pi \frac{\sigma-\sigma_1}{2\sigma}}{\pi \frac{\sigma-\sigma_1}{2\sigma} + \frac{1-\pi}{2}} \\ &= \mu + \frac{\sigma + \sigma_1}{2} \cdot \frac{\pi \frac{\sigma-\sigma_1}{2\sigma}}{\pi \frac{\sigma-\sigma_1}{2\sigma} + \frac{1-\pi}{2}}. \end{aligned}$$

From the trader's optimality above, $a_1 = \mu + \sigma_1$, which yields:

$$\begin{aligned} \mu + \sigma_1 &= \mu + \frac{\sigma + \sigma_1}{2} \cdot \frac{\pi \frac{\sigma-\sigma_1}{2\sigma}}{\pi \frac{\sigma-\sigma_1}{2\sigma} + \frac{1-\pi}{2}} \\ \Leftrightarrow \frac{\sigma_1}{2} \cdot \left(2 - \frac{\pi \frac{\sigma-\sigma_1}{2\sigma}}{\pi \frac{\sigma-\sigma_1}{2\sigma} + \frac{1-\pi}{2}} \right) &= \frac{\sigma}{2} \cdot \frac{\pi \frac{\sigma-\sigma_1}{2\sigma}}{\pi \frac{\sigma-\sigma_1}{2\sigma} + \frac{1-\pi}{2}} \\ \Leftrightarrow 2\sigma\sigma_1 - \pi\sigma_1^2 &= \pi\sigma^2 \\ \Rightarrow \sigma_1 &= \frac{1 - \sqrt{1 - \pi^2}}{\pi} \sigma \end{aligned}$$

(since the positive root yields $\sigma_1 > \sigma$, which is nonsense). Hence we have $a_1 = \mu + \sigma_1$, where σ_1 is as above

3. Similarly to the above, the informed trader buys the second unit at price a_2 only if $v > \mu + \sigma_2$, where

$\sigma_2 = a_2 - \mu$. We are looking for $\sigma_2 \in (\sigma_1, \sigma)$. The dealer's zero-profit condition now is

$$\begin{aligned} a_2 &= \mathbb{E}[v \mid x \geq 2] \\ &= \mathbb{E}[v \mid U, x \geq 2] \cdot \mathbb{P}[U \mid x \geq 2] + \mathbb{E}[v \mid I, x \geq 2] \cdot \mathbb{P}[I \mid x \geq 2] \\ &= \mu \cdot \frac{(1-\pi)\frac{\gamma}{2}}{\pi\frac{\sigma-\sigma_2}{2\sigma} + (1-\pi)\frac{\gamma}{2}} + \left(\frac{(\mu+\sigma) + (\mu+\sigma_2)}{2} \right) \cdot \frac{\pi\frac{\sigma-\sigma_2}{2\sigma}}{\pi\frac{\sigma-\sigma_2}{2\sigma} + (1-\pi)\frac{\gamma}{2}} \\ &= \mu + \frac{\sigma + \sigma_2}{2} \cdot \frac{\pi\frac{\sigma-\sigma_2}{2\sigma}}{\pi\frac{\sigma-\sigma_2}{2\sigma} + (1-\pi)\frac{\gamma}{2}}. \end{aligned}$$

Plugging in $a_2 = \mu + \sigma_2$, we get

$$\begin{aligned} \sigma_2 &= \frac{\sigma + \sigma_2}{2} \cdot \frac{\pi\frac{\sigma-\sigma_2}{2\sigma}}{\pi\frac{\sigma-\sigma_2}{2\sigma} + (1-\pi)\frac{\gamma}{2}} \\ \Leftrightarrow 0 &= \pi\sigma^2 - 2\sigma(\pi + (1-\pi)\gamma)\sigma_2 + \pi\sigma_2^2 \\ \Rightarrow \sigma_2 &= \frac{\pi + (1-\pi)\gamma - \sqrt{(2\pi + (1-\pi)\gamma)(1-\pi)\gamma}}{\pi} \sigma. \end{aligned}$$

4. Recall that in the Glosten model, a trader who submits a limit order can only condition their price on the event that their limit order executes, but not on the total order size. In our setup, there would be two limit orders on the ask side of the market for one unit each. The price of the first unit would then be $a_1 = \mathbb{E}[v \mid x \geq 1]$, and the price of the second unit $a_2 = \mathbb{E}[v \mid x \geq 2]$ – exactly the same as in the dealer model above. This serves to show that when market-order traders split their orders, the dealer has no informational advantage over the LOB traders. The market-order traders then do not care whether they trade against a dealer or a limit order book.
5. Intuitively, suppose the dealer believes that with probability λ they trade against the second order of the first trader, and with probability $1 - \lambda$ against the first order of the second trader. The competitive ask quote would then satisfy $a_2^O = \lambda\mathbb{E}[v \mid x \geq 2] + (1 - \lambda)\mathbb{E}[v \mid x \geq 1]$, so it would be $a_2^O \in (a_1, a_2)$.¹ Since $a_1 < a_2^O < a_2$, all market-order traders (who trade two units) would prefer an opaque market, since they get better prices in such a market, due to the dealer allowing for a possibility that a new uninformed trader arrives. Note, however, that this is only true for the “first” trader in the market – it is not obvious how the prices for the second and later traders would change.

A formal derivation is a_2^O is quite cumbersome, and the students were not expected to complete it; an intuitive argument like the one above would suffice. For completeness, the full derivation is presented below. We are looking for an ask price a_2^O (which is only relevant if the second order is a buy) in an opaque market conditional on the first order being a buy. Define again $\sigma_2^O \equiv a_2^O - \mu$ and guess that $\sigma_2^O > \sigma_1$ (verify at the end). The contingencies that generate a *(buy, buy)* order sequence, together with their ex ante (unconditional) probabilities, are as follows:

- (a) The first trader was uninformed and wanted to buy two units: probability $(1-\pi)\frac{\gamma}{2}$; $\mathbb{E}[v] = \mu$;²
- (b) The first trader was informed and v is high enough to justify two buys: probability $\pi\frac{\sigma-\sigma_2^O}{2\sigma}$; infer that $v \in [\mu + \sigma_2^O, \mu + \sigma]$, so $\mathbb{E}[v] = \mu + \frac{\sigma_2^O + \sigma}{2}$ in this case;
- (c) The first trader left after buying one unit, and the second trader wants to buy at least one unit. The first trader leaves if:
 - i. uninformed, only wanted to buy one unit (probability $(1-\pi)\frac{1-\gamma}{2}$); in this case the expected

¹This is not strictly correct, since $\mathbb{E}[v \mid x \geq 2]$ itself depends on a_2 .

²To be clear, notation is used loosely in this list, and the expectations $\mathbb{E}[v]$ actually mean conditional expectations, conditional on the respective cases.

value of the asset conditional on the second order being a buy order is exactly $\mathbb{E}[v] = a_1$;

- ii. informed, and v is not high enough to warrant buying second unit (probability $\pi \frac{\sigma_2^O - \sigma_1}{2\sigma}$; infer that $v \in [\mu + \sigma_1, \mu + \sigma_2^O]$). Note that in this case if the second trader is also informed, he would also not want to buy at price a_2^O . Hence a second buy order can only come from an uninformed trader (probability $\frac{1-\pi}{2}$ on top of the probability above) and would not provide any information about v beyond $v \in [\mu + \sigma_1, \mu + \sigma_2^O]$, hence in this case $\mathbb{E}[v] = \mu + \frac{\sigma_1 + \sigma_2^O}{2}$.

The total probability of all contingencies above is

$$(1 - \pi) \frac{\gamma}{2} + \pi \frac{\sigma - \sigma_2^O}{2\sigma} + (1 - \pi) \frac{1 - \gamma}{2} + \pi \frac{\sigma_2^O - \sigma_1}{2\sigma} \cdot \frac{1 - \pi}{2} = \frac{1 - \pi}{2} \cdot \left(1 + \pi \frac{\sigma_2^O - \sigma_1}{2\sigma} \right) + \pi \frac{\sigma - \sigma_2^O}{2\sigma}$$

Taking the expectation of v over the cases above, we get that

$$\begin{aligned} a_2^O = \mu + \sigma_2^O &= \frac{(1 - \pi) \frac{\gamma}{2}}{\frac{1 - \pi}{2} \cdot \left(1 + \pi \frac{\sigma_2^O - \sigma_1}{2\sigma} \right) + \pi \frac{\sigma - \sigma_2^O}{2\sigma}} \cdot \mu + \frac{\pi \frac{\sigma - \sigma_2^O}{2\sigma}}{\frac{1 - \pi}{2} \cdot \left(1 + \pi \frac{\sigma_2^O - \sigma_1}{2\sigma} \right) + \pi \frac{\sigma - \sigma_2^O}{2\sigma}} \cdot \left(\mu + \frac{\sigma_2^O + \sigma}{2} \right) \\ &\quad + \frac{(1 - \pi) \frac{1 - \gamma}{2}}{\frac{1 - \pi}{2} \cdot \left(1 + \pi \frac{\sigma_2^O - \sigma_1}{2\sigma} \right) + \pi \frac{\sigma - \sigma_2^O}{2\sigma}} \cdot a_1 + \frac{\pi \frac{\sigma_2^O - \sigma_1}{2\sigma} \cdot \frac{1 - \pi}{2}}{\frac{1 - \pi}{2} \cdot \left(1 + \pi \frac{\sigma_2^O - \sigma_1}{2\sigma} \right) + \pi \frac{\sigma - \sigma_2^O}{2\sigma}} \cdot \left(\mu + \frac{\sigma_1 + \sigma_2^O}{2} \right) \\ &= \mu + \frac{\pi \frac{\sigma - \sigma_2^O}{2\sigma} \cdot \frac{\sigma_2^O + \sigma}{2} + (1 - \pi) \frac{1 - \gamma}{2} \cdot \sigma_1 + \pi \frac{\sigma_2^O - \sigma_1}{2\sigma} \cdot \frac{1 - \pi}{2} \cdot \frac{\sigma_1 + \sigma_2^O}{2}}{\frac{1 - \pi}{2} \cdot \left(1 + \pi \frac{\sigma_2^O - \sigma_1}{2\sigma} \right) + \pi \frac{\sigma - \sigma_2^O}{2\sigma}} \\ \Rightarrow \sigma_2^O \cdot \left[\frac{1 - \pi}{2} \cdot \left(1 + \pi \frac{\sigma_2^O - \sigma_1}{2\sigma} \right) + \pi \frac{\sigma - \sigma_2^O}{2\sigma} \right] &= \frac{1 - \pi}{2} (1 - \gamma) \sigma_1 + \frac{\pi}{4\sigma} \left(\sigma^2 - \frac{1 + \pi}{2} (\sigma_2^O)^2 - \frac{1 - \pi}{2} \sigma_1^2 \right) \\ \Leftrightarrow (\sigma_2^O)^2 \cdot \frac{\pi(1 - \pi)}{4\sigma} + \sigma_2^O \cdot \left(\frac{\pi(1 - \pi)\sigma_1}{2\sigma} - 1 \right) &+ \left((1 - \pi)(1 - \gamma)\sigma_1 + \frac{\pi\sigma}{2} - \frac{\pi(1 - \pi)\sigma_1^2}{4\sigma} \right) = 0. \end{aligned}$$

The above is a quadratic equation w.r.t. σ_2^O , hence can be solved using the standard methods. Plotting the solution numerically shows that indeed $\sigma_2^O < \sigma_2$ (and demonstrates which root is relevant). For low enough π and γ it is also true that $\sigma_2^O > \sigma_1$, as assumed. If the parameters are such that this does not hold, then $a_2^O = a_1$ is the corner solution.

Problem 3: The fall of FTX

FTX, a large crypto exchange, has collapsed in the Fall of 2022. You can find the timeline of the collapse in the article from Investopedia attached at the end of this exam.³ You are to answer the following question:

“How does the fall of FTX affect the other participants of the crypto market?”

1. Consult a chatbot/AI/LLM (hereinafter referred to as LLM) on this issue.⁴ Show both your prompt and the response (and mention which LLM you used).

NOTE: for best results, you may want to phrase your own prompt, rather than just copy the question above.

2. Discuss the response you got. Does it make sense? Is it applicable to the situation at hand?
3. Is there anything you would like to add to the LLM response, using the knowledge you obtained in the

³Original article available at <https://www.investopedia.com/what-went-wrong-with-ftx-6828447>.

⁴For example, you can use openAI's ChatGPT: <https://chat.openai.com>.

course?

Solution

1. The student is evaluated based on the quality of their prompt. E.g., it should be immediate that the LLM did not take this course and likely has no idea it exists, hence mentioning the course is not constructive. At the same time, mentioning “microstructure” may in principle frame the LLM’s responses in a more relevant key. Finally, given that the collapse of FTX is a recent event, the LLMs are unlikely to have information relevant to the specific case (which, e.g., openAI usually mentions in its responses) – so it makes sense to ask more generic, counterfactual questions, such as “How would a collapse of a large cryptocurrency exchange affect other market participants?” Finally, the investopedia text is presented in the exam to give the unaware students a chance to get acquainted with the case, not to be fed to the LLM. It is implied that the text is short enough to be processed by the student without the help of the LLM – and the benefit of the LLM is in accessing the broader body of knowledge, as opposed to extracting core points from a short text.
2. In my experience, the LLM responses are typically vague, but mostly relevant. If forced to speculate on the specific case of FTX, the LLM can hypothesize about the reasons for the collapse that do not have much in common with the realized timeline (such as security issues); these can be rebuked by the student. Alternatively, if the LLM suggests hypothetical consequences that would be instantly testable using widely accessible data, the student is expected to do the fact-checking (e.g., “investors can lose confidence in crypto” or “other exchanges would gain market share by capturing orphaned traders” can both be easily tested by looking up trading volumes on competing crypto exchanges, such as Coinbase or Binance).
3. Open question. An easy answer would be to discuss the consequences of market fragmentation, which the LLMs tend to not mention (fragmentation might either increase or decrease after FTX demise, depending on where traders migrate to), and with it market transparency (the ease to aggregate information from different platforms).

Written Exam Economics summer 2023

Financial Markets Microstructure

June 8th 2023

This exam question consists of 12 pages in total

Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.

Be careful not to cheat at exams!

Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules in the Faculty of Social Science's common part of the curriculum

You can read more about the rules on exam cheating on your Study Site and in the Faculty of Social Science's common part of the curriculum.

Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

Final re-exam

Write up your responses to questions below and submit them to Digital Exam. The deadline to submit the responses is Aug 28, 21:00. No cooperation with other students is permitted.

Be concise, but show your work and explain your answers. Some questions may require you to make additional assumptions beyond those provided in the question; be clear about the assumptions you make. Some questions are open ended in that they may not have a unique correct answer. You are allowed to refer to textbooks, lecture notes, slides, problem sets, etc.

Problem 1: Spread dynamics in Glosten-Milgrom model

Consider a standard Glosten-Milgrom model:

- asset fundamental value is $v \in \{v^L, v^H\}$, the two realizations are considered equally probable ex ante;
- a competitive dealer sequentially quotes bid b_t and ask a_t prices for one unit of the asset each period;
- one trader arrives at the market per period and can submit a market buy or sell order for one unit or do nothing: $d_t \in \{-1, 0, 1\}$;
- with probability π the trader is informed and knows v and chooses d_t to maximize profit; with complementary probability $1 - \pi$ the trader is uninformed and submits either a buy or a sell order with equal probabilities regardless of v .

Answer the following questions.

1. Calculate the first-period market valuation $\mu_0 = \mathbb{E}[v]$, the ask and bid prices a_1, b_1 , and the relative spread $s_1 = \frac{a_1 - b_1}{\mu_0}$.
2. Suppose the first order was a sell: $d_1 = -1$. Calculate the second-period market valuation $\mu_1 = \mathbb{E}[v | d_1 = -1]$, the ask and bid prices a_2, b_2 , and the relative spread $s_2 = \frac{a_2 - b_2}{\mu_1}$.
3. How does s_2 compare to s_1 ? Give an intuitive explanation for why. Do you expect this trend to continue from s_2 to s_3 and onwards?

Problem 2: Dynamic LOB markets with naive traders

This problem explores a version of the Foucault/Parlour model that we have seen in class. Suppose that there is one asset, whose fundamental value v is unknown, and whose market valuation evolves according to $\mu_t = \mathbb{E}[v | \Omega_t] = \mu_{t-1} + \epsilon_t$, where $\epsilon_t \in \{-\sigma, 0, \sigma\}$ with equal probabilities is period- t news, publicly announced at the end of period t (after any period- t orders are submitted).¹ In every period t , one risk-neutral trader arrives at the market (who only knows μ_{t-1} but not ϵ_t , and has no idiosyncratic preference for the asset).

Suppose that in every period, there is one ask price $a_t = \mu_{t-1} + S$ and one bid price $b_t = \mu_{t-1} - S$, where S denotes the half-spread, constant across periods. Each arriving trader can choose between submitting a limit order for one unit at the respective price or a market order against an existing order in the limit order book. A limit order is valid for one period and is automatically cancelled if it is not traded against by the next trader.² Let $d_t \in \{\emptyset, MS, LS, LB, MB\}$ denote the order submitted by period- t trader, where $d_t = \emptyset$ means the trader abstains from trading, and the other four denote, respectively, the market sell, limit sell, limit buy, and market buy orders.

Assume first as usual that all traders are strategic and profit-maximizing.

¹Object Ω_t denotes all public information available to the market at (the end of) period t .

²To be clear: a limit order submitted in period t can **not** be cancelled or repriced when ϵ_t is revealed.

1. What is the expected utility of a period- t trader from using a limit buy order, as a function of its execution probability p_{MS} ? What about a market buy order?
2. Derive the period- t trader's optimal trading strategy as a function of ϵ_{t-1} and S .
Hint: it might be useful to consider cases $S = 0$, $S \in (0, \sigma)$, $S = \sigma$, and $S > \sigma$.
3. Explain why in equilibrium with trade it should be that $S = \sigma$. Explain intuitively how the equilibrium looks, why this should be the market-clearing price, and what the traders' equilibrium profits are.

Now, assume instead that all traders are *naive* in that they do not account for adverse selection when submitting limit orders. That is, when they submit a limit order, they expect that the asset's value conditional on trade is μ_{t-1} (on average).³

4. What is the subjective expected utility of a period- t naive trader from using a limit buy order, as a function of its execution probability p_{MS} ? What about a market buy order?
5. Derive the period- t naive trader's equilibrium trading strategy and the respective trading probabilities in an equilibrium with $S > 0$.
Bonus: characterize the set of equilibria as fully as you can.
6. Compare the equilibrium you found in part 5 to the equilibrium from part 3. Explain intuitively how they are different and what drives the difference between the two.

Problem 3: He liked the bonds

Read the article on AMI bonds attached at the end of this exam.⁴

You are to take the role of a financial market regulator in an internal discussion about this case (e.g., a SEC analyst making a presentation to your colleagues). Write a memo discussing this case, with an emphasis on the following:

1. How did the described manipulation affect market participants?
2. What kinds of remedies can you suggest to mitigate such exploits in the future?
3. What kinds of side effects could your remedies have?

NOTE: you can use the help of chatbots/AI/LLMs such as chatGPT. If you do, state clearly how they were used and which parts of the answer are mainly written by a LLM and which by you.

³The traders still estimate trading probabilities p_{MS} , p_{MB} correctly.

⁴This text is a part of a Bloomberg opinion piece, available at <https://www.bloomberg.com/opinion/articles/2023-04-10/ftx-lost-track-of-its-money>. You should ignore the text on the first page before the headline "He liked the bonds" and all text on the last page after the headline "APE Endgame".

|with no further context.

Incredibly relatable. Who among us has not saved our most precious data in a file titled “use this one”? This works, I guess, imperfectly, if your company is basically a half-dozen friends in a suite in the Bahamas. But it’s not best practices, and in particular, if the friends who run the company all get kicked out for incompetence and criminality and angry professionals come in to clean up the mess, they will be very annoyed to find all the crypto stored like this.

He liked the bonds

One fun little financial story of 2017 and 2018 involved the bonds of American Media Inc., the publisher of the National Enquirer. AMI was majority-owned by a hedge fund firm named Chatham Asset Management LLC. It had some bonds outstanding. Those bonds did not trade much, but they traded some, and they kept trading at higher and higher prices. By November 2017, some AMI bonds were trading at lower yields than the bonds of Apple Inc. [2](#) I am sure AMI is a fine tabloid publisher but it is not Apple, and its bond prices were perplexing. Also in 2018 AMI tried to raise some more money [by selling more bonds](#), and it eventually sold them at a 10.5% interest rate, which was much higher than the trading yield of its existing bonds. To the naked eye, AMI looked like a small, high-yield newspaper publisher, and when it sold new bonds the prices reflected that. But its *old* bonds traded in the secondary market at prices that made it look more creditworthy than Apple. What gives?

Everyone kind of knew the answer, which is that the AMI bonds didn’t trade that much, and they seemed to trade mostly *within*

Chatham:

Chatham, the majority equity owner of AMI's stock, also owned most of its bonds, and sometimes Chatham would sell the bonds to itself at ever-increasing prices. This didn't cost Chatham much — it was basically just overpaying itself — and made AMI's credit look good. But of course when AMI wanted to raise money by selling more bonds, Chatham was not going to buy those bonds at a 2% interest rate; it wanted a regular interest rate, like 10.5%.

This is not particularly legal, and last week Chatham and its founder Anthony Melchiorre [settled with the US Securities and Exchange Commission](#), agreeing to [pay \\$19 million](#) for doing this. *Why* did Chatham do this? The story that [the SEC tells](#) is basically that Chatham liked the bonds a lot and didn't want to let them go. Chatham ran a bunch of different accounts — some hedge funds, some liquid alternatives funds — and held large AMI bond positions in all of them. Sometimes some accounts would need to sell AMI bonds: They had concentration limits that meant they couldn't have too much of their money in AMI bonds, or customers took money out and Chatham needed to sell bonds to pay them. When this happened, Chatham did not want to let the bonds go, so it sold them to other Chatham accounts so they could stay in the family. [The SEC says](#):

Generally, when Chatham was forced to sell a high conviction AMI Bond in these circumstances, Chatham desired to purchase the AMI Bond for another Client because Chatham still believed in the merits of the investment and would not otherwise be selling the AMI Bonds if it were not for the portfolio restrictions or cash needs of the selling Clients.

As a general matter, selling bonds to yourself is allowed, but awkward: If you really believe in some bonds but have to sell

them from one account, and you want to buy them in another account, you are not *forbidden* from doing so, but there are lots of ways for it to look bad. Generally the ways for it to look bad are (1) you overpay for the bonds, making money for the selling account at the expense of the buying account (and causing the reported trading price of the bonds to be wrong) or (2) you underpay for the bonds, making money for the buying account at the expense of the selling account (and causing the reported trading price of the bonds to be wrong). The solution is generally to sell yourself the bonds at a fair market price, neither underpaying nor overpaying. Broadly speaking two ways to do that are:

1. Sell the bonds into the market at market prices, and then buy them back a bit later from the market at market prices: You don't trade with yourself at all, but only with arm's-length counterparties; or
2. Figure out a fair market price using outside sources (trading pries, pricing services, quotes from dealers, etc.) and sell the bonds to yourself at that price.

The first option is probably better, but it requires an active market; if you're the only real buyer or seller of the bonds it's hard.

Anyway Chatham sort of ... waved in the direction of doing this the right way?

Recognizing that there were legal restrictions on trading between RICs [3](#) and their affiliates, which included other Chatham Clients, Chatham and Melchiorre sought advice from a compliance consultant on how to facilitate the Rebalancing Trades. The consultant advised Chatham to conduct the trading either through a single broker over more than one day or

through multiple brokers if on the same day. The foundational principle underlying the advice was to ensure that the transactions occurred at independently-derived market prices.

Yeah I mean that's good generic advice, but when you are the market for the bonds it doesn't work out great. Also Chatham did not necessarily go all in on the *spirit* of that advice:

Around the time that Chatham began to execute the Rebalancing Trades, Melchiorre generally explained the purpose of the Rebalancing Trades to the Rebalancing Brokers. Melchiorre informed each of the Rebalancing Brokers to whom he sold a Client's AMI Bonds that he likely would have an interest in repurchasing that same AMI Bond he was selling for another Client. Over time, an understanding developed on the part of the Rebalancing Brokers that whenever Melchiorre placed an order to sell one of the AMI Bonds for a Client, he would repurchase it for another Client, either directly the following day or days, or indirectly through another broker.

The Rebalancing Brokers engaged in the Rebalancing Trades because they expected Melchiorre to repurchase the bonds. The business model of several of the Rebalancing Brokers was to "match" buy and sell orders from their customers. Those Rebalancing Brokers ordinarily did not purchase securities for their own inventory—i.e., put the firm's own capital at risk—or they did so on a very limited basis. Nonetheless, the vast majority of the Rebalancing Trades involved at least one Rebalancing Broker that purchased bonds into its firm's inventory. For example, some of the Rebalancing Brokers would at times agree to purchase securities from Chatham even though the Rebalancing Broker may not have lined up the other leg of the transaction. These brokers' willingness to do so was based on their expectation that Chatham would repurchase the

bonds, either directly or through another broker.

The purchasing Rebalancing Brokers generally did not offer the AMI Bonds to other customers in the market. Instead, in virtually every case, they resold the securities to Chatham or to another broker who they understood was purchasing for Chatham.

As Chatham's need to conduct rebalancing in its various Client Accounts increased over time, Rebalancing Trades became routine. When Melchiorre wanted to sell an AMI Bond to one particular Rebalancing Broker ("Rebalancing Broker A") and then repurchase it the following day, he would send Rebalancing Broker A a message indicating that he wanted to sell an AMI Bond in the "usual drill." Rebalancing Broker A then would purchase the AMI Bond into the firm's inventory until Melchiorre repurchased it the following day.

In the abstract, selling bonds to a broker one day and then buying them back for a different account a day or two later could be a good way to do everything at arm's-length market prices: The broker will pay, and charge, prices that reflect market levels; it won't overpay to buy from you or undercharge to sell to you.

But in practice, if you are the only buyer and the only seller and you call up a broker and say "hey it's the usual drill," you are not really getting a fair market price. The broker doesn't care what the buying price or selling price is, as long as you pay a commission. You can just pick whatever price you want:

Melchiorre proposed the price for the Rebalancing Trades and the Rebalancing Brokers agreed to it without first soliciting bids from other market participants.

When proposing a price for Rebalancing Trades, Melchiorre considered a number of factors, which included the prior day's price as reflected in prices published by a pricing service. Those

published prices would have been influenced, to some extent, by Chatham's own trading. When purchasing the AMI Bonds, Melchiorre also added a spread to compensate the Rebalancing Brokers. For example, in the case of a Rebalancing Trade executed through a single Rebalancing Broker overnight, Chatham would repurchase the AMI Bonds for a small spread above what it had sold the AMI Bonds to the broker the day before.

One consequence of this is that the compliance consultant's basic idea of selling to brokers to get market prices was not really working. Another consequence is that the prices kept going *up*: The only trades were Chatham's trades, and it kept trading at higher prices to compensate its brokers. Eventually this became absurd:

Over the Relevant Period, Chatham and Melchiorre engaged in over one hundred Rebalancing Trades in AMI Bonds, accounting for approximately 81 percent, on average, of the customer trading (i.e., not broker-to-broker trading) in such securities.

Over time, the frequent Rebalancing Trades and repeated mark-ups to compensate the Rebalancing Brokers resulted in the market price of AMI Bonds increasing at a faster rate than prices of similar securities. For example, by November 2017, two of the AMI Bonds traded in Rebalancing Trades at implied yields lower than the prevailing London Interbank Offered Rate ("LIBOR"). Such yields ordinarily would have been associated with a bond of a much higher creditworthiness than the AMI Bonds.

A third consequence is that Chatham's assets under management kept going up: It charged clients fees based on the value of its assets, and as it kept increasing the prices of these

bonds, that value kept going up. The SEC says:

Chatham was compensated for its advisory services to the [hedge] Funds with a management fee and a performance fee, and to the [liquid alts funds] with a management fee. The management fee charged to certain Clients was set at a percentage of the [net asset value] of those Clients.

In order to calculate each Fund's NAV, Chatham used an independent pricing service to determine the value of each of the portfolio securities, including the AMI Bonds. The LAF administrators calculated their own NAVs, also using the same pricing service Chatham used.

Chatham and Melchiorre understood that the pricing service that Chatham and the LAF administrators utilized was based to some extent on recent trading prices and that the Rebalancing Trades accounted for virtually all of the trading in the Bonds during the Relevant Period. Because the Rebalancing Trades at times increased the prices of the AMI Bonds, the NAVs of the Client accounts also were increased by that amount on those occasions.

Accordingly, the Clients paid Chatham an estimated \$11,000,000 in performance and/or management fees that they would not have in the absence of Chatham's Rebalancing Trades. Chatham in turn paid approximately 55 percent of such fees to Melchiorre.

Part of the \$19 million settlement is paying back that \$11 million to investors. But the SEC doesn't quite say that this is *why* Chatham did this. You could argue that this sort of thing — trading bonds back and forth with yourself in a way that raises their prices — is market manipulation, that the goal was to raise prices to charge higher fees. But the SEC doesn't say that:

It says, more or less in so many words, that the reason for the trading is that Chatham really liked the bonds, that it “still believed in the merits of the investment” even when it had to sell them, so it kept buying them back. The higher prices and higher fees were just a happy byproduct.

APE Endgame

You know the story. AMC Entertainment Holdings Inc. became a meme stock, so it sensibly sold a ton of stock to raise money and pay down debt. Eventually it [ran out of stock to sell](#): Its corporate charter authorizes about 524 million shares of common stock, and it has sold basically all of them.

Shareholders did not seem interested in amending the charter to authorize more shares, because they were worried about dilution and/or because they are retail investors who tend not to vote their shares at all. The way it works is that a majority of the *outstanding shares* need to approve the charter amendment to issue new shares, so not voting is the same as voting no.

But AMC’s charter also allows the board to issue “blank-check” preferred stock, that is, preferred stock with any terms the board wants. So AMC [started issuing a new type of preferred stock called APEs](#), AMC Preferred Equity Units, which are meant to be identical to the common stock: They have the same economic rights, same voting rights, etc. AMC did a quasi-stock-split in which shareholders got one APE for each common share they held, and then it started selling new APEs to raise more money.

Part of the plan here was just to sell APEs to raise money, but another [part of the plan](#) was to [get the APEs to vote to amend the charter](#) to allow AMC to issue more common shares. If that happens, the APEs will all be converted into common shares; since now the APEs trade at a discount to the common, this will

Final re-exam solutions

Problem 1: Spread dynamics in Glosten-Milgrom model

Consider a standard Glosten-Milgrom model:

- asset fundamental value is $v \in \{v^L, v^H\}$, the two realizations are considered equally probable ex ante;
- a competitive dealer sequentially quotes bid b_t and ask a_t prices for one unit of the asset each period;
- one trader arrives at the market per period and can submit a market buy or sell order for one unit or do nothing: $d_t \in \{-1, 0, 1\}$;
- with probability π the trader is informed and knows v and chooses d_t to maximize profit; with complementary probability $1 - \pi$ the trader is uninformed and submits either a buy or a sell order with equal probabilities regardless of v .

Answer the following questions.

1. Calculate the first-period market valuation $\mu_0 = \mathbb{E}[v]$, the ask and bid prices a_1, b_1 , and the relative spread $s_1 = \frac{a_1 - b_1}{\mu_0}$.
2. Suppose the first order was a sell: $d_1 = -1$. Calculate the second-period market valuation $\mu_1 = \mathbb{E}[v|d_1 = -1]$, the ask and bid prices a_2, b_2 , and the relative spread $s_2 = \frac{a_2 - b_2}{\mu_1}$.
3. How does s_2 compare to s_1 ? Give an intuitive explanation for why. Do you expect this trend to continue from s_2 to s_3 and onwards?

Solution

1. $\mu_0 = \mathbb{E}[v] = \frac{v^H + v^L}{2}$; $a_1 = \mathbb{E}[v|d_1 = 1] = \mu_0 + \pi \frac{v^H - v^L}{2}$; $b_1 = \mathbb{E}[v|d_1 = -1] = \mu_0 - \pi \frac{v^H - v^L}{2}$; $s_1 = \frac{a_1 - b_1}{\mu_0} = 2\pi \frac{v^H - v^L}{v^H + v^L}$.
2. After a sell order, we have $\mu_1 = b_1 = \mu_0 - \pi \frac{v^H - v^L}{2} = \frac{(1-\pi)v^H + (1+\pi)v^L}{2}$. Note in particular that $\mathbb{P}(v^H|d_1 = -1) = \frac{1-\pi}{2}$ and $\mathbb{P}(v^L|d_1 = -1) = \frac{1+\pi}{2}$. Then the quotes are (where I_2 and U_2 correspond to the events of the second trader being informed or uninformed, respectively):

$$\begin{aligned}
 a_2 &= \mathbb{E}[v|d_1 = -1, d_2 = 1] \\
 &= \mathbb{E}[v|I_2, d_1 = -1, d_2 = 1] \cdot \mathbb{P}[I_2|d_1 = -1, d_2 = 1] + \mathbb{E}[v|U_2, d_1 = -1, d_2 = 1] \cdot \mathbb{P}[U_2|d_1 = -1, d_2 = 1] \\
 &= v^H \cdot \frac{\pi \cdot \mathbb{P}(v^H|d_1 = -1)}{\mathbb{P}(d_2 = 1|d_1 = -1)} + \mu_1 \cdot \frac{(1-\pi) \cdot \frac{1}{2}}{\mathbb{P}(d_2 = 1|d_1 = -1)} \\
 &= v^H \cdot \frac{\pi}{\pi + 1} + \mu_1 \cdot \frac{1}{\pi + 1} = \frac{v^H + v^L}{2} = \mu_0,
 \end{aligned}$$

and similarly

$$\begin{aligned}
 b_2 &= v^L \cdot \mathbb{P}[I_2|d_1 = -1, d_2 = -1] + \mu_1 \cdot \mathbb{P}[U_2|d_1 = -1, d_2 = -1] \\
 &= v^L \cdot \frac{\pi \cdot \frac{1+\pi}{2}}{\pi \cdot \frac{1+\pi}{2} + (1-\pi)\frac{1}{2}} + \mu_1 \cdot \frac{(1-\pi)\frac{1}{2}}{\pi \cdot \frac{1+\pi}{2} + (1-\pi)\frac{1}{2}} \\
 &= v^H \cdot \frac{(1-\pi)^2}{2(1+\pi^2)} + v^L \cdot \frac{(1+\pi)^2}{2(1+\pi^2)}.
 \end{aligned}$$

The relative spread is hence $s_2 = \frac{2\pi}{1+\pi^2} \cdot \frac{v^H - v^L}{(1-\pi)v^H + (1+\pi)v^L}$.

3. The comparison is ambiguous and depends on the model parameters:

$$s_2 < s_1 \iff \frac{\pi}{1 + \pi^2} > \frac{v^H - v^L}{v^H + v^L}.$$

There are two countervailing forces at play here. First, it is immediate that for the absolute spreads, $S_2 \equiv a_2 - b_2 < a_1 - b_1 \equiv S_1$. Intuitively this means that the second order is less informative about the fundamental than the first one. This is because we started from the state of maximal uncertainty (v^H and v^L being equally likely), so the first order is extremely informative; after that we inevitably move towards either v^H or v^L being more likely, hence an additional signal is by definition less informative. The future absolute spreads S_t then depend on how close μ_{t-1} is to μ_0 . As discussed in class, while in the short run μ_{t-1} can fluctuate arbitrarily, in the long run the market valuation converges to the true fundamental value, $\mu_t \rightarrow v$, hence on average we move towards certainty, and the spreads should close down over time.

For relative spreads, there is a countervailing force coming from the denominator in $s_t = \frac{a_t - b_t}{\mu_{t-1}}$. The changes in the midquote directly affect the relative spread: and the lower is the midquote, the higher is the spread, and vice versa. Then the first piece of news could be so strongly negative that it crashes μ_1 and makes the smaller absolute spread S_2 actually look large in relative terms, overpowering the effect above. But the long-run intuition still suggests that as $\mu_t \rightarrow v$, the absolute spread converges to zero, and hence so should the relative spread (assuming $v^L > 0$).

Problem 2: Dynamic LOB markets with naive traders

This problem explores a version of the Foucault/Parlour model that we have seen in class. Suppose that there is one asset, whose fundamental value v is unknown, and whose market valuation evolves according to $\mu_t = \mathbb{E}[v \mid \Omega_t] = \mu_{t-1} + \epsilon_t$, where $\epsilon_t \in \{-\sigma, 0, \sigma\}$ with equal probabilities is period- t news, publicly announced at the end of period t (after any period- t orders are submitted).¹ In every period t , one risk-neutral trader arrives at the market (who only knows μ_{t-1} but not ϵ_t , and has no idiosyncratic preference for the asset).

Suppose that in every period, there is one ask price $a_t = \mu_{t-1} + S$ and one bid price $b_t = \mu_{t-1} - S$, where S denotes the half-spread, constant across periods. Each arriving trader can choose between submitting a limit order for one unit at the respective price or a market order against an existing order in the limit order book. A limit order is valid for one period and is automatically cancelled if it is not traded against by the next trader.² Let $d_t \in \{\emptyset, MS, LS, LB, MB\}$ denote the order submitted by period- t trader, where $d_t = \emptyset$ means the trader abstains from trading, and the other four denote, respectively, the market sell, limit sell, limit buy, and market buy orders.

Assume first as usual that all traders are strategic and profit-maximizing.

1. What is the expected utility of a period- t trader from using a limit buy order, as a function of its execution probability p_{MS} ? What about a market buy order?
2. Derive the period- t trader's optimal trading strategy as a function of ϵ_{t-1} and S .
Hint: it might be useful to consider cases $S = 0$, $S \in (0, \sigma)$, $S = \sigma$, and $S > \sigma$.
3. Explain why in equilibrium with trade it should be that $S = \sigma$. Explain intuitively how the equilibrium looks, why this should be the market-clearing price, and what the traders' equilibrium profits are.

Now, assume instead that all traders are *naive* in that they do not account for adverse selection when submitting limit orders. That is, when they submit a limit order, they expect that the asset's value conditional

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on trade is μ_{t-1} (on average).³

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5. Derive the period- t naive trader's equilibrium trading strategy and the respective trading probabilities in an equilibrium with $S > 0$.

Bonus: characterize the set of equilibria as fully as you can.

6. Compare the equilibrium you found in part 5 to the equilibrium from part 3. Explain intuitively how they are different and what drives the difference between the two.

Solution

1. The strategic trader's expected utility from a limit buy order is

$$\begin{aligned} U_{LB}^S &= \left(\mathbb{E}[v \mid \mu_{t-1}, d_{t+1} = MS] - b_t \right) p_{MS} \\ &= \left((\mu_{t-1} + \mathbb{E}[\epsilon_t \mid d_{t+1} = MS]) - (\mu_{t-1} - S) \right) p_{MS} \\ &= \left(\mathbb{E}[\epsilon_t \mid d_{t+1} = MS] + S \right) p_{MS}. \end{aligned} \quad (1)$$

For a market buy order we have

$$\begin{aligned} U_{MB}^S &= \mathbb{E}[v \mid \mu_{t-1}] - a_{t-1} \\ &= \mu_{t-1} - (\mu_{t-2} + S) \\ &= \epsilon_{t-1} - S. \end{aligned} \quad (2)$$

2. A market buy is only profitable (relative to abstaining from trade) if $S \leq \epsilon_{t-1}$. Since $S \geq 0$, this is only possible if $\epsilon_{t-1} \in \{0, \sigma\}$ (and if $S > 0$, then only if $\epsilon_{t-1} = \sigma$). Similarly, a market sell order is only profitable if $\epsilon_{t-1} \in \{0, -\sigma\}$, implying that $\mathbb{E}[\epsilon_t \mid d_{t+1} = MS] \in [-\sigma, 0]$. A limit buy order is only profitable if $S \geq -\mathbb{E}[\epsilon_t \mid d_{t+1} = MS] \in [0, \sigma]$. Let us consider the following cases, depending on the spread:

- If $S = 0$ then after $\epsilon_{t-1} = \sigma$ a market buy order is strictly profitable and all three other order types yield a weakly negative profit, hence MB is optimal. Similarly, after $\epsilon_{t-1} = -\sigma$ MS is optimal. Therefore, $\mathbb{E}[\epsilon_t \mid d_{t+1} = MS] < 0$ and $\mathbb{E}[\epsilon_t \mid d_{t+1} = MB] > 0$. Limit buy and limit sell thus yield a strictly negative profit (for any ϵ_{t-1}) and are never used. After $\epsilon_t = 0$ both market order types yield zero profit, so the agent is indifferent between those and doing nothing.
- If $S \in (0, \sigma)$ then market buy and market sell can only ever yield positive profit after $\epsilon_t = \sigma$ and $\epsilon_t = -\sigma$, respectively. Thus $\mathbb{E}[\epsilon_t \mid d_{t+1} = MS] = -\sigma$ and $\mathbb{E}[\epsilon_t \mid d_{t+1} = MB] = \sigma$, which implies that the profits from the limit buy and sell limit orders are given by $(S - \sigma)p_{MS}$ and $(S - \sigma)p_{MB}$, respectively, which are negative. The limit orders are thus never used, and MB/MS are indeed optimal after $\epsilon_t = \sigma, -\sigma$, respectively. After $\epsilon_{t-1} = 0$ all order types yield negative profit, hence abstinence is optimal.
- If $S = \sigma$ then the logic above implies that the limit orders yield zero profit (regardless of ϵ_{t-1}), and the same applies to market buy after $\epsilon_{t-1} = \sigma$ and market sell after $\epsilon_t = -\sigma$ (and in other cases market orders yield negative profit). The trader is thus indifferent between all the zero-profit alternatives for all respective ϵ_{t-1} .

³The traders still estimate trading probabilities p_{MS}, p_{MB} correctly.

d_t	$S = 0$	$S \in (0, \sigma)$	$S = \sigma$	$S > \sigma$
$\epsilon_{t-1} = \sigma$	MB	MB	$\{MB, LB, LS, \emptyset\}$	$\{LB, LS, \emptyset\}$
$\epsilon_{t-1} = 0$	$\{MB, MS, \emptyset\}$	$\emptyset$	$\{LB, LS, \emptyset\}$	$\{LB, LS, \emptyset\}$
$\epsilon_{t-1} = -\sigma$	MS	MS	$\{MS, LB, LS, \emptyset\}$	$\{LB, LS, \emptyset\}$

Table 1: Optimal trading strategy for a strategic trader.

- If $S > \sigma$ then market orders can never be profitable, meaning $p_{MS} = p_{MB} = 0$, so the limit orders yield zero profit, and the traders are indifferent between submitting limit orders (that never execute) and doing nothing.

The trader's resulting optimal strategy is summarized in Table 1.

3. As suggested by the previous part, if $S < \sigma$, then all traders want to take liquidity and not provide liquidity (i.e., all traders use market orders and no one uses limit orders). The exact opposite happens when $S > \sigma$. Therefore, only at $S = \sigma$ is the market balanced in terms of both market and limit orders potentially being used in equilibrium. In particular, we can consider an equilibrium, in which the trader uses MB after $\epsilon_{t-1} = +\sigma$, MS after $\epsilon_{t-1} = -\sigma$ (if there are any appropriate limit orders in the book), and mixes between LB and LS otherwise.

In such an equilibrium, all traders get zero profit, but trade is possible. The ask and bid prices on the limit orders are such that they exactly foresee the news that will arrive, and negate the risk of being picked off (think of the Glosten-Milgrom model with no uninformed traders). Given such a defensive pricing, market orders yield exactly zero profit, since all information is already internalized in the price. Limit orders also yield zero profit, since any attempt to widen the spread for profit would result in complete trade breakdown.

4. The naive trader's expected utility from a limit buy order is

$$\begin{aligned}
 U_{LB}^S &= \left(\mathbb{E}[v \mid \mu_{t-1}, d_{t+1} = MS] - b_t \right) p_{MS} \\
 &= \left(\mathbb{E}[v \mid \mu_{t-1}] - b_t \right) p_{MS} \\
 &= \left(\mu_{t-1} - (\mu_{t-1} - S) \right) p_{MS} \\
 &= S \cdot p_{MS}.
 \end{aligned} \tag{3}$$

In the above, the equality between the first and the second lines follows from the trader's naivete.

For a market buy order we have the same as for the strategic trader:

$$\begin{aligned}
 U_{MB}^S &= \mathbb{E}[v \mid \mu_{t-1}] - a_{t-1} \\
 &= \mu_{t-1} - (\mu_{t-2} + S) \\
 &= \epsilon_{t-1} - S.
 \end{aligned} \tag{4}$$

5. There is an equilibrium with trade with $S = 0$ (where traders mix between the two limit orders when $\epsilon_{t-1} = 0$ and trade via a market order otherwise), but we are not interested in it. From here onwards suppose $S > 0$.

Note from (3) that if $S > 0$ (and there is trade in equilibrium), a naive trader always expects a positive profit from a limit buy order (same for limit sell), hence providing liquidity is always better than doing nothing.

If $S > 0$ and $\epsilon_{t-1} = 0$, (4) suggests that market orders are unprofitable, so the trader submits a

random limit order. To get an equilibrium with trade, we hence want market orders to be optimal after $\epsilon_{t-1} \in -\sigma, \sigma$. If $\epsilon_{t-1} = \sigma$, a market buy is preferred to limit buy if and only if $\sigma - S \geq S \cdot p_{MS} \iff S \leq \frac{\sigma}{1+p_{MS}}$. Similarly, MS is preferred to LS after $\epsilon_{t-1} = -\sigma$ if and only if $S \leq \frac{\sigma}{1+p_{MB}}$. Hence for any $S \in (0, \frac{3}{4}\sigma]$ market orders are uniquely optimal after news, and $p_{MS} = p_{MB} = 1/3$.⁴

6. Naive traders ignore adverse selection, and are thus more eager than strategic traders to use limit orders to profit on the spread, ignoring the risk of being picked off (but still accounting for the execution risk). Such traders expect to receive positive profit from limit orders, and hence market orders should also yield positive profits (at least after news), which puts an upper bound on the spread. Worth noting that these traders' expectations regarding limit orders are incorrect, and their naivete would lead them to run at a loss.

Problem 3: He liked the bonds

Read the article on AMI bonds attached at the end of this exam.⁵

You are to take the role of a financial market regulator in an internal discussion about this case (e.g., a SEC analyst making a presentation to your colleagues). Write a memo discussing this case, with an emphasis on the following:

1. How did the described manipulation affect market participants?
2. What kinds of remedies can you suggest to mitigate such exploits in the future?
3. What kinds of side effects could your remedies have?

NOTE: you can use the help of chatbots/AI/LLMs such as chatGPT. If you do, state clearly how they were used and which parts of the answer are mainly written by a LLM and which by you.

Solution

The question is, obviously, open-ended; below are some example points that could be made in an answer.

The effects were that the investors in Chatham's funds overpaid in commissions; post-trade information available to other investors (trade prices) was distorted.

The article mentions that one remedy would have been to trade the bonds on the open market at the market price. The brokers should have the incentive to quote the fair price, but due to them knowing that Chatham would almost-certainly buy the bonds back, such an incentive is absent. It would similarly be absent for any other party that would be asked to quote a price without having to actually take a position at this price (if, e.g., we tried to force the brokers to solicit bids from other traders, such traders would not care about their bids if they expected Chatham to win regardless).

One possible solution could be to strengthen the external audit requirements for funds like Chatham, yet that would come at a significant additional cost of compliance that would be passed on to investors, so it is not clear whether such a measure would actually improve investors' welfare.

⁴There also exist equilibria for $S \in (\frac{3}{4}\sigma, \sigma]$ where traders mix between limit and market orders after news.

⁵This text is a part of a Bloomberg opinion piece, available at <https://www.bloomberg.com/opinion/articles/2023-04-10/ftx-lost-track-of-its-money>. You should ignore the text on the first page before the headline "He liked the bonds" and all text on the last page after the headline "APE Endgame".

Written Exam Economics summer 2025

Financial Markets Microstructure

June 06, 2025

This exam question consists of 7 pages in total
Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam. Please write the exam number on your exam paper as well.

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- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
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You can read more about the rules on exam cheating on your Study Site and in the Faculty of Social Science's common part of the curriculum.

Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

Final exam

Write up your responses to questions below and submit them to Digital Exam before the deadline. No cooperation with other students is permitted.

Be concise, but show your work and explain your answers. Some questions may require you to make additional assumptions beyond those provided in the question; be clear about the assumptions you make. Some questions are open ended in that they may not have a unique correct answer. You are allowed to refer to textbooks, lecture notes, slides, problem sets, etc.

Problem 1: HFT with limited liquidity supply

Consider a Biais-Foucault-Moinas model of high-frequency trading that we considered in class. We assumed in that model that fast institutions (FI) always find trading opportunities, whereas slow institutions (SI) find trading opportunities with a fixed probability $\rho < 1$.

Suppose now instead that there is a fixed supply of liquidity in the market, denoted by $\xi \in (0, 1)$. FI are the first to dip into this liquidity, and then SI split among themselves whatever liquidity is left. For example, if the share of institutions that are fast, α , is such that $\alpha \leq \xi$, then all FI find a trading opportunity. Then the $1 - \alpha$ slow institutions have to split the remaining $\xi - \alpha$ trading opportunities, so each individual SI gets to trade with probability $\rho(\alpha) = \frac{\xi - \alpha}{1 - \alpha}$.

Keep the rest of the model as presented in class. Explain how this tweak changes the incentives to invest in speed: would you expect the equilibrium FI share α^* to be higher when SI's trading probability is exogenous (ρ) or endogenous in the way described above ($\rho = \rho(\alpha)$)? Why?

Note: you do not need to provide closed-form solutions. You are, however, expected to verbally identify the main consequence(s) of such a tweak.

Problem 2: Kyle model with information acquisition

Consider a single-period Kyle model, where the speculator does not know the asset's fundamental value v perfectly, but instead decides how much to invest in a noisy signal about v . In particular, suppose that before submitting an order, the speculator chooses σ_s^2 , pays cost $c(\sigma_s^2)$, and then receives signal $s \sim \mathcal{N}(v, \sigma_s^2)$.

After that, the game proceeds as in the regular Kyle model. The speculator chooses their trade size, $x \in \mathbb{R}$, to maximize their expected profit $\Pi_I \equiv \mathbb{E}[x(v - p)]$. The noise traders submit a random market order $u \sim \mathcal{N}(0, \sigma_u^2)$. The competitive dealer observes the aggregate order imbalance $q = x + u$ and quotes a price $p(q)$ at which they are willing to absorb it. All agents have a common prior belief that $v \sim \mathcal{N}(\mu, \sigma_v^2)$.

1. Fix some signal precision σ_s^2 for the speculator and suppose they follow a strategy that is linear in the signal s : $x(s) = \beta(s - \mu)$ for some β . Derive the price schedule $p(q)$ that the dealer would offer given the equilibrium β and σ_s^2 . Specifically, show that $p(q) = \mu + \lambda q$ and provide an expression for λ .
2. Derive the speculator's optimal trading strategy $x(s)$ given σ_s^2 and the dealer's pricing schedule $p(q) = \mu + \lambda q$.
3. Calculate the speculator's expected trading profit for given σ_s^2, σ_v^2 , and λ .
4. Suppose now the speculator's information cost is given by $c(\sigma_s^2) = \frac{\gamma}{\sigma_s^2}$ for some information cost parameter γ . Derive the amount of information $\tau_s \equiv \frac{1}{\sigma_s^2}$ the speculator acquires as a function of $\lambda, \gamma, \sigma_v^2$.
5. How does the speculator's information choice depend on γ and σ_v^2 given λ ? How does it depend on λ ? How does it depend on σ_u^2 in equilibrium? Explain.

Problem 3: 0dte traders

1. Which motives for trading have we proposed throughout the course as reasons for trading in financial markets? Mention and briefly describe them.
2. Read the essay attached at the end of this exam. Which of the motives above drive the behavior of the retail traders mentioned in the essay? Are there any other motives driving them that we have not discussed?

[bloomberg.com](https://www.bloomberg.com)

The Fastest Options Are the Most Fun

Matt Levine

24–30 minutes

Single-stock Options

Some of the work in the financial industry involves finding and fostering bad counterparties. If you are a stock market maker — you buy stock from customers who want to sell, and sell to customers who want to buy — and your customers are mostly huge sophisticated hedge funds, you will have a lot of unpleasant days. They are smart and large; when they buy stock from you, it will mostly go up and you will regret selling; when they sell stock to you, it will mostly go down and you will regret buying.

On the other hand, if you have customers who like a gamble and are always wrong, that's great! When they sell stock to you, it will mostly go up, and you will make money. The obvious business model is:

1. Find more of those people.
2. Take them out to steak dinners, be nice to them, and generally make them feel like valued customers.
3. Proactively work to offer them trading opportunities that satisfy their urge to gamble, that give them more chances to be wrong, and that have a lot of profit built in for you.
4. Do as many trades as possible with them, though preferably not

so many that you blow them up entirely. You want them to have enough fun that they keep coming back for more.

This business model is obvious in part because it is the actual well-known business model of a casino: nice dinners, fun games and lots of edge.

In finance, we talk a lot about "[payment for order flow](#)," which is an important way that US equity market makers try to find bad counterparties: If you trade stock on the stock exchange, you run the risk of trading with sophisticated hedge funds, but if you trade exclusively with the customers of a retail brokerage, you know that all of your counterparties are retail traders. So market makers pay those brokerages to trade with their customers, and in fact [there is evidence](#) that the market makers discriminate even among retail customers: Some retail brokers' customers are more sophisticated than others', so market makers should pay more to trade with the unsophisticated ones.¹

But you don't have to just take your counterparties as you find them; you can work to encourage and develop bad ones. This is often a matter of product development: If you build a product that does nothing for sophisticated professionals but that is really good for noisy addicted gamblers, you will attract exactly the right sort of counterparty. This arguably explains much of crypto.

Anyway the [the Wall Street Journal reports](#):

A popular, fast-paced trade has boosted the options market to record volumes in recent years. Now, Wall Street is looking to push it even further.

Zero-day-to-expiry options let investors bet on whether a particular stock-market index will rise or fall by the end of the day. They have drawn an enthusiastic following among amateur investors, even as skeptics call them a form of gambling. They are

sometimes known by the hashtag #0dte.

So far, the #0dte boom has been limited to options tied to indexes such as the S&P 500 or Nasdaq-100. The next frontier could be options on stocks such as Tesla or Nvidia. ...

Michael McCaskill, a 48-year-old day trader and volleyball-programs coordinator in Louisville, Ky., trades short-dated options in hopes of hitting the jackpot. He's intrigued by the prospect of more-frequent expirations on single-stock options.

"The percentage gains are incredible," said McCaskill, who has previously made profitable bets on GameStop, Netflix and PayPal. "It's the short-dated options that give you that, whether it's weekly or daily."

I am sorry, I am sure he has lots of profitable trades and I don't mean to be rude, but just, statistically speaking, for equity market makers, there are few more beautiful phrases in the English language than "day trader and volleyball-programs coordinator." If you walk into the offices of Susquehanna International Group and say "hi I am a day trader and volleyball-programs coordinator from Louisville, Ky., and I love short-dated options but I wish they were shorter-dated, can you help," they will treat you like a celebrity and give you anything you ask for. You are their muse.

As it happens, Susquehanna is salivating over single-stock 0dtes but Robinhood, less so:

In closed-door industry meetings, retail brokerages such as Robinhood Markets, Schwab, Tastytrade and Morgan Stanley's E*Trade have advocated for a cautious approach, concerned they could face a customer backlash if investors' options trades blow up, the people said.

Other firms—including Susquehanna International Group, a

huge options-market maker, and Nasdaq—have actively promoted bringing daily expirations to single-stock options, the people said. Both market makers and exchanges stand to benefit from the volumes that could come from further growth in the #0dte phenomenon.

The retail brokers *basically* make money if their customers are happy; the market makers *basically* make money by trading against them. There is an overlap in their interests, but it is not complete. If Robinhood gives its customers new ways to lose money faster, that's probably bad for Robinhood in the long term, but it's very good for whoever is on the other side of those trades.

Final exam solutions

Problem 1: HFT with limited liquidity supply

Consider a Biais-Foucault-Moinas model of high-frequency trading that we considered in class. We assumed in that model that fast institutions (FI) always find trading opportunities, whereas slow institutions (SI) find trading opportunities with a fixed probability $\rho < 1$.

Suppose now instead that there is a fixed supply of liquidity in the market, denoted by $\xi \in (0, 1)$. FI are the first to dip into this liquidity, and then SI split among themselves whatever liquidity is left. For example, if the share of institutions that are fast, α , is such that $\alpha \leq \xi$, then all FI find a trading opportunity. Then the $1 - \alpha$ slow institutions have to split the remaining $\xi - \alpha$ trading opportunities, so each individual SI gets to trade with probability $\rho(\alpha) = \frac{\xi - \alpha}{1 - \alpha}$.

Keep the rest of the model as presented in class. Explain how this tweak changes the incentives to invest in speed: would you expect the equilibrium FI share α^* to be higher when SI's trading probability is exogenous (ρ) or endogenous in the way described above ($\rho = \rho(\alpha)$)? Why?

Note: you do not need to provide closed-form solutions. You are, however, expected to verbally identify the main consequence(s) of such a tweak.

Solution

While the specific answer will depend on the specific ρ, ξ , and other primitives, we can make a few conjectures.

Endogenizing the trading opportunities introduces a new negative externality of the FI share α on slow investors when $\alpha < \xi$: higher α now leads to lower $\rho(\alpha)$ for SI. This makes it relatively more appealing to invest in speed, and suggests that the equilibrium share α^* would be higher than with exogenous ρ .

Further, for $\alpha > \xi$, SI do not get to trade at all with endogenous $\rho(\alpha)$, and the access of FI to liquidity decreases in α , since there would not be enough liquidity for all FI. This introduces a negative externality of α on FI when $\alpha > \xi$, but given that SI have no access to liquidity at all, the effect described in the previous paragraph is likely to dominate.

Problem 2: Kyle model with information acquisition

Consider a single-period Kyle model, where the speculator does not know the asset's fundamental value v perfectly, but instead decides how much to invest in a noisy signal about v . In particular, suppose that before submitting an order, the speculator chooses σ_s^2 , pays cost $c(\sigma_s^2)$, and then receives signal $s \sim \mathcal{N}(v, \sigma_s^2)$.

After that, the game proceeds as in the regular Kyle model. The speculator chooses their trade size, $x \in \mathbb{R}$, to maximize their expected profit $\Pi_I \equiv \mathbb{E}[x(v - p)]$. The noise traders submit a random market order $u \sim \mathcal{N}(0, \sigma_u^2)$. The competitive dealer observes the aggregate order imbalance $q = x + u$ and quotes a price $p(q)$ at which they are willing to absorb it. All agents have a common prior belief that $v \sim \mathcal{N}(\mu, \sigma_v^2)$.

1. Fix some signal precision σ_s^2 for the speculator and suppose they follow a strategy that is linear in the signal s : $x(s) = \beta(s - \mu)$ for some β . Derive the price schedule $p(q)$ that the dealer would offer given the equilibrium β and σ_s^2 . Specifically, show that $p(q) = \mu + \lambda q$ and provide an expression for λ .
2. Derive the speculator's optimal trading strategy $x(s)$ given σ_s^2 and the dealer's pricing schedule $p(q) = \mu + \lambda q$.
3. Calculate the speculator's expected trading profit for given σ_s^2, σ_v^2 , and λ .

4. Suppose now the speculator's information cost is given by $c(\sigma_s^2) = \frac{\gamma}{\sigma_s^2}$ for some information cost parameter γ . Derive the amount of information $\tau_s \equiv \frac{1}{\sigma_s^2}$ the speculator acquires as a function of $\lambda, \gamma, \sigma_v^2$.
5. How does the speculator's information choice depend on γ and σ_v^2 given λ ? How does it depend on λ ? How does it depend on σ_u^2 in equilibrium? Explain.

Solution

1. Dealer is competitive, hence must in equilibrium get zero profit on any trade. The pricing schedule is then given as $p(q) = \mathbb{E}[v|q]$. We know that

$$q = x + u = \beta(s - \mu) + u,$$

so we can represent the volume signal as $\tilde{q} \equiv \frac{q}{\beta} + \mu = v + \varepsilon$, where $\varepsilon \sim \mathcal{N}\left(0, \frac{1}{\tau_q}\right)$ and $\tau_q = \left(\frac{\sigma_u^2}{\beta^2} + \sigma_s^2\right)^{-1}$ (recall, the idea is to express the signal about v as $v + \varepsilon$, where ε is zero-mean noise). Then we have that

$$\begin{aligned} p(q) = \mathbb{E}[v|q] &= \frac{\tau_v}{\tau_v + \tau_q} \mu + \frac{\tau_q}{\tau_v + \tau_q} \tilde{q} \\ &= \mu + \frac{\tau_q}{\tau_v + \tau_q} \cdot \frac{q}{\beta} \\ &= \mu + \frac{\beta \sigma_v^2}{\beta^2 \sigma_v^2 + \beta^2 \sigma_s^2 + \sigma_u^2} q, \end{aligned}$$

where $\tau_v = \frac{1}{\sigma_v^2}$. The price impact coefficient is then given by $\lambda = \frac{\beta \sigma_v^2}{\beta^2 \sigma_v^2 + \beta^2 \sigma_s^2 + \sigma_u^2}$.

2. The speculator's problem is the same as in the baseline Kyle model:

$$x(s) = \arg \max_x \{x(v - p)|x\} = \frac{\mathbb{E}[v|s] - \mu}{2\lambda}.$$

Since the speculator's signal precision is $\tau_s = \frac{1}{\sigma_s^2}$, we have

$$\begin{aligned} \mathbb{E}[v|s] &= \frac{\tau_v}{\tau_v + \tau_s} \mu + \frac{\tau_s}{\tau_v + \tau_s} s \\ &= \mu + \frac{\sigma_v^2}{\sigma_v^2 + \sigma_s^2} (s - \mu) \\ \Rightarrow x(s) &= \frac{1}{2\lambda} \cdot \frac{\sigma_v^2}{\sigma_v^2 + \sigma_s^2} (s - \mu), \end{aligned}$$

or $\beta = \frac{1}{2\lambda} \cdot \frac{\sigma_v^2}{\sigma_v^2 + \sigma_s^2}$.

3. The speculator's expected trading profit is

$$\begin{aligned} \mathbb{E}[x(v - p)] &= \mathbb{E}_s [\beta(s - \mu) \cdot (\mathbb{E}[v - \mu|s] - \lambda\beta(s - \mu))] \\ &= \mathbb{E}_s \left[\beta(s - \mu) \cdot \left(\frac{\sigma_v^2}{\sigma_v^2 + \sigma_s^2} (s - \mu) - \frac{\sigma_v^2}{2(\sigma_v^2 + \sigma_s^2)} (s - \mu) \right) \right] \\ &= \beta \frac{\sigma_v^2}{2(\sigma_v^2 + \sigma_s^2)} (\sigma_v^2 + \sigma_s^2) = \frac{\beta \sigma_v^2}{2} \\ &= \frac{\sigma_v^4}{4\lambda(\sigma_v^2 + \sigma_s^2)}, \end{aligned}$$

since $\mathbb{E}[(s - \mu)^2] = \sigma_v^2 + \sigma_s^2$.

4. The speculator's expected profit is given by trading profit net of information cost:

$$\frac{\sigma_v^4}{4\lambda(\sigma_v^2 + \sigma_s^2)} - \frac{\gamma}{\sigma_s^2}.$$

Maximizing that over σ_s^2 , we get the First-Order Condition

$$-\frac{\sigma_v^4}{4\lambda(\sigma_v^2 + \sigma_s^2)^2} + \frac{\gamma}{\sigma_s^4} = 0$$

$$\Rightarrow \tau_s = \frac{1}{\sigma_s^2} = \frac{1}{2\sqrt{\lambda\gamma}} - \frac{1}{\sigma_v^2}.$$

5. We can see that the speculator acquires more information (chooses higher precision τ_s /lower variance σ_s^2) when:

- γ is lower – information is cheaper;
- σ_v^2 is higher – information is more valuable since the fundamental is more uncertain;
- λ is lower – price impact is lower, so the information is easier to act on;
- σ_u^2 is higher – having more noise trades makes the market deeper (lower λ). Note that the general equilibrium effect is smaller than $\frac{\partial \tau_s}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial \sigma_u^2}$, since the market-maker would anticipate higher σ_s^2 and adjust depth accordingly:

$$\frac{d\tau_s}{d\sigma_u^2} = \frac{\partial \tau_s}{\partial \lambda} \left(\frac{\partial \lambda}{\partial \sigma_u^2} + \frac{\partial \lambda}{\partial \tau_s} \cdot \frac{d\tau_s}{d\sigma_u^2} \right) \quad \Rightarrow \quad \frac{d\tau_s}{d\sigma_u^2} = \frac{\frac{\partial \tau_s}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial \sigma_u^2}}{1 - \frac{\partial \tau_s}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial \tau_s}}.$$

Problem 3: Odte traders

1. Which motives for trading have we proposed throughout the course as reasons for trading in financial markets? Mention and briefly describe them.
2. Read the essay attached at the end of this exam. Which of the motives above drive the behavior of the retail traders mentioned in the essay? Are there any other motives driving them that we have not discussed?

Solution

1. In one of the slides, we emphasized the following main reasons for which agents trade.

Informational: investor has (or believes he has) private information relative to other agents in the market, which allows them to predict future price movements and trade at a profit.

Liquidity: investor trades due to either having excess liquidity (too much free cash – optimal to buy illiquid assets that generate higher return), or encountering a need for liquidity (sell some illiquid assets from the portfolio in exchange for liquid cash).

Hedging: investor buys or sells a given asset in order to hedge the risks in the rest of their portfolio.

We briefly discussed various other motives (e.g., exploiting arbitrage opportunities; providing liquidity can be a self-contained profitable activity; an activist investor may be buying up a company's shares in order to affect its governance), but the bulk of the course implied one of the three primary motives.

2. One could argue that out of the above, the informational motive describes traders' behavior most

accurately: they *believe* they have private information that allows them to beat the market. They may enter the market due to liquidity motive (the desire to earn a return on their free cash), but it alone would not justify intraday trading. It should be noted that there is a legitimate hedging motive in trading 0dte options too, see Adams et al. (2024).¹ However, it does not appear to be driving the behavior of the traders discussed in the essay

One factor that we did not discuss in this course that is also likely relevant in this case is risk-loving: the individuals invest in highly risky assets with zero (or negative, after all the commissions) expected return simply because they get positive expected utility from such risky gambles.

¹Adams, Greg and Fontaine, Jean-Sebastien and Ornathanalai, Chayawat, The Market for 0DTE: The Role of Liquidity Providers in Volatility Attenuation (May 03, 2024). Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4881008>

Written Exam Economics summer 2025

Financial Markets Microstructure

August 18, 2025

This exam question consists of 10 pages in total
Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam. Please write the exam number on your exam paper as well.

Use of AI tools is permitted. You must explain how you have used the tools. When text is solely or mainly generated by an AI tool, the tool used must be quoted as a source.

Be careful not to cheat at exams!

Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules in the Faculty of Social Science's common part of the curriculum

You can read more about the rules on exam cheating on your Study Site and in the Faculty of Social Science's common part of the curriculum.

Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

Final re-exam

Write up your responses to questions below and submit them to Digital Exam. The deadline to submit the responses is Aug 18, 21:00. No cooperation with other students is permitted.

Be concise, but show your work and explain your answers. Some questions may require you to make additional assumptions beyond those provided in the question; be clear about the assumptions you make. Some questions are open ended in that they may not have a unique correct answer. You are allowed to refer to textbooks, lecture notes, slides, problem sets, etc.

Problem 1: Inventory risk and demand for liquidity

Consider a Kyle model with inventory risk and no informed trading that we considered in class. How does the dealer's pricing schedule in that model depend on the variance of the incoming market order, σ_u^2 ? Explain the intuition behind this result and which modelling assumptions are responsible for this conclusion.

Problem 2: Kyle model with public information acquisition

Consider a single-period Kyle model, where the speculator does not know the asset's fundamental value v perfectly, but instead decides how much to *publicly* invest in a noisy signal about v . In particular, suppose that before submitting an order, the speculator chooses σ_s^2 , pays cost $c(\sigma_s^2)$, and then receives signal $s \sim \mathcal{N}(v, \sigma_s^2)$. All other agents in the market (specifically, the market-maker) observe the speculator's choice of σ_s^2 .

After that, the game proceeds as in the regular Kyle model. The speculator chooses their trade size, $x \in \mathbb{R}$, to maximize their expected profit $\Pi_I \equiv \mathbb{E}[x(v - p)]$. The noise traders submit a random market order $u \sim \mathcal{N}(0, \sigma_u^2)$. The competitive dealer observes the aggregate order imbalance $q = x + u$ and quotes a price $p(q)$ at which they are willing to absorb it. All agents have a common prior belief that $v \sim \mathcal{N}(\mu, \sigma_v^2)$.

One can verify that for a given signal precision σ_s^2 and speculator's strategy $x(s) = \beta(s - \mu)$ for some β , the competitive dealer's price schedule is given by $p(q) = \mu + \lambda q$ with $\lambda = \frac{\beta \sigma_v^2}{\beta^2 \sigma_v^2 + \beta^2 \sigma_s^2 + \sigma_u^2}$. One can then verify that the speculator indeed optimally trades according to $x(s) = \beta(s - \mu)$ with $\beta = \frac{1}{2\lambda} \cdot \frac{\sigma_v^2}{\sigma_v^2 + \sigma_s^2}$.

1. Give a plausible justification to the assumption that the speculator's choice of σ_s^2 is *observable* to other market participants.
2. Solve for equilibrium speculator's trading aggressiveness β and the price impact λ in terms of model parameters and σ_s^2 .
3. Calculate the speculator's expected trading profit for given σ_s^2, σ_v^2 .
4. Suppose now the speculator's information cost is given by $c(\sigma_s^2) = \frac{\gamma}{\sqrt{\sigma_s^2}}$ for some information cost parameter γ . Derive the amount of information $\tau_s \equiv \frac{1}{\sigma_s^2}$ the speculator acquires as a function of $\lambda, \gamma, \sigma_v^2$.
5. How does the speculator's information choice depend on γ, σ_v^2 , and σ_u^2 in equilibrium? Explain.
6. Answer intuitively: after committing publicly to some level of σ_s^2 , would the speculator want to secretly change σ_s^2 ? Why or why not? Explain.

Problem 3: Frozen Concentrated Orange Juice

Read a brochure about the Frozen Concentrated Orange Juice (FCOJ) future market attached at the end of this exam text. Answer the following questions.

1. According to the brochure, what are the two main goals of the FCOJ future market?
2. According to the brochure, what two types of traders participate in the FCOJ future markets? Which of these traders, do you think, are more likely to have informational advantage?
3. The figure on p.1 of the report shows that orange crop utilization for sake of producing FCOJ has been steadily declining in both absolute and relative terms during 1992–2011. The last Figure on p.3 of the report, however, shows that both trading volume and open interest in FCOJ futures has remained steady during that period. So the FCOJ market has been declining, but the FCOJ futures market has not. Propose an explanation for this discrepancy. How does it relate to market goals you identified in question 1?



Frozen Concentrated Orange Juice

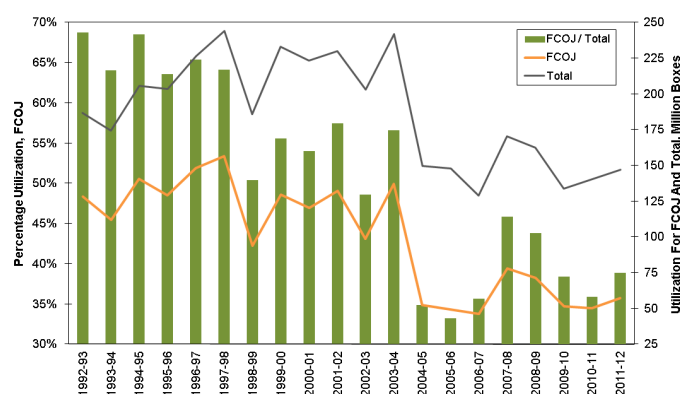
IntercontinentalExchange® (ICE®) became the center of global trading in “soft” commodities with its acquisition of the New York Board of Trade (NYBOT) in 2007. Now known as ICE Futures U.S.®, the exchange offers futures and options on futures on soft commodities including coffee, cocoa, sugar, cotton and frozen concentrated orange juice (FCOJ).

FCOJ futures have traded in New York since 1966, first on the New York Cotton Exchange, then on the successor New York Board of Trade and now on ICE Futures U.S. Options on FCOJ futures were introduced in 1985. Futures and options on futures have been used by the domestic and global citrus and juice industries to price and hedge transactions. ICE Futures U.S. is the exclusive global market for FCOJ futures and options.

THE ORANGE JUICE MARKET

FCOJ represents a combined triumph of academic research, agriculture and marketing. It was developed at the University of Florida's Citrus Research and Education Center in 1948. FCOJ competes with two other forms of juice not squeezed directly at the point of consumption: reconstituted liquid juice and not-from-concentrate, or NFC, juice. While NFC juices surpassed FCOJ in market share during the 1980s, the readily storable and easy-to-ship FCOJ remains the industry's pricing benchmark. Even though the share of Florida's orange crop going to FCOJ has declined compared to other types of juice, FCOJ remains the most visible price discovery mechanism for the industry. Ultimately, the importance of the juice industry to Florida's citrus growers cannot be overestimated; approximately 90% of the crop goes into juice in all forms.

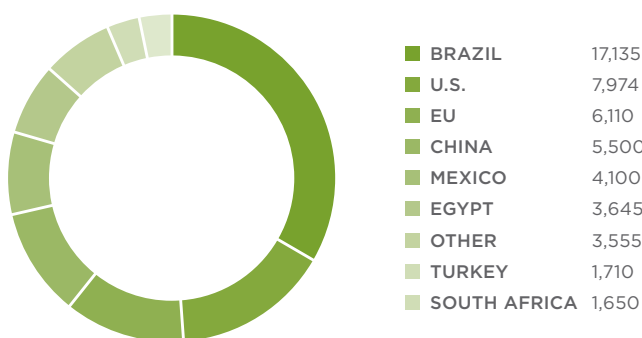
USDA FLORIDA ORANGE CROP UTILIZATION



Source: Florida Department of Citrus

While Brazil is the largest global supplier of oranges, Florida dominates U.S. production. Global demand for orange juice has attracted strong supply growth from the European Union, China and Mexico in recent years.

2006 WORLD ORANGE PRODUCTION 1,000 METRIC TONS

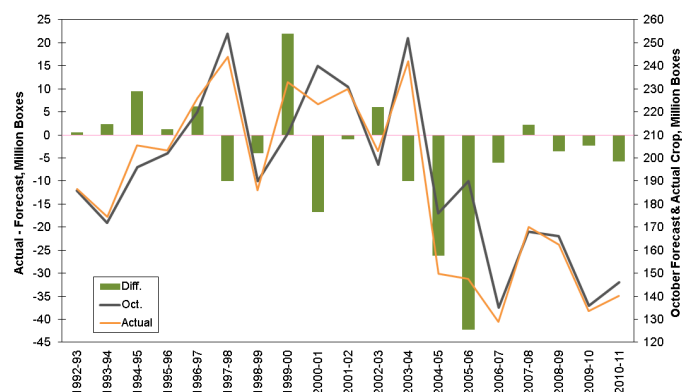


Source: Florida Department of Citrus

The citrus and orange juice markets share an odd characteristic with many other grain, oilseed and soft markets: the difficulty even experts have in assessing supply. Why this should be true in an age of satellite reconnaissance is not clear. After all, trees do not sneak into groves when no one is looking, and both the groves and the health of the trees in them are visible from both the ground and the air.

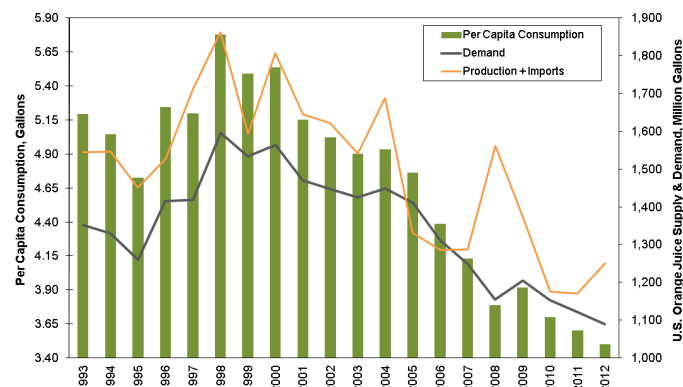
The U.S. Department of Agriculture makes a production estimate each October for the forthcoming crop year. While the Department's forecast errors from the 2003-04 crop year onward have been on the negative side (with the exception of the 2007-2008 crop year), a general decline in the crop produced and occasional supply shocks from key import sources such as Brazil have contributed to a series of price increases even though per capita consumption of orange juice has been declining steadily.

USDA FLORIDA ORANGE CROP FORECAST AND ACTUAL PRODUCTION



Source: U.S. Department of Agriculture

USDA ORANGE JUICE SUPPLY AND DEMAND ESTIMATES

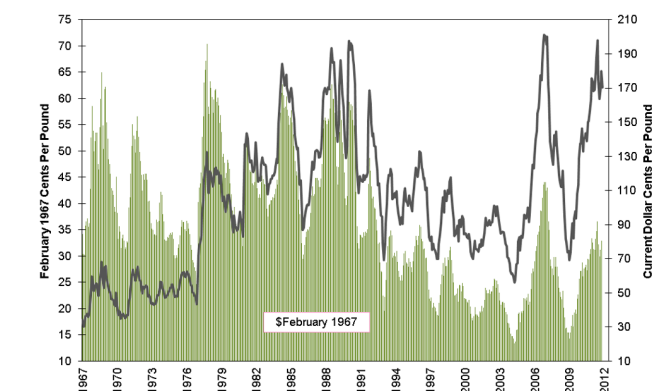


Source: U.S. Department of Agriculture

The volatile nature of FCOJ pricing is what makes this market so vital for hedgers and so interesting for speculators. The market is prone to sharp price spikes in anticipation of weather-related disruptions in supply, including freezes and hurricanes, and to retracements of those spikes when the damage was not as bad as feared initially, or when imports of FCOJ from Brazil and other suppliers enter the U.S. market. In January 2012, the market established an all-time high when the U.S. Food and Drug Administration denied entry to some Brazilian imports following discovery of traces of a fungicide prohibited for use in oranges.

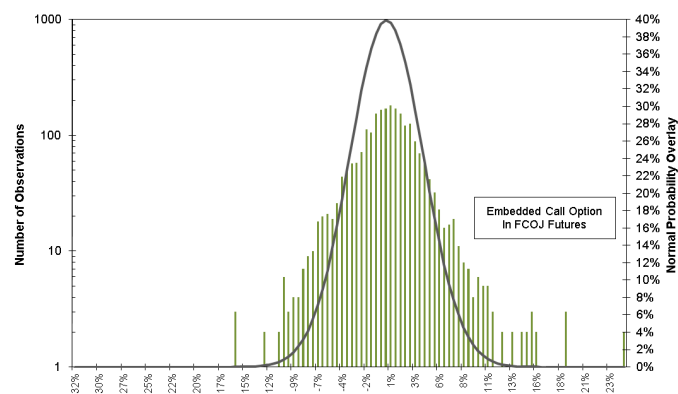
The nature of FCOJ prices over time can be viewed in two ways. The first perspective is a simple price chart in current- and constant-dollar terms. The second perspective is weekly percentage changes compared to a normal probability distribution. Here the preponderance of price spikes shows up as both the very positive "skew," or shift to the right, and as the greater-than-expected number of large positive returns. A long position in FCOJ futures is in many ways equivalent to owning a call option on a supply disruption.

EVEN CONSTANT-DOLLAR FROZEN CONCENTRATED ORANGE JUICE PRICES ARE VOLATILE



Source: CRB-Infotech CD-ROM

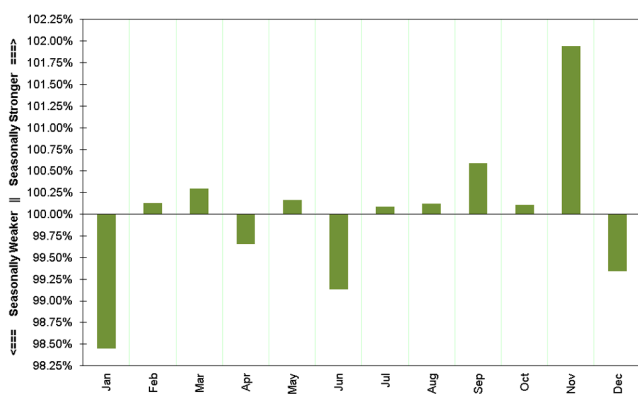
WEEKLY RETURNS SKEWED HEAVILY TO UPDATE



Source: CRB-Infotech CD-ROM

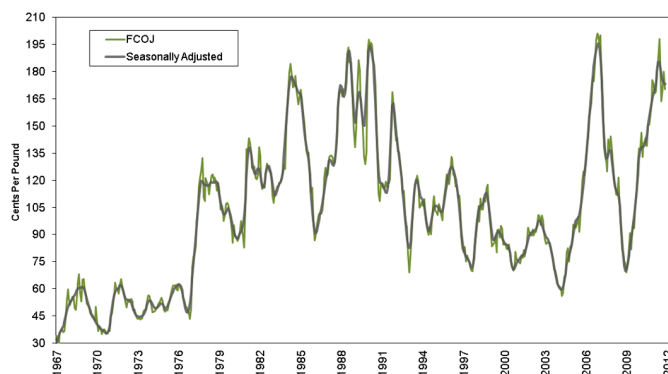
If FCOJ prices are in fact an embedded call option, do we see some seasonal distortions in their pricing structure? The answer here is rather surprising in one respect and reassuring in another. Not only are the seasonal divisors for FCOJ prices amongst the weakest of all agricultural commodities - in fact, they are not statistically significant at a rigorous 1% level - the seasonally adjusted price chart for FCOJ shows just how efficient the market has been in moderating price swings. This is what we hope for in any stored commodity. A seasonal analysis indicates the strongest month for FCOJ futures is November, the time when traders are hedging against a freeze. The seasonally weakest month is January, when many "freeze-protection" trades are lifted.

MONTHLY SEASONALITY FOR FCOJ FUTURES



Source: CRB-Infotech CD-ROM

FCOJ FUTURES MARKET EFFICIENT IN MODERATING SEASONAL SWINGS

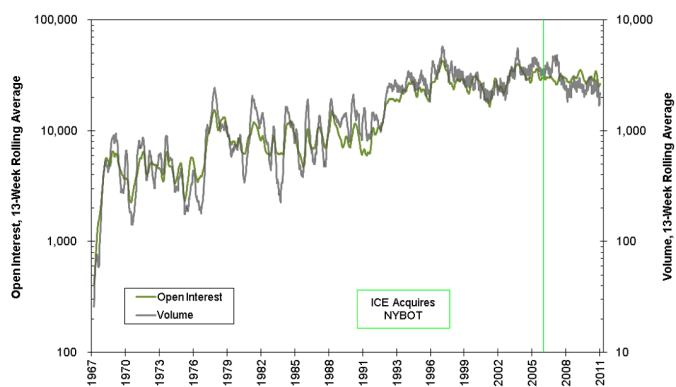


Source: CRB-Infotech CD-ROM

FCOJ TRADING AT ICE FUTURES U.S.

This price history and market structure demonstrates clearly why juice sellers such as grocery chains and juice processors should hedge price risks, and why FCOJ futures and options have been so successful over the contracts' lives. The success of the contract grew even as the physical nature of the orange juice market shifted from FCOJ to NFC. Commodity trading advisors and commodity-based hedge funds like the contract not only for its trading opportunities, but also because of its diversifying properties vis-à-vis other commodity futures.

LONG-TERM SUCCESS OF FROZEN CONCENTRATED ORANGE JUICE CONTRACT



Source: CRB-Infotech CD-ROM

ICE FUTURES U.S. FROZEN CONCENTRATED ORANGE JUICE CONTRACT

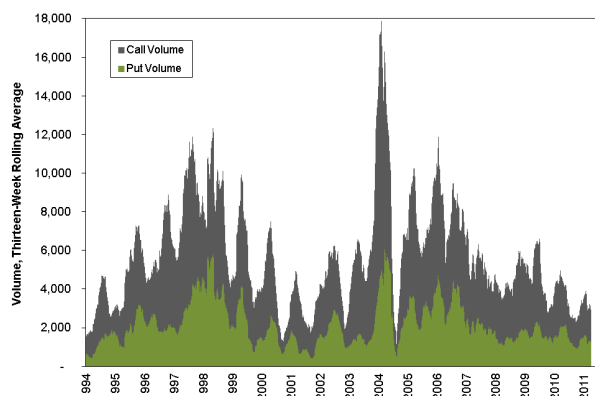
The ICE Futures U.S. FCOJ futures contract is for the physical delivery of USDA frozen concentrate, not less than 62.5° Brix, to an exchange-licensed warehouse in Florida, New Jersey or Delaware.

[Contract specifications including fees, margins and delivery standards](#)

Options on the FOCJ futures contract are also available. Each futures contract has options that settle into that contract along with a serial option expiring in the month prior to the futures contract. For example, the January future has both a December and a January option priced thereto. Option strikes are spaced 5 cents apart. The last trading day is the third Friday of the month preceding the named option month.

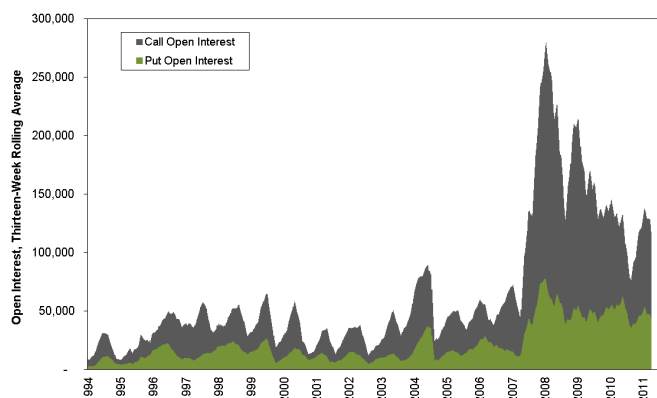
Options trading volume and open interest on the FCOJ futures contract have stayed at a high level, with surges around weather-related events such as the 2004 hurricane season in Florida. Options tend to be used by two groups of sophisticated traders. The first is commercial participants hedging their physical positions. The second is experienced speculative traders.

LONG-TERM SUCCESS OF FCOJ OPTIONS: VOLUME



Source: ICE Futures U.S.

LONG-TERM SUCCESS OF FCOJ OPTIONS: OPEN INTEREST



Source: ICE Futures U.S.

TRADING ICE FUTURES U.S. FCOJ FUTURES AND OPTIONS

Futures markets exist for the purposes of price discovery and risk transfer. Price discovery requires buyers and sellers to meet in a competitive marketplace; prices resulting from each transaction signal to other traders what a given commodity might be worth.

Anyone approved by a clearing member or futures commission merchant can participate in the price discovery process, regardless of their participation in the citrus and juice business. A market participant who is not in the citrus, juice-packing or retail business will be classified as a non-commercial or speculative trader. A market participant active in those businesses will be classified as a commercial trader or hedging trader. For a speculator, the price discovery trade is simple and straightforward; if you believe the price of FCOJ will rise, you “go long” a futures contract; if you believe the price of FCOJ will fall, you “go short” a futures contract.

These same market views can be expressed in options as well. If you believe prices will rise, you can buy a call option, sell a put option or engage in a large number of spread trades tailored to your specific price view and risk acceptance. If you believe prices will fall, you can buy a put option, sell a call option or engage in a different set of spread trades. A long call (put) option is the right, but not the obligation, to go long (short) the underlying future at the strike price at or by expiration. A short call (put) option is the obligation to deliver (take delivery) of the underlying future at or by the expiration if that option is exercised.

Hedgers use ICE FCOJ options frequently. Producers can set a floor beneath a selling price with long put options, and buyers can establish a ceiling over costs with long call options, among other strategies.

In a futures trade, you and the counterparty to your trade will post initial or original margin with your futures commission merchant or clearing member. Minimum margins are set by ICE Futures U.S., and your futures commission merchant may require additional funds. Margin schedule

There are no margin requirements for long option positions. Margin requirements for short option positions vary according to the relationship between the option strike price and the futures price.

If the market moves in your favor — higher for a long position (or commitment to take delivery of FCOJ or to offset the contract by selling it prior to delivery), or lower for a short position (or commitment to deliver FCOJ or to offset the contract by buying it prior to delivery) - the equity in your account will increase. You may withdraw these funds down to the “maintenance margin” level, depending on your account agreement.

If the market moves adversely - lower for a long position or higher for a short position - your futures commission merchant will require

you to post additional funds, called variation margin, to sustain your maintenance margin level. These “margin calls” assure both your futures commission merchant and ICE Clear U.S.®, the exchange clearing house, that you can perform according to your contractual commitment. All futures accounts are marked-to-market daily, and participants deficient in margin obligations may have positions liquidated involuntarily.

As the designated clearing house, ICE Clear U.S. serves as the counterparty to every futures contract traded on ICE Futures U.S. The clearing house clears trades matched by ICE Futures U.S. and guarantees performance in delivery even if a trader defaults. The financial integrity and anonymity provided by ICE Clear U.S. are increasingly important in the financial system.

What do the financial flows look like in a futures trade? Let's say a five-contract futures position is initiated at 217.00¢ per pound and the market rises to 218.30¢ per pound on the following trading day.

- For the long position, the gain is:
 $5 \text{ contracts} \times [218.30 - 217.00] / \text{contract} \times \$1.50 \text{ per } .01¢ = \$975$
- For the short position, the loss is equal and opposite:
 $5 \text{ contracts} \times [217.00 - 218.30] / \text{contract} \times \$1.50 \text{ per } .01¢ = -\$975$

If we reverse the price path, we reverse the gains and losses. Let's change the starting price to 216.50¢ per pound and have the market decline to 214.25¢ per pound the next day.

- For the long position, the loss is:
 $5 \text{ contracts} \times [214.25 - 216.50] / \text{contract} \times \$1.50 \text{ per } .01¢ = -\$1,687.50$
- For the short position, the gain is equal and opposite:
 $5 \text{ contracts} \times [216.50 - 214.25] / \text{contract} \times \$1.50 \text{ per } .01¢ = \$1,687.50$

Options traders see the same directional profit and loss profiles relative to price, but the actual profit and loss is subject to a range of additional factors, including market volatility, time to expiration, interest rates and the relationship between the current futures price and the option's strike price.

RISK TRANSFER

Risk transfer is the second purpose of a futures market. Any producer of FCOJ, any holder of FCOJ inventories, or any party at risk to the price of FCOJ declining can seek protection in the futures markets. These participants are long the market and can offset risk by going short a futures contract. Any juice packer or retailer at risk to the price of FCOJ increasing is short the market and can offset risk by going long a futures contract.

The mechanics and financial flows are identical to those outlined above. A FCOJ producer at risk to prices falling can acquire a financial asset, the short futures position, which will rise in value as the market declines. The opposite is true for a juice packer at risk to prices rising; there a long futures position will rise in value as the market rises.

While the financial flows should offset the economic gains and losses of the physical FCOJ position, there are two important things to remember. First, even though futures prices converge to cash prices at expiration, the convergence process is subject to what is called “basis risk” or differences resulting from changes in hedging demand and location of the FCOJ. Certified stock reports.

Second, while the economic gains on, for example, a warehouse full of FCOJ are real, they are not realized until the FCOJ is sold. If this inventory is hedged with a short futures position and the market rises, the beneficial owner will have to keep posting additional funds in the margin account.

Nothing in the above discussion of hedging indicates when or at what price to hedge. This is one of the reasons options are valuable to hedgers. While the FCOJ producer may wish to have downside protection or a price floor, that same producer probably wants to participate in any future price increases.

The producer concerned about a decline in the value of FCOJ between now and the time he expects to be able to sell his inventory could buy a January 215¢ put option, which is the right, but not the obligation, to receive a short position in a January future at 215¢ for 8.65¢, or approximately \$1,298. The purchased put guarantees the producer the right to sell the January future for an effective price of 206.35¢ per pound (the 215¢ strike price less the premium paid of 8.65¢). This right gives him protection if FCOJ prices have fallen by the expiry of the January option, but at the same time preserves his ability to profit should the price of FCOJ move higher over the period.

The juice packer wishing to cap the price of FCOJ, but not be exposed to margin calls if the price continues to rise, can do an opposite trade and buy a January 220¢ call option, which is the right, but not the obligation, to receive a long position in a January future at 220¢ for 8.40¢, or approximately \$1,260. The purchased call gives the juice packer the right to buy the January future at an effective price of 228.40¢ per pound (again, the strike price of 220¢ cents plus the premium paid of 8.40¢), offering protection against an unfavorable rise in the price of FCOJ while preserving the ability to take advantage if prices decline.

It should be noted that the risk profile for sellers of options is dramatically different than for buyers of options. For buyers, the risk of an option is limited to the premium or purchase price paid to buy the option. For sellers, the risk profile is unknown and can be potentially quite large.

Options can become complex very quickly, with trading influenced by variables including time remaining to contract expiration, underlying commodity volatility, short-term interest rates and a host of expected movements collectively called "the Greeks."

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TRADING & TECHNOLOGY

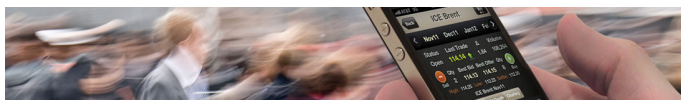
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ICE Futures	WTI	Dec14	+	2	100.23	100.25	3	101.02	101.27	101.01	101.11	101.7	0	1
ICE Futures	WTI	Dec14	+	2	91.23	91.25	1	92.02	92.27	91.01	91.11	91.7	0	1
ICE Futures	WTI	Dec14	+	2	94.41	94.43	1	95.02	95.27	94.01	94.11	94.7	0	1
ICE Futures	WTI	Jun12Dec12	+	2	94.48	94.52	2	95.02	95.27	94.01	94.11	94.7	0	1
ICE Futures	WTI	Jun12Jun13	+	12	-0.01	-0.02	9	-0.02	-0.03	-0.01	-0.01	-0.01	0	0
ICE Futures	WTI	Dec12Jun13	+	7	0.02	0.03	3	0.03	0.04	0.02	0.02	0.02	0	0
ICE Futures	WTI	Jun12Dec13	+	7	1.23	1.25	3	1.24	1.25	1.23	1.23	1.23	0	0
ICE Futures	WTI	Dec12Dec13	+	2	2.28	2.32	18	2.28	2.32	2.28	2.28	2.28	0	0
ICE Futures	WTI	Dec12Dec13	+	2	2.28	2.32	18	2.28	2.32	2.28	2.28	2.28	0	0

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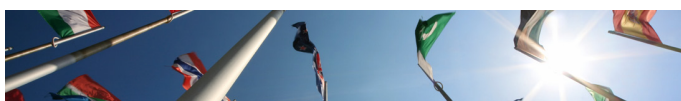
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CONTACT US

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Online: theice.com/fcoj

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Final re-exam solutions

Problem 1: Inventory risk and demand for liquidity

Consider a Kyle model with inventory risk and no informed trading that we considered in class. How does the dealer's pricing schedule in that model depend on the variance of the incoming market order, σ_u^2 ? Explain the intuition behind this result and which modelling assumptions are responsible for this conclusion.

Solution

The dealer's pricing schedule in the mentioned model does not depend on σ_u^2 (see slide deck L7, slide 23). This is because the dealer's pricing decisions are guided by the premium they require for holding on to the inventory given the uncertainty in the asset valuation. There is no informed trading that attempts to hide between noise traders' orders. This model further assumes that the dealer maximizes their one-period-ahead wealth, which abstracts away from how the question of how the dealer will unravel their inventory. Finally, the dealers are assumed to be competitive. Therefore, there is no reason for the variance of liquidity demand to affect the pricing offered by the dealer. If, however, any of the aforementioned assumptions were altered, the result would be different.

Problem 2: Kyle model with public information acquisition

Consider a single-period Kyle model, where the speculator does not know the asset's fundamental value v perfectly, but instead decides how much to *publicly* invest in a noisy signal about v . In particular, suppose that before submitting an order, the speculator chooses σ_s^2 , pays cost $c(\sigma_s^2)$, and then receives signal $s \sim \mathcal{N}(v, \sigma_s^2)$. All other agents in the market (specifically, the market-maker) observe the speculator's choice of σ_s^2 .

After that, the game proceeds as in the regular Kyle model. The speculator chooses their trade size, $x \in \mathbb{R}$, to maximize their expected profit $\Pi_I \equiv \mathbb{E}[x(v - p)]$. The noise traders submit a random market order $u \sim \mathcal{N}(0, \sigma_u^2)$. The competitive dealer observes the aggregate order imbalance $q = x + u$ and quotes a price $p(q)$ at which they are willing to absorb it. All agents have a common prior belief that $v \sim \mathcal{N}(\mu, \sigma_v^2)$.

One can verify that for a given signal precision σ_s^2 and speculator's strategy $x(s) = \beta(s - \mu)$ for some β , the competitive dealer's price schedule is given by $p(q) = \mu + \lambda q$ with $\lambda = \frac{\beta \sigma_v^2}{\beta^2 \sigma_v^2 + \beta^2 \sigma_s^2 + \sigma_u^2}$. One can then verify that the speculator indeed optimally trades according to $x(s) = \beta(s - \mu)$ with $\beta = \frac{1}{2\lambda} \cdot \frac{\sigma_v^2}{\sigma_v^2 + \sigma_s^2}$.

1. Give a plausible justification to the assumption that the speculator's choice of σ_s^2 is *observable* to other market participants.
2. Solve for equilibrium speculator's trading aggressiveness β and the price impact λ in terms of model parameters and σ_s^2 .
3. Calculate the speculator's expected trading profit for given σ_s^2, σ_v^2 .
4. Suppose now the speculator's information cost is given by $c(\sigma_s^2) = \frac{\gamma}{\sqrt{\sigma_s^2}}$ for some information cost parameter γ . Derive the amount of information $\tau_s \equiv \frac{1}{\sigma_s^2}$ the speculator acquires as a function of $\lambda, \gamma, \sigma_v^2$.
5. How does the speculator's information choice depend on γ, σ_v^2 , and σ_u^2 in equilibrium? Explain.
6. Answer intuitively: after committing publicly to some level of σ_s^2 , would the speculator want to secretly change σ_s^2 ? Why or why not? Explain.

Solution

1. We can think of the speculator as a hedge fund, and of their information acquisition efforts as the size of their research department, in terms of headcount and funding. The funds would likely advertise information like this to attract clients, so it would not be a stretch to assume that other market participants can readily observe it.
2. We have

$$\begin{aligned}\lambda &= \frac{\beta \sigma_v^2}{\beta^2 \sigma_v^2 + \beta^2 \sigma_s^2 + \sigma_u^2} & \beta &= \frac{1}{2\lambda} \cdot \frac{\sigma_v^2}{\sigma_v^2 + \sigma_s^2} \\ \Rightarrow \lambda &= \frac{\sigma_v^2}{2\sqrt{\sigma_u^2(\sigma_v^2 + \sigma_s^2)}} & \beta &= \sqrt{\frac{\sigma_u^2}{\sigma_v^2 + \sigma_s^2}}\end{aligned}$$

3. The speculator's expected trading profit is

$$\begin{aligned}\mathbb{E}[x(v-p)] &= \mathbb{E}_s [\beta(s-\mu) \cdot (\mathbb{E}[v-\mu|s] - \lambda\beta(s-\mu))] \\ &= \mathbb{E}_s \left[\beta(s-\mu) \cdot \left(\frac{\sigma_v^2}{\sigma_v^2 + \sigma_s^2}(s-\mu) - \frac{\sigma_v^2}{2(\sigma_v^2 + \sigma_s^2)}(s-\mu) \right) \right] \\ &= \beta \frac{\sigma_v^2}{2(\sigma_v^2 + \sigma_s^2)}(\sigma_v^2 + \sigma_s^2) = \frac{\beta \sigma_v^2}{2} \\ &= \frac{\sigma_v^2}{2} \cdot \sqrt{\frac{\sigma_u^2}{\sigma_v^2 + \sigma_s^2}},\end{aligned}$$

since $\mathbb{E}[(s-\mu)^2] = \sigma_v^2 + \sigma_s^2$.

4. The speculator's expected profit is given by trading profit net of information cost:

$$\frac{\sigma_v^2}{2} \cdot \sqrt{\frac{\sigma_u^2}{\sigma_v^2 + \sigma_s^2}} - \frac{\gamma}{\sqrt{\sigma_s^2}}.$$

Maximizing that over σ_s^2 , we get the First-Order Condition

$$\begin{aligned}-\frac{\sigma_v^2 \sigma_u}{4(\sigma_v^2 + \sigma_s^2)^{\frac{3}{2}}} + \frac{\gamma}{2(\sigma_s^2)^{\frac{3}{2}}} &= 0 \\ \Rightarrow \tau_s = \frac{1}{\sigma_s^2} &= \left(\frac{\sigma_u^2}{4\gamma^2 \sigma_v^2} \right)^{\frac{1}{3}} - \frac{1}{\sigma_v^2}.\end{aligned}$$

5. We can see that the speculator acquires more information (chooses higher precision τ_s /lower variance σ_s^2) when:
 - γ is lower – information is cheaper;
 - σ_u^2 is higher – having more noise trades makes the market deeper (lower λ);
 - σ_v^2 is “average” – information is more valuable when the fundamental is more uncertain, but this is offset by the higher price impact that a more informed speculator faces. Therefore, as σ_v^2 increases, in equilibrium it sometimes pays off for the speculator to commit to a lower σ_s^2 in order to increase the market depth and mitigate the price impact.
6. Yes. On top of direct costs $c(\tau_s)$, the speculator in this model incurs indirect costs of τ_s , which stem from λ . Specifically, τ_s is factored by the market-maker into the depth of the pricing schedule the

speculator faces when trading – i.e., costs stem from the observability of τ_s or, in other words, from the market maker's expectation of the speculator's τ_s . In turn, the benefits of τ_s come from the actually chosen amount of information (not the other agents' expectation of it) – more private information about v means larger scope for profitable trades. Therefore, secretly increasing τ_s brings additional benefit but does not increase the “indirect costs”.

Problem 3: Frozen Concentrated Orange Juice

Read a brochure about the Frozen Concentrated Orange Juice (FCOJ) future market attached at the end of this exam text. Answer the following questions.

1. According to the brochure, what are the two main goals of the FCOJ future market?
2. According to the brochure, what two types of traders participate in the FCOJ future markets? Which of these traders, do you think, are more likely to have informational advantage?
3. The figure on p.1 of the report shows that orange crop utilization for sake of producing FCOJ has been steadily declining in both absolute and relative terms during 1992–2011. The last Figure on p.3 of the report, however, shows that both trading volume and open interest in FCOJ futures has remained steady during that period. So the FCOJ market has been declining, but the FCOJ futures market has not. Propose an explanation for this discrepancy. How does it relate to market goals you identified in question 1?

Solution

1. Price discovery and risk transfer.
2. Commercial/hedging traders (related to citrus, juice-packing, or retail business) and speculative traders (everyone else). Speculators only have access to public information, while commercial traders have relevant insider information from their business activities. (It is, however, possible, that speculators are better at processing publicly available information than commercial traders, so they would have a different kind of informational advantage.)
3. While the relevance of FCOJ as a commodity has declined, this financial asset “remains the most visible price discovery mechanism for the [citrus] industry”. I.e., this asset is relevant to other forms of juices and other types of citrus, and serves as a proxy asset for hedgers seeking to insure against risks in citrus/citrus juice markets more broadly, and enables price discovery in those markets.

FINANCIAL MARKETS MICROSTRUCTURE: PROBLEM SET 1

Egor Starkov

Københavns Universitet, Spring 2025

Problem 1

Exercise 4 in chapter 3 (pp. 125-126):

Consider the one-period Glosten-Milgrom model with $v \in \{v^L, v^H\}$ and $\mathbb{P}(v^H) = 1/2$. The dealer is competitive and risk neutral, so prices are equal to expected values. With probability $1 - \pi$ the trader is a noise trader who buys and sells with equal probability, with probability π he is a potential insider. The potential insider can, before submitting a trade, privately acquire information that perfectly reveals v at some fixed cost $c > 0$. If no information is acquired, the potential insider has no private information about v . As usual, suppose the dealer cannot distinguish the traders.

- (a). Suppose an insider decided to not acquire any information. What trading strategy maximizes his expected profit?
- (b). Compute the bid and ask prices set by the dealer, assuming that he believes that the insider acquires information with probability $\varphi \in [0, 1]$.
- (c). Given these prices, determine the trading profit of an insider who has decided to acquire information.
- (d). There exists $\bar{c}$ such that if $c > \bar{c}$, then the potential insider never acquires information (i.e., in equilibrium, $\varphi = 0$). Calculate $\bar{c}$.
- (e). There exists $\underline{c}$ such that if $c < \underline{c}$, then the potential insider always acquires information (i.e., in equilibrium, $\varphi = 1$). Calculate $\underline{c}$.
- (f). Suppose now that $c \in (\underline{c}, \bar{c})$. Find the mixed-strategy equilibrium in which the potential insider learns v with positive probability smaller than one. Describe the equilibrium fully.

Problem 2

This problem considers an issue left open in Section 4.3 in the textbook, which explores inventory risk within the Kyle's model. At the outset of this section, it is assumed that there is no informed trading, i.e., $x = 0$ and $q = u$. Let us now relax this assumption and try to analyze the effects of dealer's risk aversion in Kyle's model in the presence of adverse selection.

So, consider a combination of the two versions of the Kyle model we have seen in class: we now have one informed trader, a pool of noise traders, and a representative competitive risk-averse dealer with mean-variance preferences, risk aversion ρ , initial asset holding z_0 , and initial cash holding c_0 . I.e., the dealer's utility from a [random] next period's wealth $w_{t+1} = z_{t+1}p_{t+1} + c_{t+1}$ is $U(w_{t+1}) = \mathbb{E}[w_{t+1}] - \frac{\rho}{2}\mathbb{V}(w_{t+1})$.

We are again looking for a linear equilibrium, where the insider submits an order $x = \beta v - x_0$, and the pricing schedule quoted by the dealer is $p = p_0 + \lambda q$. Now, there are four endogenous parameters, β, x_0, p_0, λ , with $\beta > 0$ and $\lambda > 0$.

- (a). Take for granted some arbitrary parameters β, x_0 . The dealer observes $q = x + u$. Use the result from Section 4.2 or lecture slides to find $\mathbb{E}[v|q]$ and $\mathbb{V}[v|q]$ as functions of β and the model parameters.

(Note that it is no longer the case that $q = \beta(v - \mu) + u$, but rather $q + x_0 - \beta\mu = \beta(v - \mu) + u$.)

- (b). The competitive dealer takes p as given. In equilibrium, $p = p_0 + \lambda q$, so the price reveals q – same logic as in Section 4.2.4. Rewrite $\mathbb{E}[v|q]$ and $\mathbb{V}[v|q]$ from (a) as functions of p instead of q , when $q = (p - p_0)/\lambda$. Let's call these functions $\mathbb{E}[v|p]$ and $\mathbb{V}[v|p]$.

- (c). Calculate the dealer's asset supply curve and show that the amount supplied at price p is

$$y(p) = z_0 + \frac{p - \mathbb{E}[v|p]}{\rho \mathbb{V}[v|p]}$$

(A reminder: To solve this problem, instead of the zero-profit condition you should invoke another feature of competitive agents, namely price-taking. Use the following algorithm to obtain the dealer's price schedule: (1) fix some arbitrary price p ; (2) find supply amount $y(p)$ that maximizes the dealer's profit given this p ; (3) invert the supply function $y(p)$ to obtain a pricing schedule $p(q)$.)

- (d). When the market clears at price p , then $y(p) = q$. Can you see if there exists a $\lambda > 0$ and a number p_0 , such that market clearing fits with our conjecture that $p = p_0 + \lambda q$? I.e. try to solve the following equation

$$(y(p) =) \quad z_0 + \frac{p - \mathbb{E}[v|p]}{\rho \mathbb{V}[v|p]} = \frac{p - p_0}{\lambda} \quad (= q)$$

w.r.t. λ and p_0 . Note that they must be such that the equality holds for all p .

- (e). Go back to the informed trader's problem. Assuming that $p = p_0 + \lambda q$, argue that the insider chooses x to maximize $x(v - p_0 - \lambda x)$. Show that the relation is of the form $x = \beta v - x_0$ and try to relate parameters β, λ, x_0, p_0 to each other. Is there a solution for all four parameters? If the algebra is impossible, try a numerical solution.
- (f). In case $\rho = 0$, the dealer is risk neutral, so we are back to the case of pure adverse selection. In this limit case, do you get back the equilibrium found in Section 4.2/in class? How do you find that changes in ρ affect market depth in equilibrium?

Problem 3

One of the news articles assigned earlier in the course argued that corporate bond markets are significantly less liquid than stock markets. This is partly due to a more opaque trading tradition: instead of the dealers posting their quotes openly, the bond markets operate via Requests For Quotes (RFQ). In particular, an interested trader must contact the dealer with an RFQ, and then the dealer would respond with a quote. This process is somewhat time-consuming for traders, and thus costly. However, the costs of this opaqueness can be far greater than just these time costs.

Your goal is to build a model of search costs and to examine how they affect market outcomes: prices, spread and/or market depth, volume of trade if applicable. In particular, take any dealer model we considered (I suggest Glosten-Milgrom, but Kyle can probably work as well). Assume that instead of the dealers posting quotes openly, any arriving trader must approach dealers individually, paying cost c per dealer to learn that dealer's quotes. Solve your model as best you can and answer the following questions based on your results:

- How do prices (and thus liquidity/depth) in your model compare to the baseline model, in which dealers post quotes openly?
- How do prices depend on c ?
- How do prices depend on the number of dealers in the market?
- In this situation, could a dealer profit by announcing quotes publicly?

FINANCIAL MARKETS MICROSTRUCTURE: PROBLEM SET 1

Egor Starkov

Københavns Universitet, Spring 2025

Problem 1

Exercise 4 in chapter 3 (pp. 125-126):

Consider the one-period Glosten-Milgrom model with $v \in \{v^L, v^H\}$ and $\mathbb{P}(v^H) = 1/2$. The dealer is competitive and risk neutral, so prices are equal to expected values. With probability $1 - \pi$ the trader is a noise trader who buys and sells with equal probability, with probability π he is a potential insider. The potential insider can, before submitting a trade, privately acquire information that perfectly reveals v at some fixed cost $c > 0$. If no information is acquired, the potential insider has no private information about v . As usual, suppose the dealer cannot distinguish the traders.

- (a). Suppose an insider decided to not acquire any information. What trading strategy maximizes his expected profit?

Solution: Since $a > \mu > b$ in equilibrium (you can verify this by solving part (b) for arbitrary trading strategy of the uninformed insider), a profit-maximizing uninformed insider would choose to abstain from trading, since it yields a payoff of zero, and expected payoff from any trade in the absence of information is negative.

- (b). Compute the bid and ask prices set by the dealer, assuming that he believes that the insider acquires information with probability $\varphi \in [0, 1]$.

Solution: A potential insider who does not acquire information will always abstain (as long as there is a positive bid-ask spread) whereas an insider who observes v^H (v^L) will buy (sell). (You can verify this in the end.)

Use Bayes' Rule to calculate the probability that the trader is an *insider* given a buy order is

$$\mathbb{P}(I|\text{buy}) = \frac{\mathbb{P}(I) \cdot \mathbb{P}(\text{buy}|I)}{\mathbb{P}(\text{buy})} = \frac{\pi\varphi\frac{1}{2}}{\pi\varphi\frac{1}{2} + (1-\pi)\frac{1}{2}} = \frac{\pi\varphi}{\pi\varphi + (1-\pi)}.$$

Let $\alpha_\varphi = \frac{\pi\varphi}{\pi\varphi + (1-\pi)}$. Similarly, we can deduce that $\mathbb{P}(I|\text{sell}) = \alpha_\varphi$. Letting $m = \frac{v^H + v^L}{2}$, the ask (a_φ) and bid (b_φ) prices are

$$\begin{aligned} a_\varphi &= \alpha_\varphi v^H + (1 - \alpha_\varphi)m, \\ b_\varphi &= \alpha_\varphi v^L + (1 - \alpha_\varphi)m. \end{aligned} \tag{1}$$

That is, the ask price is the probability that the trader is an (informed) insider times the value given that he is informed, plus the probability that the trader is a noise trader, times the expected value given that he is a noise trader (which is just the prior expectation m).

- (c). Given these prices, determine the trading profit of an insider who has decided to acquire information.

Solution: The trader's profit from acquiring information depends on the belief of the dealer, i.e. the probability that the dealer attaches to the trader acquiring information. Suppose the dealer's belief attaches probability φ to the trader being an informed insider. A trader who acquires information then has the following expected profit:

$$u(\varphi) = \frac{1}{2}(v^H - a_\varphi) + \frac{1}{2}(b_\varphi - v^L) - c = (1 - \alpha_\varphi)\frac{v^H - v^L}{2} - c.$$

Notice that the dealer's beliefs enters only through the prices.

- (d). There exists $\bar{c}$ such that if $c > \bar{c}$, then the potential insider never acquires information (i.e., in equilibrium, $\varphi = 0$). Calculate $\bar{c}$.

Solution: Notice that $u'(\cdot) < 0$, i.e. the trader's potential profits are decreasing in the dealer's belief of the probability that he is informed. Thus, the best potential case for acquiring information is when $\varphi = 0$, that is, when the market does not expect the trader to be an informed insider. As a consequence, the trader will never acquire information when $u(0) < 0$, which corresponds to

$$(1 - \alpha_0)\frac{v^H - v^L}{2} - c < 0 \Leftrightarrow c > \frac{v^H - v^L}{2} \equiv \bar{c}.$$

The higher the value volatility, as measure by the distance between the high and the low value, the greater the incentive to acquire information.

- (e). There exists $\underline{c}$ such that if $c < \underline{c}$, then the potential insider always acquires information (i.e., in equilibrium, $\varphi = 1$). Calculate $\underline{c}$.

Solution: Similarly to the previous question, the worst potential case for acquiring information is when $\varphi = 1$. Hence, the trader will always acquire information when $u(1) > 0$, which corresponds to

$$(1 - \alpha_1) \frac{v^H - v^L}{2} - c > 0 \Leftrightarrow c < (1 - \pi) \frac{v^H - v^L}{2} \equiv \underline{c}.$$

- (f). Suppose now that $c \in (\underline{c}, \bar{c})$. Find the mixed-strategy equilibrium in which the potential insider learns v with positive probability smaller than one. Describe the equilibrium fully.

Solution: Let the probability with which the trader acquires information (his strategy) be denoted by σ (recall that φ is the dealer's belief about σ). In equilibrium, we must have $\sigma^* = \varphi^* \in (0, 1)$, i.e. the beliefs must be correct (otherwise, the dealer would change his beliefs). Since the trader is playing a strictly mixed information acquisition strategy, we know from standard game theory considerations that he must then be indifferent between acquiring and not acquiring information: $u(\varphi^*) = 0$. Thus,

$$(1 - \alpha_{\varphi^*}) \frac{v^H - v^L}{2} - c = 0 \Rightarrow \sigma^* = \varphi^* = \frac{1 - \pi}{\pi} \left(\frac{v^H - v^L}{2c} - 1 \right). \quad (2)$$

The equilibrium is thus such that the dealer believes that the potential insider acquires information with probability φ^* given by (2) and quotes bid and ask prices $a_{\varphi^*}, b_{\varphi^*}$ as given by (1); and the potential insider learns v with probability σ^* given by (2), submits a buy order if learned that $v = v^H$, a sell order if $v = v^L$, and no order if learned nothing.

Inspecting the expression, we can make the following observations about equilibrium information acquisition in the mixed-strategy equilibrium. Higher π means smaller probability of acquiring information. This is because more adverse selection leads to a greater spread, and therefore less profitable trades. Higher fundamental value volatility ($v^H - v^L$) leads to higher probability of acquiring information. This is because information is worth more when values are volatile. Higher cost c leads to a smaller probability of acquiring information, since information is more expensive.

Problem 2

This problem considers an issue left open in Section 4.3 in the textbook, which explores inventory risk within the Kyle's model. At the outset of this section, it is assumed that there is no informed trading, i.e., $x = 0$ and $q = u$. Let us now relax this assumption and try to analyze the effects of dealer's risk aversion in Kyle's model in the presence of adverse selection.

So, consider a combination of the two versions of the Kyle model we have seen in class: we now have one informed trader, a pool of noise traders, and a representative competitive risk-averse dealer with mean-variance preferences, risk aversion ρ , initial asset holding z_0 , and initial cash holding c_0 . I.e., the dealer's utility from a [random] next period's wealth $w_{t+1} = z_{t+1}p_{t+1} + c_{t+1}$ is $U(w_{t+1}) = \mathbb{E}[w_{t+1}] - \frac{\rho}{2}\mathbb{V}(w_{t+1})$.

We are again looking for a linear equilibrium, where the insider submits an order $x = \beta v - x_0$, and the pricing schedule quoted by the dealer is $p = p_0 + \lambda q$. Now, there are four endogenous parameters, β, x_0, p_0, λ , with $\beta > 0$ and $\lambda > 0$.

- (a). Take for granted some arbitrary parameters β, x_0 . The dealer observes $q = x + u$. Use the result from Section 4.2 or lecture slides to find $\mathbb{E}[v|q]$ and $\mathbb{V}[v|q]$ as functions of β and the model parameters.

(Note that it is no longer the case that $q = \beta(v - \mu) + u$, but rather $q + x_0 - \beta\mu = \beta(v - \mu) + u$.)

Solution: The dealer wealth at the end of the trading day is

$$w = v(z_0 - q) + c_0 + pq$$

To simplify the problem, we will define a new variable that will have some desirable properties (you can solve the problem without doing this as well). Let

$$z = q + x_0 - \beta\mu. \tag{3}$$

Notice that z is normally distributed with $\mathbb{V}[z] = \mathbb{V}[q] = \beta^2\sigma_v^2 + \sigma_u^2$ and $\mathbb{C}[v, z] = \mathbb{C}[v, q] = \beta\sigma_v^2$. We can then use the lecture notes to get $\mathbb{E}[v|z] = \mu + \frac{\mathbb{C}[v, z]}{\mathbb{V}[z]}z$. Thus:

$$\mathbb{E}[v|z] = \mu + \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2}z = \mu + \alpha z,$$

where $\alpha = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2}$, and

$$\mathbb{V}[v|z] = \frac{\sigma_v^2\sigma_u^2}{\beta^2\sigma_v^2 + \sigma_u^2}.$$

Substituting (3) into these two expressions, we get

$$\mathbb{E}[v|q] = (\mu + \alpha x_0 - \alpha\beta\mu) + \alpha q,$$

and

$$\mathbb{V}[v|q] = \frac{\sigma_v^2 \sigma_u^2}{\beta^2 \sigma_v^2 + \sigma_u^2}.$$

- (b). The competitive dealer takes p as given. In equilibrium, $p = p_0 + \lambda q$, so the price reveals q – same logic as in Section 4.2.4. Rewrite $\mathbb{E}[v|q]$ and $\mathbb{V}[v|q]$ from (a) as functions of p instead of q , when $q = (p - p_0)/\lambda$. Let's call these functions $\mathbb{E}[v|p]$ and $\mathbb{V}[v|p]$.

Solution: Since we are conditioning on the same information, given that p reveals q but nothing else, the conditional expectation and variance do not change. We merely substitute $q = (p - p_0)/\lambda$ into the expressions from the previous question. This yields

$$\mathbb{E}[v|p] = (\mu + \alpha x_0 - \alpha\beta\mu) + \alpha \frac{p - p_0}{\lambda},$$

and

$$\mathbb{V}[v|p] = \frac{\sigma_v^2 \sigma_u^2}{\beta^2 \sigma_v^2 + \sigma_u^2}.$$

- (c). Calculate the dealer's asset supply curve and show that the amount supplied at price p is

$$y(p) = z_0 + \frac{p - \mathbb{E}[v|p]}{\rho \mathbb{V}[v|p]}$$

(A reminder: To solve this problem, instead of the zero-profit condition you should invoke another feature of competitive agents, namely price-taking. Use the following algorithm to obtain the dealer's price schedule: (1) fix some arbitrary price p ; (2) find supply amount $y(p)$ that maximizes the dealer's profit given this p ; (3) invert the supply function $y(p)$ to obtain a pricing schedule $p(q)$.)

Solution: Conditional on p , choose supply q to maximize utility

$$\begin{aligned} U(w|p) &= \mathbb{E}[w] - \frac{\rho}{2} \mathbb{V}[w] \\ &= \mathbb{E}[v|p](z_0 - q) + c_0 + pq - \frac{\rho}{2} \mathbb{V}[v|p](z_0 - q)^2. \end{aligned}$$

Take the FOC wrt. q

$$-\mathbb{E}[v|p] + p + \rho \mathbb{V}[v|p](z_0 - q) = 0.$$

Solving this for q gives the desired result.

- (d). When the market clears at price p , then $y(p) = q$. Can you see if there exists a $\lambda > 0$ and a number p_0 , such that market clearing fits with our conjecture that $p = p_0 + \lambda q$? I.e. try to solve the following equation

$$(y(p) =) \quad z_0 + \frac{p - \mathbb{E}[v|p]}{\rho \mathbb{V}[v|p]} = \frac{p - p_0}{\lambda} \quad (= q)$$

w.r.t. λ and p_0 . Note that they must be such that the equality holds for all p .

Solution: First, notice that

$$z_0 + \frac{p - \mathbb{E}[v|p]}{\rho \mathbb{V}[v|p]} = z_0 + \frac{p(\lambda - \alpha) + \alpha p_0 - \lambda(\mu + \alpha x_0 - \alpha \beta \mu)}{\lambda \rho \mathbb{V}[v|p]}.$$

We therefore have to solve $z_0 + \frac{p(\lambda - \alpha) + \alpha p_0 - \lambda(\mu + \alpha x_0 - \alpha \beta \mu)}{\lambda \rho \mathbb{V}[v|p]} = \frac{p - p_0}{\lambda}$, which can be rewritten as

$$\left[\frac{\lambda - \alpha}{\lambda \rho \mathbb{V}[v|p]} \right] p + \left[z_0 + \frac{\alpha p_0 - \lambda(\mu + \alpha x_0 - \alpha \beta \mu)}{\lambda \rho \mathbb{V}[v|p]} \right] = \left[\frac{1}{\lambda} \right] p + \left[-\frac{p_0}{\lambda} \right].$$

Matching first the coefficients for p , we get

$$\frac{\lambda - \alpha}{\lambda \rho \mathbb{V}[v|p]} = \frac{1}{\lambda},$$

which can be solved for

$$\lambda = \alpha + \rho \mathbb{V}[v|p]. \quad (4)$$

Matching the constants

$$z_0 + \frac{\alpha p_0 - \lambda(\mu + \alpha x_0 - \alpha \beta \mu)}{\lambda \rho \mathbb{V}[v|p]} = -\frac{p_0}{\lambda},$$

which can be solved for

$$\begin{aligned} p_0 &= \frac{\lambda}{\alpha + \rho \mathbb{V}[v|p]} ((\mu + \alpha x_0 - \alpha \beta \mu) - \rho \mathbb{V}[v|p] z_0) \\ &= (\mu + \alpha x_0 - \alpha \beta \mu) - \rho \mathbb{V}[v|p] z_0. \end{aligned} \quad (5)$$

Equations (4) and (5) give us the optimal strategy of the dealer, conditional on the strategy of the traders.

- (e). Go back to the informed trader's problem. Assuming that $p = p_0 + \lambda q$, argue that the insider chooses x to maximize $x(v - p_0 - \lambda x)$. Show that the relation is of the form $x = \beta v - x_0$ and try to relate parameters β, λ, x_0, p_0 to each other. Is there a solution for all four parameters? If the algebra is impossible, try a numerical solution.

Solution: The trader is a risk-neutral expected-utility maximizer. Conditional on the dealer's strategy $p = p_0 + \lambda q = p_0 + \lambda(x + u)$. Thus, he will maximize $\mathbb{E}[x(v - p)|x] = x(v - p_0 - \lambda x)$. This quadratic can be solved to give $x = (v - p_0)/(2\lambda)$, so in order for our conjecture about the strategy to be right, we must have

$$\beta = \frac{1}{2\lambda} \quad (6)$$

and

$$x_0 = \beta p_0. \quad (7)$$

Using this latter condition in (5) we get

$$p_0 = (\mu + \alpha\beta p_0 - \alpha\beta\mu) - \rho\mathbb{V}[v|p]z_0, \quad (8)$$

which is solved for

$$p_0 = \mu - \frac{\rho\mathbb{V}[v|p]z_0}{1 - \alpha\beta}. \quad (9)$$

We also have two conditions on λ , one coming from the dealer's optimality (4) and one from the trader's optimality (6). Use them together to get

$$\lambda = \alpha + \rho\mathbb{V}[v|p] = \frac{\beta\sigma_v^2 + \rho\sigma_u^2\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_u^2} = \frac{2\lambda\sigma_v^2 + 4\lambda^2\rho\sigma_u^2\sigma_v^2}{\sigma_v^2 + 4\lambda^2\sigma_u^2}.$$

This is a quadratic equation in λ with a unique positive solution:

$$\lambda = \frac{\rho\sigma_v^2 + \sqrt{\rho^2\sigma_v^4 + \sigma_v^2/\sigma_u^2}}{2}. \quad (10)$$

Plugging into (6) this yields

$$\beta = \frac{1}{\rho\sigma_v^2 + \sqrt{\rho^2\sigma_v^4 + \sigma_v^2/\sigma_u^2}}. \quad (11)$$

Together, equations (7), (9), (10), and (11) pin down the four parameters.

- (f). In case $\rho = 0$, the dealer is risk neutral, so we are back to the case of pure adverse selection. In this limit case, do you get back the equilibrium found in Section 4.2/in class? How do you find that changes in ρ affect market depth in equilibrium?

Solution: When $\rho = 0$, this becomes the book's solution for λ , and we also recover $p_0 = \mu$. In general, λ is increasing in risk aversion ρ .

Problem 3

One of the news articles assigned earlier in the course argued that corporate bond markets are significantly less liquid than stock markets. This is partly due to a more opaque trading tradition: instead of the dealers posting their quotes openly, the bond markets operate via Requests For Quotes (RFQ). In particular, an interested trader must contact the dealer with an RFQ, and then the dealer would respond with a quote. This process is somewhat time-consuming for traders, and thus costly. However, the costs of this opaqueness can be far greater than just these time costs.

Your goal is to build a model of search costs and to examine how they affect market outcomes: prices, spread and/or market depth, volume of trade if applicable. In particular, take any dealer model we considered (I suggest Glosten-Milgrom, but Kyle can probably work as well). Assume that instead of the dealers posting quotes openly, any arriving trader must approach dealers individually, paying cost c per dealer to learn that dealer's quotes. Solve your model as best you can and answer the following questions based on your results:

- How do prices (and thus liquidity/depth) in your model compare to the baseline model, in which dealers post quotes openly?
- How do prices depend on c ?
- How do prices depend on the number of dealers in the market?
- In this situation, could a dealer profit by announcing quotes publicly?

FINANCIAL MARKETS MICROSTRUCTURE: PROBLEM SET 2

Egor Starkov

Københavns Universitet, Spring 2023

Problem 1

Trading at exchanges sometimes breaks down due to technical problems. E.g., The Economist reported on August 31, 2013: “On August 26th trading on Eurex, the main German derivatives exchange, opened as usual; 20 minutes later it shut down for about an hour. Four days earlier the shares of every company listed on NASDAQ, an American stock exchange, ceased trading for three hours”.

What are the implications of such breakdowns for liquidity risk? How do they affect asset prices? How does competition among exchanges affect breakdown frequency?

Problem 2 [Ch.7, ex.4]

This problem deals with competition between limit order markets with uniformly distributed market orders. Consider the model of section 7.4.2 (“Glosten model with fragmented market” from Lectures) and assume that the size of the market order, x , has a uniform distribution $[0, \bar{X}]$. That is, $F(x) = x/\bar{X}$. We denote by $q_{jk}(\gamma)$ the cumulative depth posted at the ask price $A_k = \mu + k\Delta$ in market $j \in \{I, E\}$ when the fraction of investors submitting market orders in both markets I and E is γ , and by c_j be the submission cost in market j .

- (a). Assume that $2c_I \leq \Delta$ and that $\gamma = 0$. Show that the equilibrium cumulative depth at price A_1 is¹

$$q_{I1}(0) = \bar{X} \left(1 - \frac{2c_I}{\Delta} \right).$$

Hint: Use (7.13).

- (b). Now suppose that γ is high enough and that the other parameters are such that $q_{I1}(\gamma) > 0$, $q_{E1}(\gamma) > 0$, but $q_{I1}(\gamma) + q_{E1}(\gamma) < \bar{X}$. Compute $q_{I1}(\gamma)$ and $q_{E1}(\gamma)$ as a function of γ .

¹The A_1 is misprinted as A_k in the problem text in the book.

Deduce further from the result that the conditions $q_{I1}(\gamma) > 0$ and $q_{E1}(\gamma) > 0$ are satisfied if $\frac{4c_I}{\Delta(2-\gamma)+2c_E} < 1$ and $\frac{4c_E}{\Delta+2c_I} < \gamma$. Moreover, deduce that the condition $q_{I1}(\gamma) + q_{E1}(\gamma) < \bar{X}$ is satisfied if $4(\gamma c_I + (2 - \gamma)c_E) > (2 - \gamma)\gamma\Delta$.

Hint: You need the equations (7.11), (7.12), (7.14) and (7.15) to get the system of equations that pins down $q_I \equiv q_{I1}(\gamma)$ and $q_E \equiv q_{E1}(\gamma)$. You can then either solve the algebra by muscle or use some computer algebra system.

- (c). Deduce from question (b) that the two markets can coexist even if their order submission costs differ and $\gamma = 1$.

Hint: first think about the case where $c_I = c_E = c$. This will give you an interval for Δ in which the markets can coexist. Then argue that there exist some $c_I \neq c_E$ such that this is true as well.

- (d). Why does the cumulative depth at price A_1 in one market decrease with the order submission cost in this market but increase with the cost in the competing market?
- (e). Consider the case $\gamma = 1$ and suppose that $4(c_I + c_E) < \Delta$ and $4c_I < \Delta$. Compute $q_{I1}(1)$ and $q_{E1}(1)$.

Hint: Notice that now we are violating one of the conditions given in (b). What effect does this have on $F(q_I + q_E)$? Take account of this when writing up (7.14) and (7.15).

- (f). Under the assumptions in question (e), what is the number of shares offered at price $A_k > A_1$? Is the result different when $\gamma = 0$?

Hint: Look at the values of q_{I1} in the two cases.

Problem 3

MiFID II, the recent European financial market regulation, requires that “firms shall disclose to the client information on the payment or benefit concerned, in a manner that is comprehensive, accurate and understandable” (in accordance with the second paragraph of Article 24(9) of MiFID II). Evaluate the possible effects of this regulation.

In particular, suppose that some asset is traded at multiple exchanges. One of the exchanges offers one of the banks a payment for directing order flow originating from bank’s clients towards this exchange. This relates either to all order flow, or to order flow from retail investors. How would the bank’s obligation to be transparent about this fee towards its clients affect market outcomes?

FINANCIAL MARKETS MICROSTRUCTURE: PROBLEM SET 2

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Problem 1

Trading at exchanges sometimes breaks down due to technical problems. E.g., The Economist reported on August 31, 2013: “On August 26th trading on Eurex, the main German derivatives exchange, opened as usual; 20 minutes later it shut down for about an hour. Four days earlier the shares of every company listed on NASDAQ, an American stock exchange, ceased trading for three hours”.

What are the implications of such breakdowns for liquidity risk? How do they affect asset prices? How does competition among exchanges affect breakdown frequency?

Solution: Think about this in terms of the Liquidity CAPM model we saw in class. It relates the nominal return R_j and (il)liquidity s_j of asset j to market return R_M and market liquidity s_M as

$$\begin{aligned}\mathbb{E}[R_j - s_j] &= r_f + \lambda_M \beta_j, \\ \text{where } \lambda_M &= \mathbb{E}[R_M - s_M] - r_f \\ \text{and } \beta_j &= \frac{\mathbb{C}(R_j - s_j, R_M - s_M)}{\mathbb{V}(R_M - s_M)}.\end{aligned}$$

It is most natural to think of such market breakdowns as temporarily reducing liquidity to zero ($s = \infty$) – i.e., increasing liquidity risk. You can take one of two interpretations here: either (1) most traders multi-home to different exchanges, so when one exchange breaks down, it affects s_j for assets j traded on that exchange only, or (2) most traders trade primarily on one exchange (not necessarily the same), so for them exchange breakdowns affect market liquidity s_M and liquidity s_j of all individual assets.

In the former interpretation, adding a small risk of $s_j = \infty$ implies that R_j should increase correspondingly – investors will require a larger nominal return on asset j if it suffers from larger liquidity risk. (This implicitly assumes that the breakdown risk is uncorrelated with R_j, R_M, s_M ,

which is reasonable – contrary, for example, to the case of trading halts, when the trade in a given asset is stopped after when short-term R_j become extreme.)

In the latter interpretation, the final effect is ambiguous, since both asset j and market as a whole become less appealing to invest in. Assuming exogenous R_M , it is not clear whether R_j would increase or decrease. You are welcome to work this out with specific assumptions on distributions.

To think about competition between exchanges, use the arguments we discussed when talking about trading costs in fragmented markets. To the extent that exchanges trade in the same stocks, they will be competing on quality of service in the same way as they do in trading fees. Competition may also induce exchanges to invest in better equipment to prevent trade breakdowns. Further, if one exchange breaks down, you can probably trade on the other – i.e., the effective liquidity risk that traders face is lower under competition even in the absence of such investments. Thus, competition between exchanges should lower the liquidity premium.

Problem 2 [Ch.7, ex.4]

This problem deals with competition between limit order markets with uniformly distributed market orders. Consider the model of section 7.4.2 (“Glosten model with fragmented market” from Lectures) and assume that the size of the market order, x , has a uniform distribution $[0, \bar{X}]$. That is, $F(x) = x/\bar{X}$. We denote by $q_{jk}(\gamma)$ the cumulative depth posted at the ask price $A_k = \mu + k\Delta$ in market $j \in \{I, E\}$ when the fraction of investors submitting market orders in both markets I and E is γ , and by c_j be the submission cost in market j .

Solution: General comment. In this question, we are investigating the exact same model as in 7.4.2, but just making a specific assumption about $F(\cdot)$. When you get questions like these that build directly on a model in the book, the best way to proceed is to follow closely the steps of the book (or the slides). In the hints I posted, I gave you relevant equations. Thus, it is a question of updating these equations and then using the results to answer the questions.

- (a). Assume that $2c_I \leq \Delta$ and that $\gamma = 0$. Show that the equilibrium cumulative depth at price A_1 is¹

$$q_{I1}(0) = \bar{X} \left(1 - \frac{2c_I}{\Delta} \right).$$

Hint: Use (7.13).

¹The A_1 is misprinted as A_k in the problem text in the book.

Solution: With $\gamma = 0$ there is effectively only one market, I . Let $q_I \equiv q_{I1}(0)$, which is the volume supplied at A_1 . This is given by equation (7.13). (Notice parallels to (6.6).) Here, the display cost is c_I . The assumption $2c_I \leq \Delta$ assures that $q_I \leq \bar{X}$, and hence $1 - F(q_I) \leq 1$. I.e. the assumption assures that the probability is well-defined. Thus, (7.13) yields

$$\frac{1}{2} \left(1 - \frac{q_I}{\bar{X}}\right) (A_1 - \mu) = c_I.$$

Solve for q_I and substitute $\Delta = A_1 - \mu$ to get the result.

- (b). Now suppose that γ is high enough and that the other parameters are such that $q_{I1}(\gamma) > 0$, $q_{E1}(\gamma) > 0$, but $q_{I1}(\gamma) + q_{E1}(\gamma) < \bar{X}$. Compute $q_{I1}(\gamma)$ and $q_{E1}(\gamma)$ as a function of γ . Deduce further from the result that the conditions $q_{I1}(\gamma) > 0$ and $q_{E1}(\gamma) > 0$ are satisfied if $\frac{4c_I}{\Delta(2-\gamma)+2c_E} < 1$ and $\frac{4c_E}{\Delta+2c_I} < \gamma$. Moreover, deduce that the condition $q_{I1}(\gamma) + q_{E1}(\gamma) < \bar{X}$ is satisfied if $4(\gamma c_I + (2 - \gamma)c_E) > (2 - \gamma)\gamma\Delta$.

Hint: You need the equations (7.11), (7.12), (7.14) and (7.15) to get the system of equations that pins down $q_I \equiv q_{I1}(\gamma)$ and $q_E \equiv q_{E1}(\gamma)$. You can then either solve the algebra by muscle or use some computer algebra system.

Solution: We use equations (7.14) and (7.15) to describe the equilibrium, and refer to (7.11) and (7.12) to get the execution probabilities. In particular, (7.14) becomes

$$\frac{1}{2} \left[\left(1 - \gamma + \frac{\gamma}{2}\right) \left(1 - \frac{q_I}{\bar{X}}\right) + \frac{\gamma}{2} \left(1 - \frac{q_I}{\bar{X}} - \frac{q_E}{\bar{X}}\right) \right] = \frac{c_I}{\Delta}. \quad (1)$$

This is linear in q_I and q_E , thanks to the uniform distribution. Notice that the assumption that $q_{I1}(\gamma) + q_{E1}(\gamma) < \bar{X}$ implies that $1 - F(q_{I1}(\gamma) + q_{E1}(\gamma)) > 0$, that is to say, since the supply at the first tick A_1 is smaller than the largest possible order ($\bar{X}$), then there is always a probability that the marginal order at A_1 executes, even if the market is the second to get served. If $q_{I1}(\gamma) + q_{E1}(\gamma) \geq \bar{X}$, then the marginal order at A_1 is only executed if the market is the first to get served (i.e. $1 - F(q_{I1}(\gamma) + q_{E1}(\gamma)) = 0$ and the second term in the square brackets in (1) drops out).

Rewrite (1) to get

$$\bar{X} \left(1 - \frac{2c_I}{\Delta}\right) = q_I + \frac{\gamma}{2} q_E. \quad (2)$$

You can already from this deduce that q_I will be lower than in (a). We now repeat the exercise with E 's equilibrium condition. Now, (7.15) becomes

$$\frac{\gamma}{4} \left[\left(1 - \frac{q_E}{\bar{X}}\right) + \left(1 - \frac{q_I}{\bar{X}} - \frac{q_E}{\bar{X}}\right) \right] = \frac{c_E}{\Delta}, \quad (3)$$

rewritten as

$$\bar{X} \left(1 - \frac{2c_E}{\gamma\Delta} \right) = \frac{1}{2}q_I + q_E. \quad (4)$$

This leaves us with two conditions, (2) and (4), which have a unique solution for any $\gamma \in (0, 1)$. This solution is given by

$$q_I = \frac{2\bar{X}}{4-\gamma} \left(2 - \gamma + \frac{2c_E - 4c_I}{\Delta} \right) \text{ and } q_E = \frac{2\bar{X}}{4-\gamma} \left(1 + \frac{2\gamma c_I - 4c_E}{\gamma\Delta} \right).$$

We can then check that the necessary conditions given in the question are true: The conditions for $q_I > 0$ and $q_E > 0$ are straightforward. The third condition, $q_I + q_E < \bar{X}$, is equivalent to

$$\frac{2}{4-\gamma} \left(3 - \gamma + \frac{(2\gamma - 4)c_E - 3\gamma c_I}{\gamma\Delta} \right) < 1,$$

which can be reduced to

$$(4\gamma - 8)c_E - 4\gamma c_I < (\gamma - 2)\gamma\Delta.$$

- (c). Deduce from question (b) that the two markets can coexist even if their order submission costs differ and $\gamma = 1$.

Hint: first think about the case where $c_I = c_E = c$. This will give you an interval for Δ in which the markets can coexist. Then argue that there exist some $c_I \neq c_E$ such that this is true as well.

Solution: When $\gamma = 1$, market I no longer has 'priority'. The three conditions from (b) correspond to $4c_I < \Delta + 2c_E$, $4c_E < \Delta + 2c_I$, and $4(c_I + c_E) > \Delta$. If $c_I = c_E = c$ they are all satisfied when $2c < \Delta < 8c$. If Δ is an interior point of the interval $(2c, 8c)$, continuity allows us to vary c_E and c_I around c whilst satisfying all the conditions.

NOTE: the third condition ($q_I + q_E > \bar{X}$) is per se not necessary for such coexistence, but it is necessary for the first two conditions to look the way they do. As we shall see in part (e), if the third condition does not hold, the conditions for $q_I > 0$ and $q_E > 0$ look differently.

- (d). Why does the cumulative depth at price A_1 in one market decrease with the order submission cost in this market but increase with the cost in the competing market?

Solution: In equilibrium, the expected profit from the marginal limit order must be zero in both markets. Thus, the higher the cost of posting orders in a market, the higher the execution probability must be to 'compensate' traders. Higher execution probability requires fewer orders.

Furthermore, the two markets are connected in that whenever an order is coming in, even if one market initially 'wins' the order, the other market will fill the remaining part of the order if the first market is not deep enough. Therefore, increasing the posting cost of the first market makes it shallower. The shallower the first market is, the more attractive the second market is, because the execution probability is higher. The equilibrium response to this is to post more limit orders on the second market.

- (e). Consider the case $\gamma = 1$ and suppose that $4(c_I + c_E) < \Delta$ and $4c_I < \Delta$. Compute $q_{I1}(1)$ and $q_{E1}(1)$.

Hint: Notice that now we are violating one of the conditions given in (b). What effect does this have on $F(q_I + q_E)$? Take account of this when writing up (7.14) and (7.15).

Solution: Now the condition $q_I + q_E < \bar{X}$ is no longer satisfied, implying that $1 - F(q_I + q_E) = 0$. I.e., the marginal limit order never executes if the competing market gets the first part of the order. Substituting this and $\gamma = 1$ into (7.14) and (7.15) we get

$$\frac{1}{2} \left[\frac{1}{2} \left(1 - \frac{q_I}{\bar{X}} \right) \right] = \frac{c_I}{\Delta} \text{ and } \frac{1}{4} \left[\left(1 - \frac{q_E}{\bar{X}} \right) \right] = \frac{c_E}{\Delta}.$$

Thus,

$$q_I = \bar{X} \left(1 - \frac{4c_I}{\Delta} \right) \text{ and } q_E = \bar{X} \left(1 - \frac{4c_E}{\Delta} \right).$$

Since the execution probability of the marginal order is independent of the competing market, the posting cost of the competing platform now has no effect on market depth. In fact, the market depth is almost the same as in the 'monopoly' case in (a), except it takes account for the fact that traders initially make a choice of platform. Notice however that in general, the expected profits of limit orders are not independent across the two platforms: the execution probability of earlier orders (i.e. those that are not marginal) may still depend on the competing market.

- (f). Under the assumptions in question (e), what is the number of shares offered at price $A_k > A_1$? Is the result different when $\gamma = 0$?

Hint: Look at the values of q_{I1} in the two cases.

Solution: In (e), the assumptions made imply $q_I + q_E \geq \bar{X}$. Recall that here, q_I and q_E are shorthand for $q_{I1}(1)$ and $q_{E1}(1)$, the orders posted at ask price A_1 . Thus the total number of orders posted at A_1 is $q_{I1}(1) + q_{E1}(1) \geq \bar{X}$, implying that even the largest possible order ($\bar{X}$) will be filled at price A_1 . So there can be no (profitable) orders at higher ask prices.

If $\gamma = 0$, then our result from (a) tells us that $q_{I1} < \bar{X}$, and thus we can still have profitable orders at higher ask prices.

Problem 3

MiFID II, the recent European financial market regulation, requires that “firms shall disclose to the client information on the payment or benefit concerned, in a manner that is comprehensive, accurate and understandable” (in accordance with the second paragraph of Article 24(9) of MiFID II). Evaluate the possible effects of this regulation.

In particular, suppose that some asset is traded at multiple exchanges. One of the exchanges offers one of the banks a payment for directing order flow originating from bank’s clients towards this exchange. This relates either to all order flow, or to order flow from retail investors. How would the bank’s obligation to be transparent about this fee towards its clients affect market outcomes?

Solution: There are obviously many ways in which you can answer this question. I mention here some angles, from which you can look at this issue (but note that this is *not* a benchmark solution). To impose some structure, let us set up an informal model with some explicit timing:

1. investor selects which bank to use as a broker;
2. bank may (has to under the regulation) disclose to its clients the forwarding fee it receives from the exchange;
3. liquidity providers (dealers or limit traders) select into markets and set quotes without knowing any of the above;
4. investor submits a market order (he may or may not be aware of market quotes when doing so);
5. the bank chooses which exchange to forward this market order to;
6. the bank and the investor split gains from trade.

Now proceed by backward induction and look at what can change at each step.

- At step 6 if the investor knows that the bank has received a rebate from the exchange, he realizes that the gains from trade was higher than if the bank concealed this information.

The investor can then bargain more aggressively and obtain better terms, while the bank's profit-per-trade declines on paid-to-forward orders.

- At step 5 the bank will then be more reluctant (under transparency) to forward the orders to exchanges that pay for it. Therefore, payment for order flow will be less effective as an instrument for attracting trading volume, which will make it harder for new trading platforms to enter the market and may thus give more market power to existing platforms, potentially increasing order processing fees.
- At step 4 investors will be more eager to trade due to higher anticipated profits.
- At step 3 more liquidity will be supplied (if we assume that investors from step 4 are mostly uninformed) in markets that pay for order flow.
- At step 2 in the absence of regulation the bank has a choice of whether to disclose forwarding fees to its clients. The trade-off is non-trivial, since disclosure reduces profit-per-trade (step 6) but increases the clients' desire to trade (step 4). In principle, the bank may disclose its fees even in the absence of regulation – in which case it has no effect. A formal model could tell you whether, e.g., voluntary disclosure is more likely from banks with high or low bargaining power.
- At step 1 the effect is not immediate. If you think that banks are heterogeneous and some were more likely than others to reveal forwarding fees to its clients in the absence of regulation, then regulation would likely shift market demand towards banks that were known to be secretive before the regulation. Welfare implications of this shift are not obvious without a more careful analysis.

As you can see, even a very surface-level analysis identifies a lot of potential effects of such regulation, giving arguments to both supporters and opponents of such regulation. A proper welfare analysis, however, would require a more careful approach, which at the ex ante stage (before the regulation is implemented) is only really possible via teoretical modeling.

Financial Markets Microstructure

Exercises after L1

Egor Starkov

Københavns Universitet
Spring 2025

This lecture:

1 Let's talk about GameStop

2 Nikola who?

3 Problems from Lecture 1

- 1 What kind of traders are involved on both sides? Based on that fact: why has this story made the news?
- 2 Would now be a good time to buy GameStop stock? Why/why not?
- 3 Robinhood (trading app) halted trading in GameStop stocks at some point. Is there a scope for some regulatory intervention to prevent this kind of exclusion of retail traders?

This lecture:

1 Let's talk about GameStop

2 Nikola who?

3 Problems from Lecture 1

Is Hindenburg Research an insider? Is it an informed trader?

This lecture:

1 Let's talk about GameStop

2 Nikola who?

3 Problems from Lecture 1

Lecture 1

From Lecture 1: problems 1-3 from FPR ch.1 (pp. 44-45)

Orders

We have a collection of limit orders and market orders

- Limit buy orders:

- 100 shares at bid price 3
- 200 shares at bid price 4
- 200 shares at bid price 3.5
- 500 shares at bid price 2.5

- Limit sell orders:

- 500 shares at ask price 5
- 600 shares at ask price 3
- 500 shares at ask price 4

- Market buy order: 500

- Market sell order: 200

Call auction

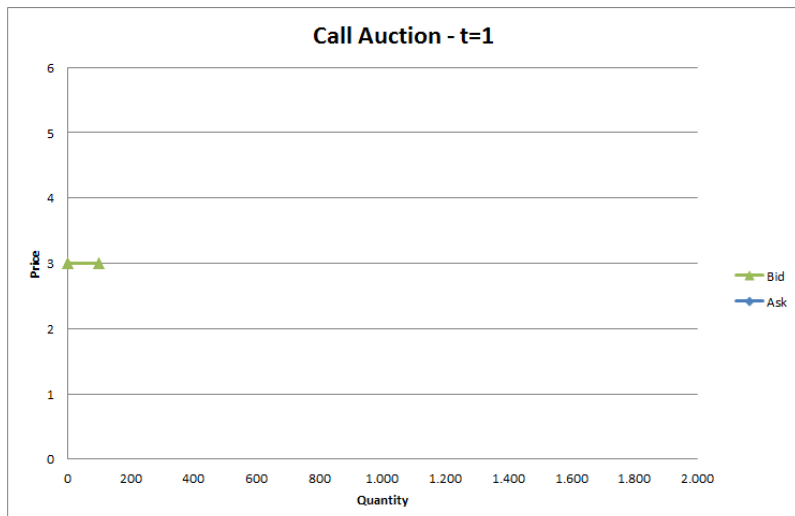
Recall idea of a call auction:

- Orders are collected (in a 'batch') and cleared at a certain frequency
- The price is chosen so as to maximize the number of executed orders
- **Uniform price**: all orders in a batch trades at same price

Let's build the auction step by step

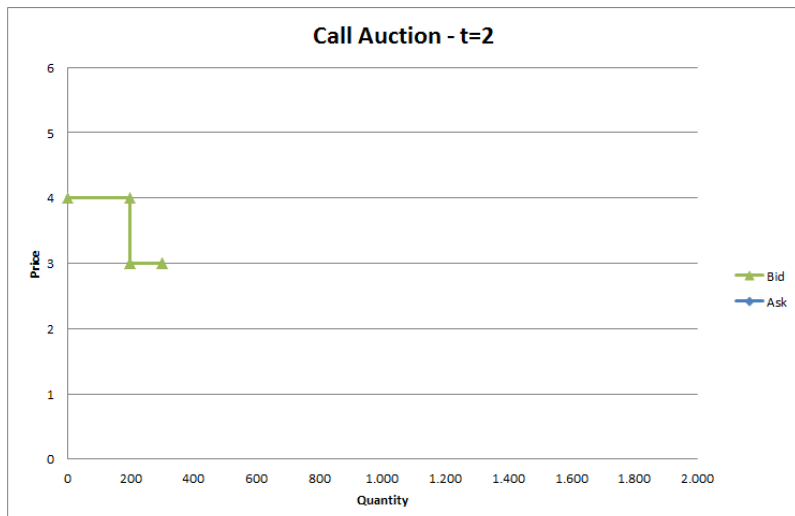
Call auction

Limit buy order of 100 shares at bid price 3



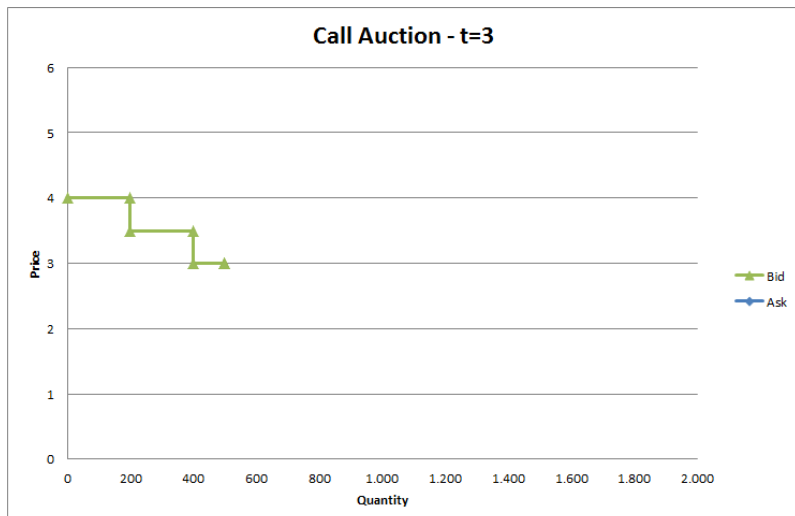
Call auction

Limit buy order of 200 shares at bid price 4



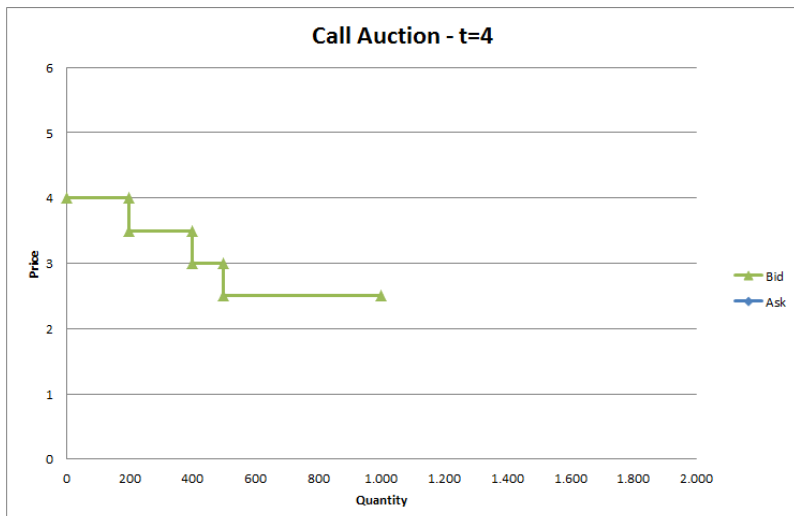
Call auction

Limit buy order of 200 shares at bid price 3.5



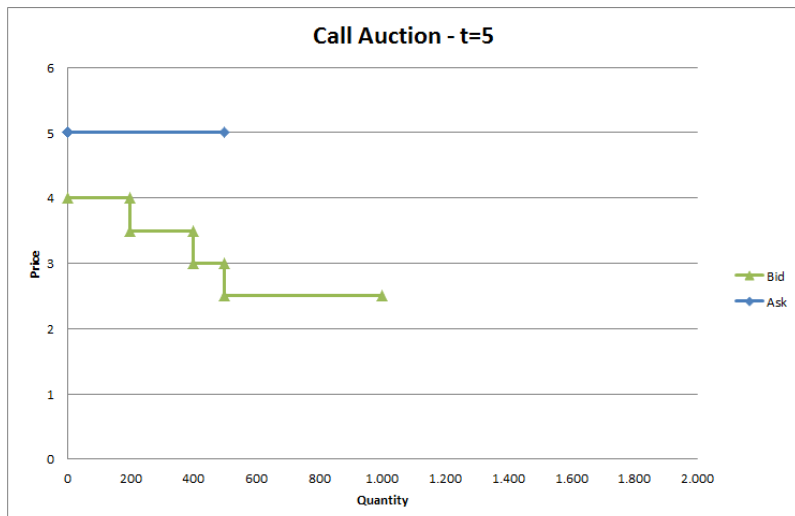
Call auction

Limit buy order of 500 shares at bid price 2.5



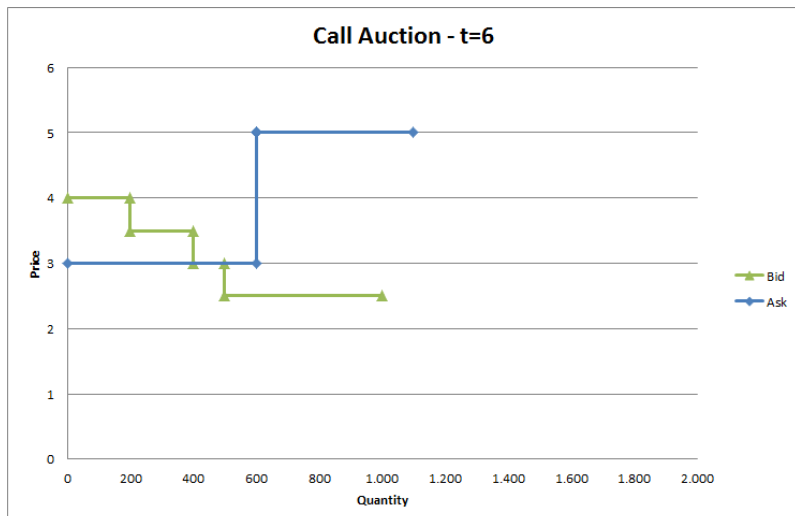
Call auction

Limit sell order of 500 shares at ask price 5



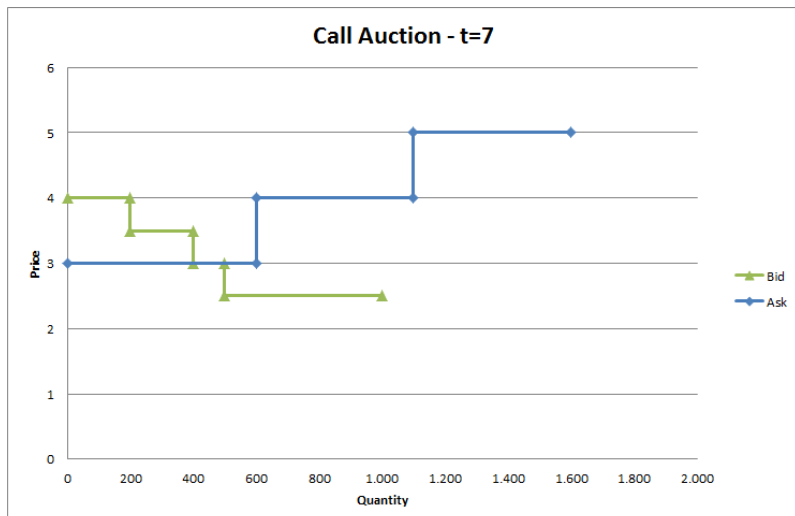
Call auction

Limit sell order of 600 shares at ask price 3



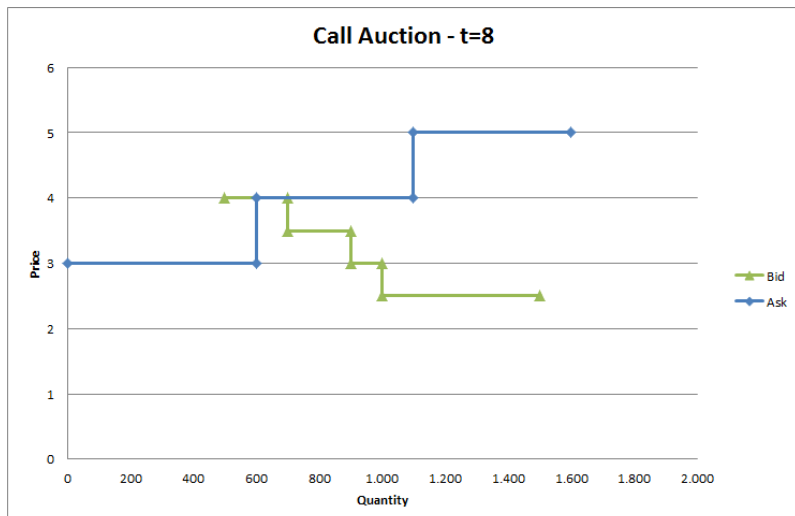
Call auction

Limit sell order of 500 shares at ask price 4



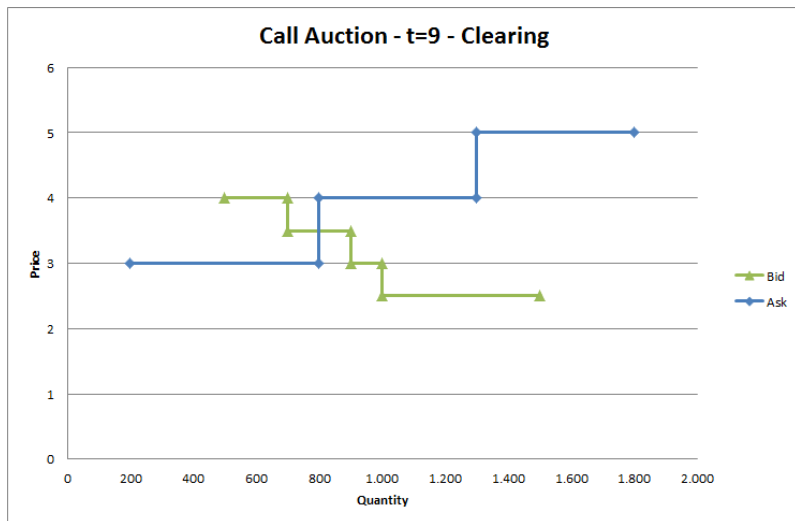
Call auction

Market buy order of 500 shares



Call auction

Market sell order of 200 shares



Call auction

Market clears at price 3.5, with 800 units being sold Filled buy orders:

- The market order is filled completely
- The limit order for 200 shares at price 4 is filled completely
- The limit order for 200 shares at price 3.5 is filled partially (100 shares)

Filled sell orders:

- The market order is filled completely
- The limit order for 600 shares at price 3 is filled completely

Thus: *Excess demand* at the clearing price

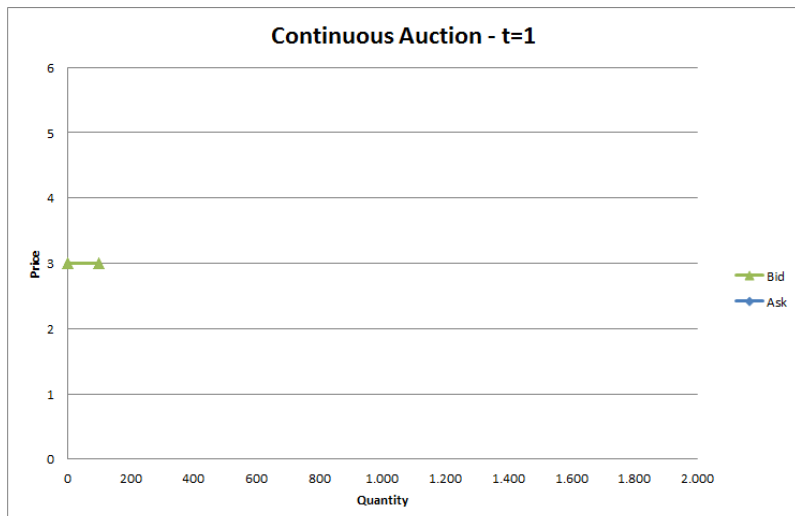
Continuous auction

Recall idea of a continuous auction:

- Some traders post limit orders (provide liquidity)
- Some traders post market orders (take liquidity)
- Trades occur either through
 - 1 a limit order 'match' (somebody posts a bid price above the lowest ask price), or
 - 2 through market orders
- **Discriminatory price:** depends on how your order is matched

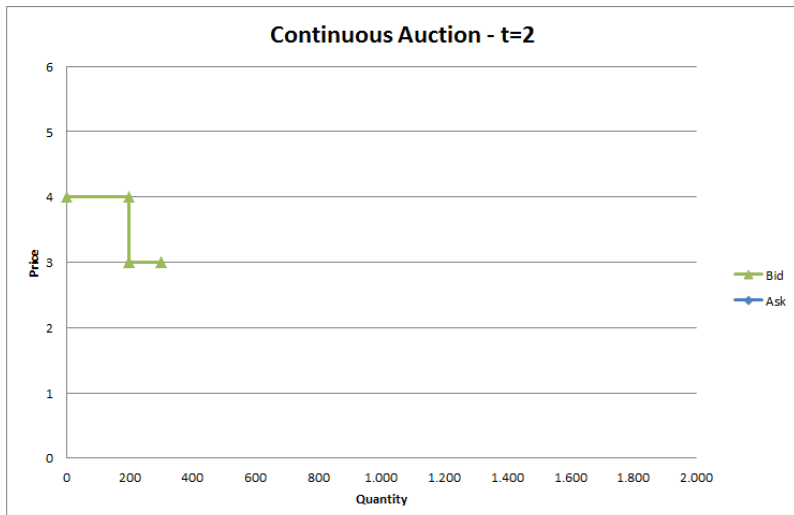
Continuous auction

Limit buy order of 100 shares at bid price 3



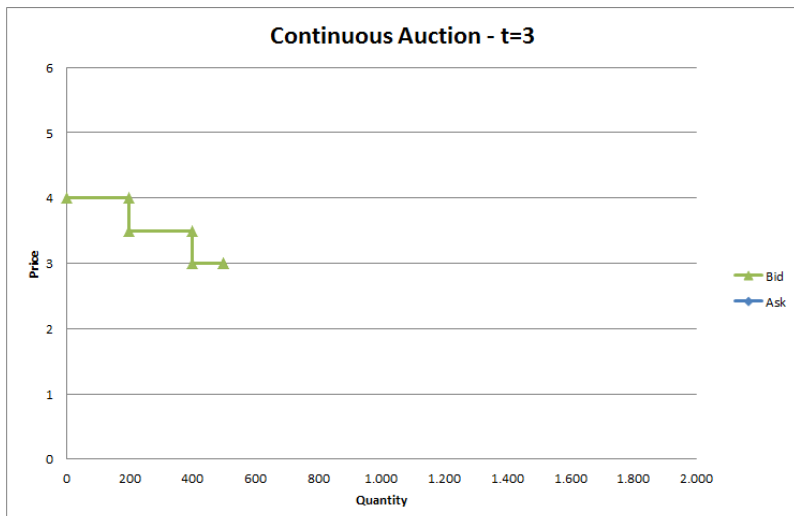
Continuous auction

Limit buy order of 200 shares at bid price 4



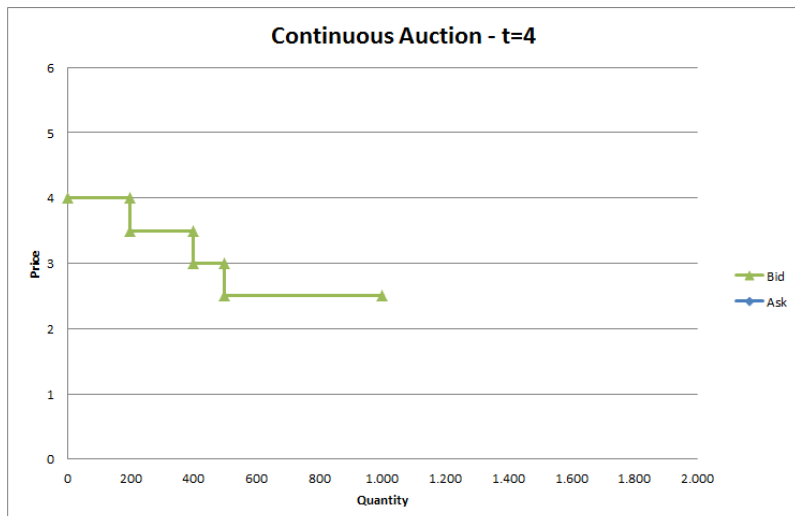
Continuous auction

Limit buy order of 200 shares at bid price 3.5



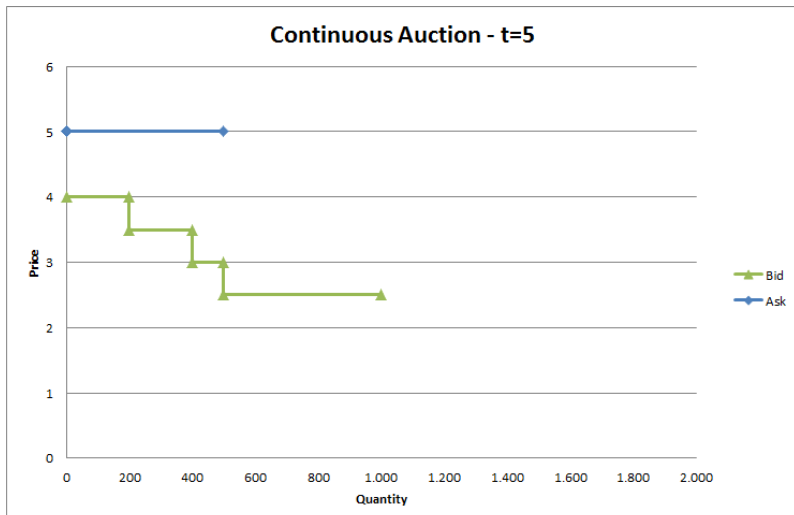
Continuous auction

Limit buy order of 500 shares at bid price 2.5



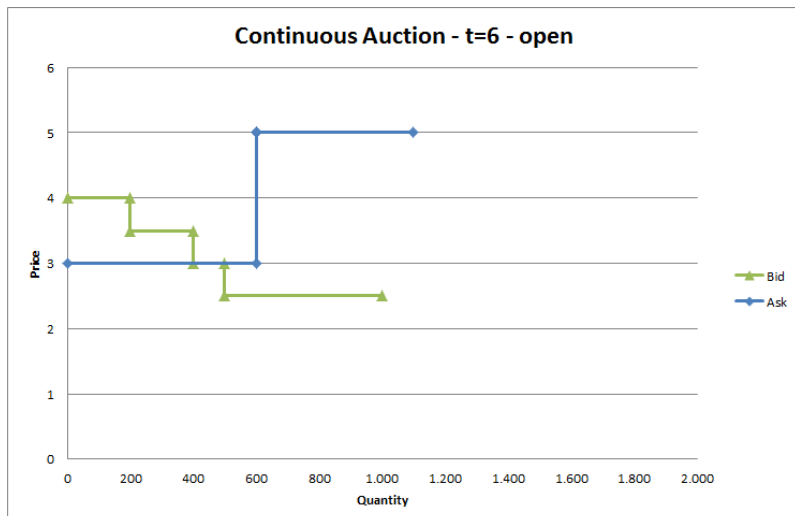
Continuous auction

Limit sell order of 500 shares at ask price 5. Spread = 1



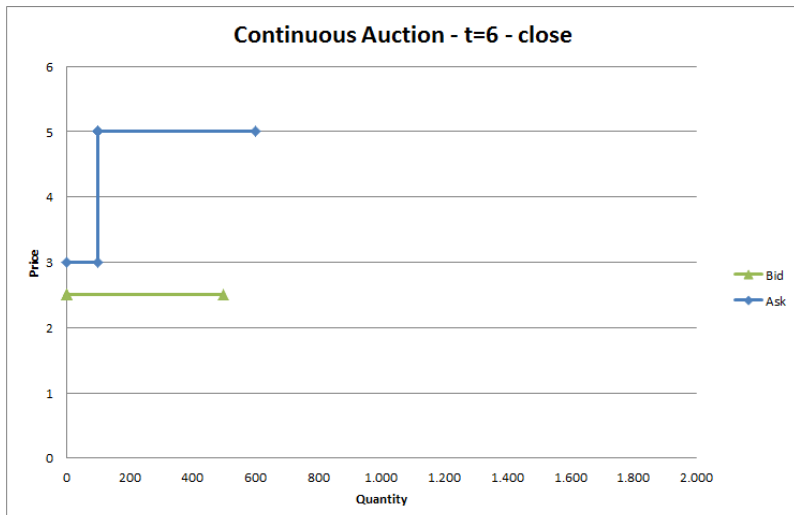
Continuous auction

Limit sell order of 600 shares at ask price 3



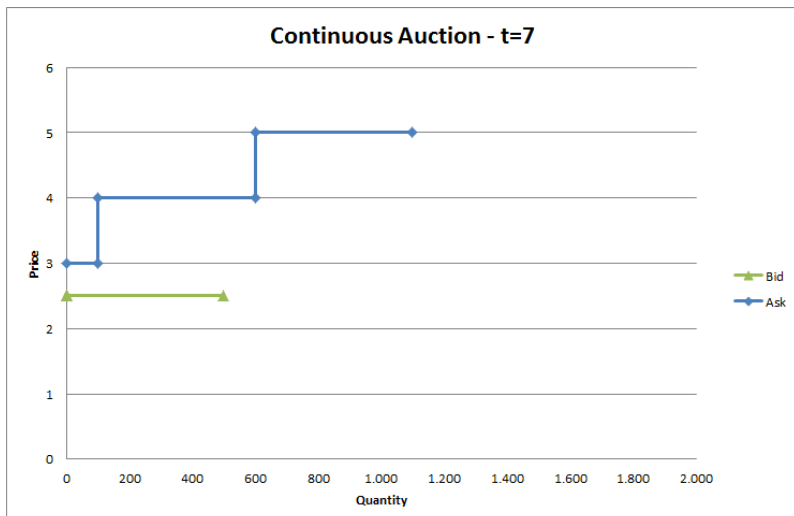
Continuous auction

Seller executes 200 shares at price 4, 200 at price 3.5, and 100 at price 3. Spread = 0.5



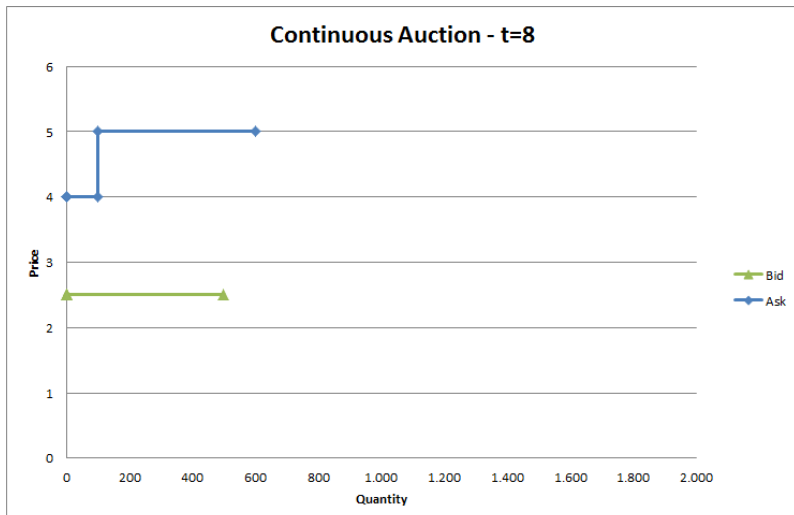
Continuous auction

Limit sell order of 500 shares at ask price 4. Spread = 0.5



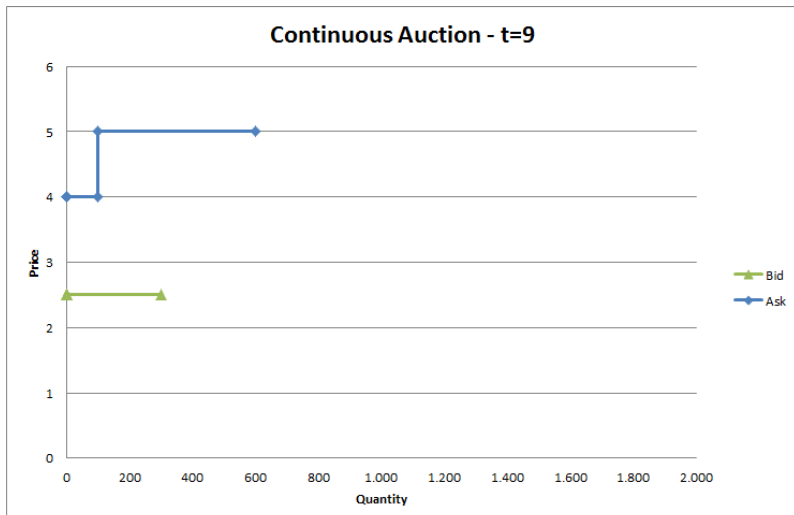
Continuous auction

Market buy for 500 shares – buys 100 shares at price 3, and 400 at 4. Spread = 1.5



Continuous auction

Market sell order of 200 shares. Executes against 200 shares at price 2.5. Spread = 1.5



Efficiency

Suppose limit order traders' valuation is equal to their price, market order buyers have valuation 10, market order sellers have valuation 0

Trader		Valuation	Desired trade	Actual trade	
				Call	Continuous
T1	B	3	100	0	100
T2	B	4	200	200	200
T3	B	3,5	200	100	200
T4	B	2,5	500	0	200
T5	S	5	-500	0	0
T6	S	3	-600	-600	-600
T7	S	4	-500	0	-400
T8	B	10	500	500	500
T9	S	0	-200	-200	-200

Welfare

We can also do a welfare calculation, using the post market holdings of each type

Trader		Valuation	Position (pre)	Desired trade	Post-market position	
					Call	Continuous
T1	B	3	0	100	0	100
T2	B	4	0	200	200	200
T3	B	3,5	0	200	100	200
T4	B	2,5	0	500	0	200
T5	S	5	0	-500	0	0
T6	S	3	0	-600	-600	-600
T7	S	4	0	-500	0	-400
T8	B	10	0	500	500	500
T9	S	0	0	-200	-200	-200
		Welfare	0		4350	3900

Note there's too much trade in cont auction: T1 buys 100 (valuation 3), T4 buys 200 (val 2.5), T7 sells 400 (val 4) – if these three met, they would reverse trades.

Clarifications

- To clarify: in the previous slide, I assumed that every trader starts with zero position.
- Welfare calculation just multiplies every trader's position (before or after trade) by their valuation for the asset, and sums over traders.

Conclusions

- The two formats result in different sets of trades.
- **Call auction** is **welfare-maximizing** given the orders, is also **Pareto-optimal** – there are no profitable trades left to conduct after market closes
- **Continuous auction** acts based on limited info (about future orders), results in **too much trade and less welfare**.
 - Some of the traders might regret their trades after seeing other trades in the market; there is scope for further welfare-improving trades that reverse some of those that happened.
 - But continuous auction is faster, gives more trading opportunities so may attract more traders

Financial Markets Microstructure

Exercises after L3

Egor Starkov

Københavns Universitet
Spring 2025

This lecture:

1 Problems from Lecture 2

Lecture 2

From Lecture 2:

- problem 8 from FRP ch.2 (pp.72-76)
- reproduce graphs from lecture ($\approx$ problems 6-7 from FPR ch.2) – we did that in lecture

Problem 8: Implementation Shortfall

Your client wants to buy q shares in company XYZ at time 0 and hold them until time t . Midprice at time 0 is m_0 . Market is not perfectly liquid, so realized [average] purchase price will be $\bar{p} = m_0 + \lambda k q$, where $k \in [0, 1]$ is the share of the order fulfilled, λ price pressure parameter. Expected midprice at time t is m_t (independent of k).

Choose k to minimize the resulting implementation shortfall.

Problem 8: Implementation Shortfall

Implementation shortfall:

$$\begin{aligned}IS_t &= q(m_t - m_0) - kq(m_t - \bar{p}) \\&= kq(\bar{p} - m_0) + (1 - k)q(m_t - m_0) \\&= \lambda(kq)^2 + (1 - k)q(m_t - m_0).\end{aligned}$$

Problem 8: Implementation Shortfall

Implementation shortfall:

$$\begin{aligned}IS_t &= q(m_t - m_0) - kq(m_t - \bar{p}) \\&= kq(\bar{p} - m_0) + (1 - k)q(m_t - m_0) \\&= \lambda(kq)^2 + (1 - k)q(m_t - m_0).\end{aligned}$$

Find min:

$$\begin{aligned}\frac{dIS_t(k)}{dk} &= 2\lambda q^2 k - q(m_t - m_0) = 0 \\ \Leftrightarrow k &= \frac{m_t - m_0}{2\lambda q}\end{aligned}$$

(don't forget to check the second order condition: $\frac{d^2 IS_t(k)}{dk^2} > 0$)

Problem 8: Implementation Shortfall

$$k = \frac{m_t - m_0}{2\lambda q}$$

- Increasing in $m_t - m_0$:
 - if asset price is expected to increase, the opportunity cost (regret) of not buying the asset is high.
- Decreasing in λ :
 - the more sensitive is the price, the less you can buy cheaply enough
- Decreasing in q :
 - (same as above) large orders move prices by more – costlier to fulfill fully.

Problem 8: Implementation Shortfall

- If you were solving this problem in reality, you would not know m_t .
- Instead, treat m_t as your **expectation** of the future price, choose trading strategy to minimize the **expected implementation shortfall**.

This lecture:

1 Glosen-Milgrom model

- ex 3
- GM with Uniform value

- FPR chapter 3 exercises (p. 124-125):
 - exercise 3 about the GM model where speculators are not perfectly informed, but instead receive a signal about the value of the asset
- Bonus problem: GM model with uniform v

Exercise 3

- $v \in \{v^H, v^L\}$ w.p. $\frac{1}{2}$;
- dealer competitive, risk-neutral, does not know v ;
- trader uninformed w.p. $1 - \pi$, then buys and sells w.p. $\frac{1}{2}$;
- trader informed w.p. π , then observes **signal about state**
 - signal accurate w.p. $\rho \in (\frac{1}{2}, 1]$.

Exercise 3.a

(a) Write dealer's expected profits upon receiving buy/sell order, assuming informed trader follows his signal.

$$\mathbb{E}[\Pi_D | Buy] =$$

$$\mathbb{E}[\Pi_D | Sell] =$$

Exercise 3.a

(a) Write dealer's expected profits upon receiving buy/sell order, assuming informed trader follows his signal.

$$\begin{aligned}\mathbb{E}[\Pi_D | Buy] &= \frac{\pi \frac{1}{2} \rho (a - v^H) + \pi \frac{1}{2} (1 - \rho) (a - v^L) + (1 - \pi) \frac{1}{2} (a - \mu)}{\pi \frac{1}{2} \rho + \pi \frac{1}{2} (1 - \rho) + (1 - \pi) \frac{1}{2}} \\ &= \pi \rho (a - v^H) + \pi (1 - \rho) (a - v^L) + (1 - \pi) (a - \mu)\end{aligned}$$

$$\begin{aligned}\mathbb{E}[\Pi_D | Sell] &= \frac{\pi \frac{1}{2} \rho (v^L - b) + \pi \frac{1}{2} (1 - \rho) (v^H - b) + (1 - \pi) \frac{1}{2} (\mu - b)}{\pi \frac{1}{2} \rho + \pi \frac{1}{2} (1 - \rho) + (1 - \pi) \frac{1}{2}} \\ &= \pi \rho (v^L - b) + \pi (1 - \rho) (v^H - b) + (1 - \pi) (\mu - b)\end{aligned}$$

where $\mu = \frac{v^H + v^L}{2}$.

Exercise 3.b

(b) Compute the bid and ask prices set by a risk-neutral competitive dealer.

$$a =$$

$$b =$$

Exercise 3.b

(b) Compute the bid and ask prices set by a risk-neutral competitive dealer.

$$a = \pi \rho v^H + \pi(1 - \rho)v^L + (1 - \pi)\mu$$

$$b = \pi \rho v^L + \pi(1 - \rho)v^H + (1 - \pi)\mu$$

from $\mathbb{E}[\Pi_D|Buy] = \mathbb{E}[\Pi_D|Sell] = 0$.

Exercise 3.c

(c) Derive bid-ask spread as a function of signal's informativeness ρ . When is the market more or less liquid? Why?

$$S = a - b$$

Exercise 3.c

(c) Derive bid-ask spread as a function of signal's informativeness ρ . When is the market more or less liquid? Why?

$$\begin{aligned} S &= a - b \\ &= \pi(2\rho - 1)(v^H - v^L) \end{aligned}$$

Higher $\rho \Leftrightarrow$ larger $S \Leftrightarrow$ less liquid market because of larger adverse selection costs (same as larger π).

Exercise 3.d

(d) Verify that the speculator's strategy (buy after signal H , sell after signal L) is optimal.

Consider signal H :

$$\mathbb{E}[\Pi_I | H, Buy] =$$

$$\mathbb{E}[\Pi_I | H, Sell] =$$

Exercise 3.d

(d) Verify that the speculator's strategy (buy after signal H , sell after signal L) is optimal.

Consider signal H :

$$\begin{aligned}\mathbb{E}[\Pi_I|H, Buy] &= \rho(v^H - a) + (1 - \rho)(v^L - a) \\ &= (\rho v^H + (1 - \rho)v^L - \mu)(1 - \pi) > 0\end{aligned}$$

$$\begin{aligned}\mathbb{E}[\Pi_I|H, Sell] &= \rho(b - v^H) + (1 - \rho)(b - v^L) \\ &= (1 - \pi) [\mu - (\rho v^H + (1 - \rho)v^L)] - \pi(2\rho - 1)(v^H - v^L) < 0\end{aligned}$$

Same for signal L .

GM with Uniform value

- **Uniform outcome:** Suppose v is uniformly distributed on $[0,1]$
- **Prior value:** Prior density $f(v) = 1$, and $\mu = \mathbb{E}[v] = \int_0^1 vf(v)dv = 1/2$
- Look for an ask price $a < 1$:
 - Speculator buys if $v > a$ and sells if $v < a$ ($v = a$ has zero prob.). Thus:

$$\mathbb{P}(Buy|v) = \begin{cases} (1 - \pi)\beta_B + \pi & \text{if } v > a; \\ (1 - \pi)\beta_B & \text{if } v < a. \end{cases}$$

- Recall Bayes' Rule: $f(v|Buy) = \frac{f(v)\mathbb{P}(Buy|v)}{\mathbb{P}(Buy)}$. Then,

$$f(v|Buy) = \begin{cases} \frac{(1 - \pi)\beta_B + \pi}{(1 - \pi)\beta_B + (1 - a)\pi} & \text{if } v > a; \\ \frac{(1 - \pi)\beta_B}{(1 - \pi)\beta_B + (1 - a)\pi} & \text{if } v < a. \end{cases}$$

GM with Uniform value (2)

Now we explicitly have a on both sides of $a = \mathbb{E}[v|Buy]$. Must solve:

$$\begin{aligned} a &= \mathbb{E}[v|Buy] \\ &= \int_0^a \frac{(1-\pi)\beta_B \cdot v}{(1-\pi)\beta_B + (1-a)\pi} dv + \int_a^1 \frac{[(1-\pi)\beta_B + \pi] \cdot v}{(1-\pi)\beta_B + (1-a)\pi} dv \\ &= \frac{(1-\pi)\beta_B \cdot a^2/2}{(1-\pi)\beta_B + (1-a)\pi} + \frac{[(1-\pi)\beta_B + \pi] \cdot (1-a^2)/2}{(1-\pi)\beta_B + (1-a)\pi} \\ &= \frac{(1-\pi)\beta_B + \pi - \pi a^2}{2(1-\pi)\beta_B + 2(1-a)\pi} = \frac{1}{2} + \frac{\pi a(1-a)}{2(1-\pi)\beta_B + 2(1-a)\pi} \end{aligned}$$

This is a quadratic equation in a . For $\pi = \beta_B = 1/2$, solution is $a = \frac{3}{2} \pm \frac{\sqrt{3}}{2}$. Since we assumed $a < 1$, equilibrium in this case is $a = \frac{3}{2} - \frac{\sqrt{3}}{2}$.

Lecture 9

- solve exercise 3 in chapter 4 (p.159) which explores competition among speculators

Exercise 4.3

- Consider a Kyle model with many speculators $i \in \{1, 2, \dots, N\}$.
- Every speculator uses a linear strategy

$$x_i = \beta(v - \mu)$$

- Everything else is same as in lecture

Exercise 4.3.a

(a) Find the equilibrium aggressiveness β , determine how it depends on N and explain why.

Speculator i maximizes the expected profit $\mathbb{E}[x_i(v - p)]$.

The clearing price is given by the zero-profit condition:

$$p = \mu + \lambda[x_i + (N - 1)\beta(v - \mu) + u]$$

Plugging it into the expected profit expression and maximizing it w.r.t. x_i yields

$$x_i = \underbrace{\frac{1 - \lambda\beta(N - 1)}{2\lambda}}_{=\beta}(v - \mu)$$
$$\Rightarrow \beta = \frac{1}{\lambda(N + 1)}$$

Exercise 4.3.a

$$\beta = \frac{1}{\lambda(N+1)}$$

- β decreasing in N : the more traders, the smaller is one trader's share (as in Cournot oligopoly)
- $N\beta$ increasing in N : total order size increases in N because with high N , each trader suffers less from higher price (due to lower output)

Exercise 4.3.b

(b) Derive the price impact coefficient λ from the dealer's zero-profit condition

$$\lambda = \frac{\mathbb{C}(v, q)}{\mathbb{V}(q)} = \frac{N\beta\sigma_v^2}{(N\beta)^2\sigma_v^2 + \sigma_u^2}$$

Combining with the expression for β , we get

$$\lambda = \frac{\sqrt{N}}{N+1} \frac{\sigma_v}{\sigma_u} \qquad \beta = \frac{1}{\sqrt{N}} \frac{\sigma_u}{\sigma_v}$$

Exercise 4.3.c

(c) What is the market depth in equilibrium, and how is it affected by an increase in the number of informed traders, N ? What is the economic intuition for this result? Do you think this result is robust?

- Depth is $\frac{1}{\lambda} = \frac{N+1}{\sqrt{N}} \frac{\sigma_u^2}{\sigma_v^2}$
- It is increasing in N because traders become more aggressive (relative to their market share) in the presence of competition – submit larger orders $|x_i|$ given any fixed v , – price reacts less to any fixed order size.
- The conclusion is not 100% robust...

Exercise 4.3.d

(d) Compute the ex-ante expected profit of each informed investor. What is the effect of an increase in N on the aggregate profit of informed investors?

$$\mathbb{E}[x_i(v - p)] = \frac{\sigma_u \sigma_v}{(N + 1)\sqrt{N}}$$

- predictably, the profits of each speculator decrease in N
- aggregate profit of all speculators $\frac{\sigma_u \sigma_v \sqrt{N}}{N+1}$ also decreases in N (because total aggressiveness $N\beta$ increases away from the monopolistic optimum)

Exercise 6.1

This exercise considers a two-period model of an LOB.

- **Security:** Value $v \sim g(v) = \frac{1}{2\sigma} e^{-\frac{|v-\mu|}{\sigma}}$
 - Useful fact: $\mathbb{E}[v|v \geq z] = z + \sigma$.
- **Trader:**
 - Type *I*. With prob. π_0 , knows v and maximizes exp. utility
 - Type *N*. With prob. $1 - \pi_0$, buys/sells (with equal prob.) x_N shares, with $x_N \sim f(x_N) = \frac{1}{2}\theta e^{-\theta|x_N|}$
- **Tick:** Tick size is zero, prices are continuous

Exercise 6.1.a

(a) Let $q(A)$ be the cumulative depth up to ask price A in the book and A^* be the lowest ask price in the LOB. Show that when $v \geq A^*$, the optimal strategy $x^*(v)$ of the informed trader is to buy $x^*(v) = q(v)$ shares.

Exercise 6.1.a

(a) Let $q(A)$ be the cumulative depth up to ask price A in the book and A^* be the lowest ask price in the LOB. Show that when $v \geq A^*$, the optimal strategy $x^*(v)$ of the informed trader is to buy $x^*(v) = q(v)$ shares.

- Let $p(q) \equiv \{A : x = q(A)\}$ be the marginal price schedule.

Note that it's just the inverse of $q(A)$: $p(q(A)) = A$ and $q(p(q')) = q'$.

- The trader's profit from buying x units is $\int_0^x (v - p(q)) dq$.
- Take the FOC wrt. x

$$v - p(x^*) = 0 \Leftrightarrow p(x^*) = v.$$

- Denote the optimal strategy by $x^*(v)$. Then: $q(p(x^*(v))) = x^*(v) = q(v)$.

Exercise 6.1.b

(b) Using part (a) and the zero-profit condition show that in eqm:

$$q(A) = \frac{1}{\theta} \left[\ln \left(\frac{1-\pi_0}{\pi_0} \right) + \ln \left(\frac{A-\mu}{\sigma} \right) + \frac{A-\mu}{\sigma} \right] \text{ if } A > A^*.$$

Exercise 6.1.b

(b) Using part (a) and the zero-profit condition show that in eqm:

$$q(A) = \frac{1}{\theta} \left[\ln \left(\frac{1-\pi_0}{\pi_0} \right) + \ln \left(\frac{A-\mu}{\sigma} \right) + \frac{A-\mu}{\sigma} \right] \text{ if } A > A^*.$$

Zero-profit condition is $p(q) = \mathbb{E}[v|x \geq q]$ (x is the incoming mkt order), can be expanded into:

$$p(q) = \pi(q)\mathbb{E}[v|v \geq p(q)] + (1 - \pi(q))\mathbb{E}[v],$$

where the first term incorporates the $v = p(x^*(v))$ shown in (a), and

$$\begin{aligned} \pi(q) &\equiv \mathbb{P}(I|x \geq q) = \frac{\mathbb{P}(I)\mathbb{P}(x \geq q|I)}{\mathbb{P}(x \geq q)} \\ &= \frac{\pi_0 \mathbb{P}(v \geq p(q))}{\pi_0 \mathbb{P}(v \geq p(q)) + (1 - \pi_0) \mathbb{P}(x_N \geq q)}. \end{aligned}$$

Exercise 6.1.b (2)

- Compute the ingredients of $\pi(q)$:

$$\mathbb{P}(v \geq p(q)) = \int_{p(q)}^{\infty} g(v) dv = \frac{1}{2} e^{-\frac{p(q)-\mu}{\sigma}}$$
$$\mathbb{P}(x_N \geq q) = \int_q^{\infty} f(x_N) dx_N = \frac{1}{2} e^{-\theta q}$$

- Plug them back into $\pi(q)$:

$$\begin{aligned}\pi(q) &= \frac{\pi_0 \frac{1}{2} e^{-\frac{p(q)-\mu}{\sigma}}}{\pi_0 \frac{1}{2} e^{-\frac{p(q)-\mu}{\sigma}} + (1 - \pi_0) \frac{1}{2} e^{-\theta q}} \\ &= \frac{e^{\theta q - \frac{p(q)-\mu}{\sigma}}}{e^{\theta q - \frac{p(q)-\mu}{\sigma}} + \frac{1 - \pi_0}{\pi_0}}\end{aligned}$$

Exercise 6.1.b (3)

- Going back to the marginal price, we had

$$p(q) = \pi(q)\mathbb{E}[v|v \geq p(q)] + (1 - \pi(q))\mathbb{E}[v]$$

- Recall the hint: $\mathbb{E}[v|v \geq p(q)] = p(q) + \sigma$. Plug this and $\pi(q)$ into the expression above:

$$\begin{aligned} p(q) &= \pi(q)(p(q) + \sigma) + (1 - \pi(q))\mu \\ \Leftrightarrow \frac{1 - \pi_0}{\pi_0} \cdot \frac{p(q) - \mu}{\sigma} &= e^{\theta q - \frac{p(q) - \mu}{\sigma}} \\ \Leftrightarrow \ln\left(\frac{1 - \pi_0}{\pi_0}\right) + \ln\left(\frac{p(q) - \mu}{\sigma}\right) &= \theta q - \frac{p(q) - \mu}{\sigma} \end{aligned}$$

Finally, solve for q (recall $p(q(A)) = A$) to get the result.

Exercise 6.1.c

(c) Show that the book becomes thinner on the ask side when (i) π_0 increases or (ii) σ increases. What is the economic intuition for this result.

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(c) Show that the book becomes thinner on the ask side when (i) π_0 increases or (ii) σ increases. What is the economic intuition for this result.

$$q(A) = \frac{1}{\theta} \left[\ln \left(\frac{1 - \pi_0}{\pi_0} \right) + \ln \left(\frac{A - \mu}{\sigma} \right) + \frac{A - \mu}{\sigma} \right] \text{ if } A > A^*.$$

■ **Share of informed traders.**

$$\frac{\partial q(A)}{\partial \pi_0} = \frac{1}{\theta} \left[-\frac{\pi_0}{1 - \pi_0} \cdot \frac{1}{\pi_0^2} \right] = \frac{1}{\theta} \left[-\frac{1}{\pi_0(1 - \pi_0)} \right] < 0$$

More informed trading implies greater risk when posting orders

Exercise 6.1.c

(c) Show that the book becomes thinner on the ask side when (i) π_0 increases or (ii) σ increases. What is the economic intuition for this result.

$$q(A) = \frac{1}{\theta} \left[\ln \left(\frac{1 - \pi_0}{\pi_0} \right) + \ln \left(\frac{A - \mu}{\sigma} \right) + \frac{A - \mu}{\sigma} \right] \text{ if } A > A^*.$$

■ **Value uncertainty.**

$$\frac{\partial q(A)}{\partial \sigma} = \frac{1}{\theta} \left[-\frac{\sigma}{A - \mu} \frac{A - \mu}{\sigma^2} - \frac{A - \mu}{\sigma^2} \right] = \frac{1}{\theta} \left[-\frac{1}{\sigma} - \frac{A - \mu}{\sigma^2} \right] < 0$$

More value uncertainty implies greater risk when posting orders

Make/take fees and bid-ask spreads.

Consider the Parlour model. Trading platforms often charge different fees for market and limit orders.

- Let f_{mo} be the fee per share paid by a market order placer and
- f_{lo} the fee per share for a limit order placer when the limit order executes (there is no entry fee for limit orders).
- Finally let f be the total fee earned by the platform on each trade, $f = f_{mo} + f_{lo}$.

Foucault model: lecture ver

- **Exogenous prices.** Bid and ask prices exogenously given as $A > v > B$
- **Traders.** Arriving trader chooses btw limit or market order (one unit)
 - Limit order valid one period. Choice depends on prob. of limit order being executed, i.e. 'hit' by a market order from the next trader
 - Valuation: $v + y$. y is uniformly distributed on $(-Y, Y)$, unobserved and independent across traders. v is known and common to all.
- **Profits.** Let $P_M^B(P_M^S)$ be prob. of next-period market buy (sell) order

Foucault/(Parlour) model: lecture vs book ver

- Model in lecture differs slightly from model in the book (6.4.1-2):
 - we made A and B exogenous, the book derives them;
 - we let $y_i \sim U[-L, L]$, the book assumes $y_i \in \{-L, L\}$;
 - in 6.4.1 the textbook initially sets up a much more general model, but never actually solves it;
 - in lecture, we implicitly assumed that if LOB is empty on either side, it is filled by MM at same prices. Don't really need to
- For the problem today we stick to the textbook version (6.4.2)

Ch 6, ex 5, part a

(a) Compute bid and ask quotes in equilibrium

How do?

(a) Compute bid and ask quotes in equilibrium

How should traders behave in equilibrium?

- If $y_i = L$ then buy
 - indifferent between market buy (if available) and limit buy
- If $y_i = -L$ then sell
 - indifferent between market sell (if available) and limit sell

Ch 6, ex 5, part a

Consider $y_i = L$.

- Profit from market buy is $v + L - A - f_{mo}$.
- Profit from limit buy is $(v + L - B - f_{lo})P_M^S$.
- Indifference $\Rightarrow$ the two are equal. This gives a condition on A, B given $v, L, f_{mo}, f_{lo}, P_M^S$.
- But P_M^S is uncertain – even if next trader has $y_i = -L$, how does he choose between MS and LS?
 - In equilibrium: if $t + 1$ -trader *can* trade with t -trader, then will always choose so.
 - Idea: limit trader at t can set a price that is ϵ -better for $t + 1$ than submitting a limit order. So anything different from the above cannot be an equilibrium.

Ch 6, ex 5, part a

So indifference condition is:

$$v + L - A - f_{mo} = (v + L - B - f_{lo})1/2$$

Same for trader with $y_i = -L$:

$$B - (v - L) - f_{mo} = (A - (v - L) - f_{lo})1/2$$

Solve the two for A, B to get:

$$A = v + \frac{1}{3}(L + f_{lo} - 2f_{mo})$$

$$B = v - \frac{1}{3}(L + f_{lo} - 2f_{mo})$$

(b) Show that the bid-ask spread decreases in f_{mo} and increases in f_{lo} . Explain.

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$$S = \frac{2}{3}(L + f_{lo} - 2f_{mo})$$

- if limit orders expensive then the price improvement from LO compared to MO (=spread) must be large to offset this cost, make LO competitive with MO
- vice versa for f_{mo}

(c) Trading platforms often subsidize traders who submit limit orders. That is, they set $f_{lo} < 0$ and $f_{mo} > 0$, maintaining that this practice ultimately helps to narrow the spread and benefits traders submitting market orders. Holding the total trading fee fixed, is this argument correct?

(c) Trading platforms often subsidize traders who submit limit orders. That is, they set $f_{lo} < 0$ and $f_{mo} > 0$, maintaining that this practice ultimately helps to narrow the spread and benefits traders submitting market orders. Holding the total trading fee fixed, is this argument correct?

This **does** narrow down the nominal spread, but does **NOT** benefit market traders.

Consider a MB order. Trader pays

$$A + f_{mo} = v + \frac{1}{3}(L + f)$$

which only depends on total f and not on how it is split between f_{lo} and f_{mo} .

Payments for order flow.

- One risky security, pays either $v^H = \mu + \sigma$ or $v^L = \mu - \sigma$ with equal probabilities at $t = 1$.
- At $t = 0$ investor gives his broker an order to buy or sell one unit.
 - W.p. ϕ retail investor is retail investor, buys and sells randomly 50/50.
 - W.p. $1 - \phi$ institutional investor, informed w.p. α . If uninformed, buys and sells randomly 50/50.
- Three risk-neutral dealers 1, 2, and 3 post quotes before the broker contacts them.
 - Broker cannot split his order among dealers
 - Dealers have no private information on the payoff of the security

Ch 7, ex 3, part a

(a) Assume that there is no payment for order flow between the broker and the three dealers. In this case, the broker randomly selects one dealer among those posting the best price for his order. Compute the bid and ask quotes posted by the dealers.

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If any single dealer posts quotes at a profit, the other two can undercut and attract order flow $\Rightarrow$ we are down to standard Glosten-Milgrom with competitive dealers.

$$\begin{aligned}a &= \mathbb{E}[v|\text{buy}] = \mu + \mathbb{P}(\text{informed}|\text{buy})\sigma \\&= \mu + \frac{(1 - \phi)\alpha/2}{(1 - \phi)\alpha/2 + [\phi + (1 - \phi)(1 - \alpha)]/2}\sigma \\&= \mu + (1 - \phi)\alpha\sigma \\b &= \mu - (1 - \phi)\alpha\sigma\end{aligned}$$

Ch 7, ex 3, part b

(b) Assume now that dealer 1 has a payment for order flow arrangement under which the broker gives dealer 1 all orders from retail investors and the dealer commits to execute all these orders at the best quotes (i.e., the ask and bid price set by the remaining dealers, 2 and 3). Other orders are sent to dealer 2 or 3 as in question a. What are the quotes posted by dealers 2 and 3? Deduce that the bid-ask spread is higher in this case than where there is no payment for the order flow.

Ch 7, ex 3, part b

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Dealers 2 and 3 are still competing with each other $\Rightarrow$ post zero-profit quotes

$$\begin{aligned}a &= \mu + \mathbb{P}(\text{informed}|\text{buy,institutional})\sigma \\&= \mu + \frac{(1-\phi)\alpha/2}{(1-\phi)\alpha/2 + (1-\phi)(1-\alpha)/2}\sigma \\&= \mu + \alpha\sigma > \mu + (1-\phi)\alpha\sigma \\b &= \mu - \alpha\sigma < \mu - (1-\phi)\alpha\sigma\end{aligned}$$

Ch 7, ex 3, part c

(c) Let P be the payment of dealer 1 to the broker. What is the largest possible value of P ?

(c) Let P be the payment of dealer 1 to the broker. What is the largest possible value of P ?

- Dealer 1's per-order profit is:
 - $a - \mu = \alpha\sigma$ from buy orders
 - $\mu - b = \alpha\sigma$ from sell orders
- so dealer 1 would be willing to pay up to $\alpha\sigma$ per order.

(d) Is payment for order flow beneficial or detrimental to investors?

(d) Is payment for order flow beneficial or detrimental to investors?

- All investors trade at worse prices under order flow payments – bad.
- Dealer 1 and broker profit from order flow payments.
- If many brokers compete with each other, they may transmit part of the profit to (some?) investors.

Price discovery and transparency. Consider the model of [post-trade transparency](#) described in section 8.2. Consider the time-averaged expected squared deviation between the transaction price and the true value of the security, that is

$$\frac{E[(p_1^k - v)^2]}{2} + \frac{E[(p_2^k - v)^2]}{2},$$

where p_t^k is the transaction price in period $t = 1, 2$ in regime $k = T, O$ (transparent, opaque). Show that price discovery is more efficient in the transparent market. You may limit your analysis to the case $\pi > \frac{1}{2}$ in which the equilibrium first-period spread is positive.

Post-trade transparency

- If orders arrive sequentially, what effect does information about **past orders** have?
- **Value:** high v^H or low v^L with equal probability
 - Mean: $\mu = (v^H + v^L)/2$
 - **Denote** $\sigma = (v^H - v^L)/2$
- **Dealers:** set quotes, competitive, risk neutral
- **Traders:** two traders arrive, submit unit market orders
 - With prob. π : both are informed
 - With prob. $(1 - \pi)/2$: both liquidity traders; first seller, then buyer
 - With prob. $(1 - \pi)/2$: both liquidity traders; first buyer, then seller
- **Transparent market:** All dealers observe the first order y_1
 - Set $a_1 = \mu + \pi(v^H - \mu)$ and $a_{2y_1} = \mathbb{E}[v|y_1, buy]$

Post-trade transparency: Period 2

Opaque market: First dealer gains informational advantage. Focus on **ask side**

- **Period 2.** Denote the dealer who observed period-1 trade by I , and the other dealer by U .
 - For technical reasons, suppose I sets price after observing U 's quote
 - **Dealer U :** How to quote if you didn't see the first trade and second trade is buy?
 - If you set ask $a_2^U < v^H$ you will be undercut by I if first order was a sell
 - You only get to trade if first order was buy: lose $v^H - a_2^U$
 - Thus, uninformed dealers need to quote $a_2^U = v^H$
 - **Dealer I :** Suppose you saw the first trade, and second trade is a buy:
 - Set price at $a_{2s}^I = v^H$ if first trade was a sell, and $a_{2b}^I = v^H - \epsilon$ if buy
 - I wins period-2 buy order if y_1 was a sell (otherwise can undercut U)
 - U wins period-2 buy order if y_1 was a buy, since I knows that asset value is high

Post-trade transparency: Period 1

- **Period 1.** The sequential information advantage uncovered in the previous slide can make dealers bid keenly for the first order (Forex dealers often said to quote negative spread to large traders)
 - In second period, I 's profit is $(1 - \pi)(v^H - v^L)/2$. U 's profit is zero
 - Competition leads the first period half-spread to be reduced by this amount, to $(2\pi - 1)(v^H - v^L)/2$ (dealers undercut each other to obtain information contained in first order)
 - The uninformed's aggregate trading cost is $\pi(v^H - v^L)$ - double the cost under transparency. Why is this?
- Would dealers commit to transparency?
 - No, there is always an incentive to hide your orders (section 8.4.2)
 - May explain the rise of less transparent trading venues

- Transparency, $t = 1$:

$$a_1^T = \mu + \pi\sigma$$

$$b_1^T = \mu - \pi\sigma$$

$$\begin{aligned} E[(p_1^T - v)^2] &= \frac{1}{2} \left[\pi(a_1^T - v^H)^2 + (1 - \pi)\frac{1}{2}(a_1^T - v^H)^2 + (1 - \pi)\frac{1}{2}(b_1^T - v^H)^2 \right] + \\ &\quad + \frac{1}{2} \left[\pi(b_1^T - v^L)^2 + (1 - \pi)\frac{1}{2}(b_1^T - v^L)^2 + (1 - \pi)\frac{1}{2}(a_1^T - v^L)^2 \right] \\ &= (1 - \pi^2)\sigma^2 \end{aligned}$$

- $t = 2$:

- $p_2 = v$ if informed at $t = 1$,
- $p_2 = \mu$ if uninformed at $t = 1$;

$$\Rightarrow E[(p_2^T - v)^2] = \pi \cdot 0 + (1 - \pi)\sigma^2$$

- Averaging over time:

$$\left[\frac{1}{2}(1 - \pi^2) + \frac{1}{2}(1 - \pi) \right] \sigma^2 = (1 - \pi) \left(1 + \frac{\pi}{2} \right) \sigma^2$$

- **Opaqueness**, $t = 1$ (assuming $\pi > 1/2$ to avoid crossed quotes):

$$a_1^O = \mu + (2\pi - 1)\sigma$$

$$b_1^O = \mu - (2\pi - 1)\sigma$$

$$E[(p_1^O - v)^2] = 2(1 - \pi)\sigma^2$$

- **Opaqueness**, $t = 1$ (assuming $\pi > 1/2$ to avoid crossed quotes):

$$a_1^O = \mu + (2\pi - 1)\sigma$$

$$b_1^O = \mu - (2\pi - 1)\sigma$$

$$E[(p_1^O - v)^2] = 2(1 - \pi)\sigma^2$$

- $t = 2$:

$$E[(p_2^O - v)^2] = 2(1 - \pi)\sigma^2$$

- so the average is also $2(1 - \pi)\sigma^2$

Comparison:

$$\begin{aligned}\frac{E[(p_1^T - v)^2]}{2} + \frac{E[(p_2^T - v)^2]}{2} &< \frac{E[(p_1^O - v)^2]}{2} + \frac{E[(p_2^O - v)^2]}{2} \\ (1 - \pi) \left(1 + \frac{\pi}{2}\right) \sigma^2 &< 2(1 - \pi) \sigma^2 \\ 1 + \frac{\pi}{2} &< 2\end{aligned}$$

Transparency yields better price discovery

Liquidity premium in the presence of dividend income. Consider a stock with a dividend yield d per period and with fundamental value μ_t at date t (equal to its midprice m_t at that date). Investors hold the stock for one period and can trade it at a constant percentage bid-ask spread s in each period. Their required rate of return on the stock is given, equal to r .

Ch 9, ex 1, part a

(a) Define the gross-of-transaction-cost return $1 + R$ in terms of μ_t , μ_{t+1} , and d .

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$$1 + R = \frac{\mu_{t+1}}{\mu_t} + d$$

Note that “gross-of-transaction-cost return” interpretation of R differs from how we interpreted R in lectures (which was “rate of price growth”).

Ch 9, ex 1, part b

(b) Determine the equilibrium gross return $1 + R$ as a function of r , s , and d alone.

Ch 9, ex 1, part b

(b) Determine the equilibrium gross return $1 + R$ as a function of r , s , and d alone.

- Asset bought at t at price $a_t = (1 + s/2)\mu_t$,
- sold at $t + 1$ at $b_{t+1} = (1 - s/2)\mu_{t+1}$,
- yields d in the interim.
- Required return is $1 + r = \frac{b_{t+1} + d\mu_t}{a_t}$.
- Plugging in and rearranging:

$$1 + R = \frac{1}{1 - \frac{s}{2}} \left[(1 + r) \left(1 + \frac{s}{2} \right) - \frac{s}{2} d \right]$$

(c) How does the liquidity premium respond to an increase in the dividend yield d ? What is the intuitive reason for this result?

(c) How does the liquidity premium respond to an increase in the dividend yield d ? What is the intuitive reason for this result?

- Liquidity premium is

$$R - r = \left(1 + r - \frac{d}{2}\right) \frac{s}{1 - \frac{s}{2}}$$

- Decreasing in d

- dividends do not suffer from stock illiquidity
- if larger share of the return is generated by the dividend, investors lose less from illiquidity and require smaller liquidity premium.